King Fahd University of Petroleum & Minerals

Department of Mathematical Sciences

Semester II (042) (2004-05)

MATH 102-15 FINAL EXAM

CODE 002

Instructor: Dr. A. Shawky Ibrahim **Duration: 150 Minutes**

Name: ______________________________	**Serial:** ______________
ID #: ______________________________	**Section:** ______________

Calculators Are Not Allowed in this Exam

This Exam consists of Two Parts:

	Marks
Part I (48 points) **Multiple Choice Problems**	

Part II (24-points)
Written Problems

			Marks
1	______________________________		**9**
2	**(a)** ______________________________		**5**
	(b) ______________________________		**3**
	(c) ______________________________		**3**
	(d) ______________________________		**4**
	24		**24**

Total ______________
72

Part (I) (48 points)

1. Which one of the following series is absolutely convergent?

(a) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k}}$

(b) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k k}{k+1}$

(c) $\displaystyle\sum_{k=1}^{\infty} \cos(\pi k)$

(d) $\displaystyle\sum_{k=1}^{\infty} (-1)^k \pi^k$

(e) $\displaystyle\sum_{k=1}^{\infty} (-1)^k e^{-k}$

2. The value of $\displaystyle\lim_{\max \Delta x_k \to 0} \sum_{k=1}^{n} (\tan x_k^*)^2 (\sec x_k^*)^4 \Delta x_k$ over the interval $\left[0, \frac{\pi}{4}\right]$ is equal to

(a) $\frac{2}{15}$

(b) $\frac{11}{15}$

(c) $\frac{8}{15}$

(d) $\frac{13}{15}$

(e) $\frac{14}{15}$

3. The radius of convergence R and the interval of convergence I of the power series $\sum_{k=0}^{\infty}(-1)^k\frac{(x-5)^k}{3^k}$ are

(a) $R=\frac{5}{3}$ and $I=(2,8)$

(b) $R=3$ and $I=(2,8)$

(c) $R=3$ and $I=(2,8]$

(d) $R=\frac{5}{3}$ and $I=[2,8)$

(e) $R=3$ and $I=[2,8)$

4. If the area enclosed by the curves $y^2=x$ and $y=x-2$ is revolved about the line $x=4$, then the volume of the solid generated is equal to

(a) $\pi\int_0^4[(4-x)^2-(2-x)^2]dx$

(b) $\pi\int_{-1}^2[(2+y-y^2)^2dy$

(c) $2\pi\int_1^4(4-x)(\sqrt{x}-x+2)dx$

(d) $2\pi\int_1^4(4-x)(2-\sqrt{x}+x)dx$

(e) $\pi\int_{-1}^2[(4-y^2)^2-(2-y)^2]dy$

5. The sequence $\frac{\ln 3}{\ln 5}, \frac{\ln 6}{\ln 7}, \frac{\ln 9}{\ln 9}, \frac{\ln 12}{\ln 11}, \ldots$

(a) converges to 3/2

(b) converges to 1

(c) converges to 2/3

(d) diverges

(e) oscillates between 2 and 3

6. If the region enclosed by $y = \sin x$ and the x-axis from $x = 0$ to $x = \pi$ is revolved about the vertical line $x = -1$, then the volume of the solid generated is equal to

(a) $2\pi(\pi - 4)$

(b) $2\pi^2$

(c) $2\pi(\pi - 1)$

(d) $2\pi(\pi + 2)$

(e) $2\pi(\pi + 4)$

7. The sequence $\left\{\tan^{-1} n\right\}_{n=1}^{+\infty}$ is

(a) increasing (not strictly) with an upper bound

(b) strictly increasing **without** an upper bound

(c) increasing (not strictly) **without** an upper bound

(d) strictly increasing with an upper bound

(e) neither increasing nor decreasing

8. If the portion of the curve $y = 2\sqrt{x}$ between $x = 0$ and $x = 8$ is revolved about the x-axis, then the area of the resulting surface is equal to

(a) 104π

(b) $\dfrac{208\pi}{3}$

(c) $\dfrac{215\pi}{3}$

(d) $\dfrac{103\pi}{3}$

(e) 16π

9. The series $\sum_{k=1}^{\infty} 2^{3k} \cdot 3^{1-2k}$

(a) converges and has the sum 8/9

(b) converges and has the sum 3/4

(c) oscillates between 2 and 3

(d) diverges

(e) converges and has the sum 24

10. If $\left\{S_n\right\}_{n=1}^{+\infty}$ is the sequence of partial sums of the series $\frac{1}{3.5}+\frac{1}{5.7}+\frac{1}{7.9}+\cdots$, then S_n is equal to

(a) $\frac{n+1}{6(2n+3)}$

(b) $\frac{2}{(2n+1)(2n+3)}$

(c) $\frac{n}{3(2n+3)}$

(d) $\frac{6n}{2n+3}$

(e) $\frac{n}{2n+1}$

11. If the integral test is used in the interval $[0, \infty)$ for the series $\sum_{k=0}^{\infty} \frac{1}{1+9k^2}$, then the value of the integral is

(a) $+\infty$, and the series diverges

(b) $\frac{\pi}{9}$, and the series diverges

(c) $\frac{\pi}{6}$, and the series converges

(d) 9π, and the series converges

(e) $\frac{9\pi}{2}$, and the series converges

12. $\int \frac{3x^2 + x + 4}{x^3 + 4x} dx =$

(a) $\ln|x(x^2+4)| + \frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right) + c$

(b) $\ln|x\sqrt{x^2+4}| + c$

(c) $\ln|x\sqrt{x^2+4}| + \frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right) + c$

(d) $\ln|x(x^2+4)| + \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + c$

(e) $\ln|x\sqrt[3]{x^2+4}| + \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + c$

13. If $a_k = \dfrac{19}{\sqrt[3]{27k^2 + 5k}}$, $b_k = \dfrac{1}{k^{2/3}}$, and $\lim_{k\to\infty} \dfrac{a_k}{b_k} = L$, then the series $\sum_{k=1}^{\infty} a_k$

(a) converges because $L > 1$ and finite

(b) diverges because $L < 1$ and finite

(c) converges because $L < 1$ and finite

(d) diverges because $L = +\infty$

(e) diverges because $L > 1$ and finite

14. The improper integral $\int_0^e x \ln x \, dx$

(a) diverges

(b) converges and has the value e^2

(c) converges and has the value $4e^2$

(d) converges and has the value $\frac{1}{4} e^2$

(e) oscillates between e^2 and $4e^2$

15. If $a_k = \dfrac{(k!)^2(3^k)}{(2k)!}$ and $\lim\limits_{k\to\infty} \dfrac{a_{k+1}}{a_k} = L$, then the series $\sum\limits_{k=1}^{\infty} a_k$

(a) diverges because $L = \frac{4}{3} > 1$

(b) diverges because $L = 12 > 1$

(c) converges because $L = \frac{3}{4} < 1$

(d) converges because $L = \frac{1}{12} < 1$

(e) may converge or diverge because $L = 1$

16. $\displaystyle\int_0^{\ln 2} \frac{\sinh x}{1+\cosh x}\,dx =$

(a) $\ln(9/8)$

(b) $\ln(4/5)$

(c) $\ln(12/7)$

(d) $\ln(15/8)$

(e) $\ln 36$

Part II (24 points) Written Part

1. (9-points) Evaluate $\displaystyle\int \frac{x}{\sqrt{5+8x-4x^2}}\,dx.$ (Show your steps)

2. Let $f(x) = \dfrac{1}{1-2x}$.

(a) (5-points) Find the first five terms of the Maclaurin series for f, then find its k-th term and write the series in sigma notation. (Show your steps)

(b) (3-points) Find the radius and the interval of convergencve of the series obtained in part (a).

(c) (3-points) Use Parts (a), (b), and integration to find the Maclaurin series for $\ln(1-2x)$.

(d) (4-points) Use part (a) and multiplication to find the first four nonzero terms of the Maclaurin series for the function $f(x) = \dfrac{1+2x}{1-2x}$. $\left[\text{Hint}: \dfrac{1+2x}{1-2x} = (1+2x)\cdot\dfrac{1}{1-2x}\right]$.