

**MATH 101-01, 03 / Major Exam#2/ Time allowed=2 hours**

1. [5 marks]

Assume that  $f$  has the property  $f(x + w) = f(x)f(w)$ , for all values of  $x$  and  $w$ , and that  $f(0) = 1$ ,  $f'(0) = 2$ . By using the definition of the derivative, show that  $f'(x) = 2f(x)$ . Give an example of such a function.

2. [5 marks]

Find  $f'(x)$  if a)  $f(x) = x^{\tan(x^2)}$ , b)  $f(x) = \log_{\log_x(e)}(e)$ .

3. [5 marks]

Find all points  $(x, y)$  on the curve  $x^6 + y^6 = 65$  at which the tangent lines are parallel to the line  $y = 32x$ .

4. [5 marks]

Sand is pouring from a chute forms a conical pile whose height  $h$  is always equal twice its diameter  $D$ . If the sand is pouring at a constant rate of  $6.28 \text{ ft}^3/\text{min}$ , at what rate is the height increasing when the pile is  $4 \text{ ft}$  high? (Take  $\pi \approx 3.14$ ).

5. [5 marks]

Use the local linear approximation to estimate  $\ln(1.007)$  and  $(1.003)^{-1.003}$ .

6. [5 marks]

Find  $\lim_{x \rightarrow 1} (2 - x)^{\frac{1}{x-1}}$  and  $\lim_{x \rightarrow 0} \frac{5^x - 1}{x}$ .

7. [5 marks]

a) Find  $f^{-1}(x)$  if  $f(x) = x^3 - 6x^2 + 12x - 8$ .

b) Show that  $g(x) = x^3 - x^2 + 1$  is not one-to-one on  $(-\infty, +\infty)$ . Then find an interval on which its restriction is one-to-one.

8. [5 marks]

a) Find the exact value of  $\log_5(49) \cdot \log_7(125)$ .

b) Solve for  $x$  the equation  $\log_6(x^3) + \log_6(x^4) = 7$ .