

Department of Mathematics and Statistics
King Fahd University of Petroleum and Minerals
Dr. Raja Mohammad Latif

Name: _____

I.D.:# _____

Stat 319-02Ch1,2,3,4.Exan I 063: Summer Semester 31June-25Aug2007

Date: Tuesday 31 July 2007. Marks:100

Time: 4.10PM-6.10PM
(4.00PM-4.10PM reading time)

Instructions to the candidates:

1. The first 10 minutes is reading time. Do not write in your answer book until this time is complete.
2. Maximum marks for each part/question are shown in brackets.
3. Answer all questions very carefully.
4. You are not allowed to leave the examination hall during the first hour.
5. The questions may not carry equal number of marks.
6. The questions are not in any order of difficulty at all.
7. You are not allowed to use any type of mobiles or pagers.
8. Show complete work for full credit.
9. Write your name and I.D. # on each page of the examination.
10. Count that you have 21 questions and 24 pages of this examination.

Materials required:

1. Examination Paper
2. Formulae Sheet
3. Scientific calculator

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Q.1. 129McClave129. A supplier of kerosene has a 200 – gallon tank filled at the beginning of each week.

His weekly demands show a relative frequency behavior that increases steadily up to 100 gallons, and then levels of between 100 and 200 gallons.

Letting X denote weekly demand in hundreds of gallons, suppose the relative frequencies for demand are modeled adequately by

$$f(x) = \begin{cases} = 0 & \text{if } x < 0 \\ = x & \text{if } 0 \leq x \leq 1 \\ = \frac{1}{2} & \text{if } 1 < x \leq 2 \\ = 0 & \text{if } x > 2 \end{cases}$$

(a) Find the probability $P(0.5 < X < 1.5)$ that demand will be between 50 gallons and 150 gallons on a given week.

[5 marks]

$P(0.5 < X < 1.5) =$ _____.

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(Continue Question # 1 from the Last Page)

(b) Let \mathbf{F} be a cumulative distributive function.
Find $F(x) = P(X \leq x)$ for this random variable.

[5 marks]

$$\mathbf{F}(x) = \begin{cases} \text{-----} & \text{if } x < 0 \\ \text{-----} & \text{if } 0 \leq x \leq 1 \\ \text{-----} & \text{if } 1 < x \leq 2 \\ \text{-----} & \text{if } x > 2 \end{cases}$$

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Q.2. Theorem 3.2.1 (Larsen/Marks 117). If X is a continuous random variable with pdf (probability density function) $f(x)$ and cdf (cumulative distribution function) $F(x)$, then

$$f(x) = \frac{d}{dx} [F(x)]$$

provided $F'(x) = \frac{d}{dx} [F(x)]$ exists at all but a finite number of points.

Use the above Theorem to answer the next question.

Question. Suppose that a random variable has the cdf (cumulative distribution function) given below.

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{3x}{4} & \text{if } 0 \leq x \leq 1 \\ \frac{3}{4} & \text{if } 1 < x \leq 2 \\ \frac{x}{4} + \frac{1}{4} & \text{if } 2 < x \leq 3 \\ 1 & \text{if } x > 3 \end{cases}$$

Find the corresponding pdf (probability density function) by differentiating $F(x)$.

[5 marks]

$$f(x) = \begin{cases} \text{-----} & \text{if } x < 0 \\ \text{-----} & \text{if } 0 \leq x \leq 1 \\ \text{-----} & \text{if } 1 < x \leq 2 \\ \text{-----} & \text{if } 2 < x \leq 3 \\ \text{-----} & \text{if } x > 3 \end{cases}$$

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Q.3.71Dougherty2.51. Of the programmers in a particular department,
20 are fluent in Pascal,
18 in FORTRAN,
9 in COBOL,
15 in both Pascal and FORTRAN,
7 in both Pascal and COBOL,
6 in both FORTRAN and COBOL,
and 4 in all three languages.

Assume that there are 50 employees in the department.

Make a Venn Diagram below and then answer the questions given on the next page:

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(Continue Q#3 from the last page and answer the following questions)

Find the probability that an employee chosen at random will possess the following language proficiencies:

(a) Proficient in all three languages.

(a) .Answer. Probability: = _____

[1 marks]

(b) Proficient in none (not in any) of the three languages.

(b) .Answer. Probability: = _____

[1 marks]

(c) Proficient in exactly one of the languages.

(c) .Answer. Probability: = _____

[1 marks]

(d) Proficient in exactly two of the languages.

(d) .Answer. Probability: = _____

[1 marks]

(e) Proficient in at least (*minimum*) one of the languages.

(e) .Answer. Probability: = _____

[1 marks]

(f) Proficient in FORTRAN but not in COBOL.

(f) .Answer. Probability: = _____

[1 marks]

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Q.4.262HB26. Fill in the blank.

The total number of 5 – digit numbers made with the digits 1 through 8 is equal to _____ if no digit is repeated. [Hint : Examples of such 5 – digit numbers: 53271, 82514, 38741, 65438, 23567, 42367, 25482, etc.]

[3 marks]

		POSSIBLE ANSWER CHOICE
A		1680
A		56
B		40320
C		120
C		32768
D		6720
D		390625
E		15120
E		3024
F		126
F		1680
G		70
G		26544
H		None of the above choices is Correct.
H		
I		
I		
J		
J		
K		
K		
L		
L		
M		
M		
(N)		

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Q.5.109Rinaman3.10. Let X be a discrete random variable that can assume any value in the set $\{1, 2, 3, 4, 5\}$, and let the probability mass function be

$$p(x) = \begin{cases} \frac{x^2}{3K+1} & , \quad x = 1, 2, 3, 4, 5 \\ 0 & , \quad \textit{otherwise.} \end{cases}$$

Find the value of K to make $p(x)$ a probability mass function(*pmf*).

[3 marks]

		<i>POSSIBLE ANSWER CHOICE</i>
<i>A</i>		14
<i>A</i>		
<i>B</i>		18
<i>B</i>		
<i>C</i>		$\frac{121}{9}$
<i>C</i>		
<i>D</i>		16
<i>D</i>		
<i>E</i>		20
<i>E</i>		
<i>F</i>		28
<i>F</i>		
<i>G</i>		44
<i>G</i>		
<i>H</i>		$3\sqrt{5} - 1$
<i>H</i>		
<i>I</i>		45
<i>I</i>		
<i>J</i>		14
<i>J</i>		
<i>K</i>		$3\sqrt{5} + 1$
<i>K</i>		
<i>L</i>		90
<i>L</i>		
<i>M</i>		$\frac{1}{44}$
<i>M</i>		
(<i>N</i>)		None of the above choices is Correct.

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Q.6.2.265HB211Q.2. (Marks : 4). Fill in the blank.

In a third-grade class, there are 10 boys and 12 girls.

Then the number of ways of making groups of 7 children is equal to _____ if a group contains exactly 3 boys and exactly 4 girls.

[3 marks]

POSSIBLE ANSWER CHOICE	
A	615
B	8553600
C	184800
D	46200
E	6652800
F	12600
G	170544
H	85941760
I	3991680
J	604800
K	59400
L	8855
M	108900
(N)	None of the above choices is Correct.

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Q.7.(271T9.4.12B). Suppose that a pair of fair dice are rolled. Find the probability that the SUM of dots appearing equal to 5 is:

[3 marks]

		POSSIBLE ANSWER CHOICE
A		$\frac{1}{36}$
A		$\frac{2}{36}$
B		$\frac{3}{36}$
B		$\frac{4}{36}$
C		$\frac{5}{36}$
C		$\frac{6}{36}$
D		$\frac{7}{36}$
D		$\frac{8}{36}$
E		$\frac{9}{36}$
E		$\frac{12}{36}$
F		$\frac{15}{36}$
F		$\frac{18}{36}$
G		$\frac{30}{36}$
G		$\frac{30}{36}$
H		$\frac{30}{36}$
H		$\frac{30}{36}$
I		$\frac{30}{36}$
I		$\frac{30}{36}$
J		$\frac{12}{36}$
J		$\frac{12}{36}$
K		$\frac{15}{36}$
K		$\frac{15}{36}$
L		$\frac{18}{36}$
L		$\frac{18}{36}$
M		$\frac{30}{36}$
M		$\frac{30}{36}$
(N)		None of the above choices is Correct.

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Q.8.412Tan30. Course Enrollment. Among 800 freshmen pursuing a business degree at a university, 333 are enrolled in a **Economics** course, 222 are enrolled in a **Mathematics** course, and 155 are enrolled in **BOTH** an **Economics** and a **Mathematics** course. What is probability that a freshman selected at random from this group is enrolled in an **Economics** course and/or a **Mathematics** course?

[4 marks]

		POSSIBLE ANSWER CHOICE
A		0.10
A		
B		0.15
B		
C		0.20
C		
D		0.30
D		
E		0.40
E		
F		0.44
F		
G		0.50
G		
H		0.55
H		
I		0.60
I		
J		0.70
J		
K		0.75
K		
L		0.80
L		
M		0.95
M		
(N)		None of the above choices is Correct.

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Q.9.89Tan65TB. The following table reports how many new and returning customers are either satisfied or unsatisfied:

	SATISFIED (S)	UNSATISFIED (U)	TOTAL
NEW (N)	400	200	600
RETURNING (R)	500	100	600
TOTAL	900	300	1200

(a). Let N represent the event that the customer is a new customer, R represent the event that the customer is a returning customer, S represent the event that the customer is a satisfied customer, and U represent the event that the customer is an unsatisfied customer. Find the conditional probability $P(S|N)$ that the customer is satisfied given that the customer is a new one.

[2 marks]

	POSSIBLE ANSWER CHOICE
A	0.333333333
B	0.50
C	0.20
D	0.40
E	0.666666667
F	0.60
G	0.80
H	0.555555556
I	0.444444444
J	0.25
K	0.70
L	0.90
M	0.95
(N)	None of the above choices is Correct.

(b) Find the following probabilities:

[3 marks]

(i) $P(R) =$ _____

(ii) $P(U) =$ _____

(iii) $P(R \cap U) =$ _____

(iv) Are the events R and U independent?

TRUE OR FALSE

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Q.10.(M131T434B13. Cola Preference. A survey was taken among Coca drinkers to see which of two popular brands people preferred.

**It was found that 45% like brand A,
40% like brand B,
and 20% like both brands.**

Suppose that a person in the survey is randomly selected.

(a) Find the probability that the person liked brand A, given that he or she liked brand B.

[1 marks]

$P(A | B) =$ _____.

(b) Find the probability that the person liked brand B, given that he or she liked brand A.

[1 marks]

$P(B | A) =$ _____.

(c) Are the events A and B independent?

TRUE OR FALSE

[1 marks]

Justify your answer.

(d) Find $P(A \cup B) =$ _____.

[1 marks]

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Q.11.(71DeGroot8). (Bayes' Theorem). In a certain city,
30 % percent of the people are Conservatives,
50 percent are Liberals,
and 20 % are independents.

Records show that in particular election,
65 % percent of the Conservatives voted,
82 % percent of the Liberals voted,
and 50 % of the Independents voted.

If a person in the city is selected at random and it is learned that he did not vote in the last election,

what is the probability that he is a Liberal?

[5 marks]

$P(\text{The person is Liberal} | \text{The person did not Vote}) = \text{_____}.$

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Q.12. 134McClave 4.5. The weekly demand X for kerosene at a certain supply station has a probability density function (*pdf*) $f(x)$ given by

$$f(x) = \begin{cases} x & \text{if } 0 \leq x \leq 1 \\ \frac{1}{2} & \text{if } 1 < x \leq 2 \\ 0 & \text{if elsewhere} \end{cases}$$

(a) Find the expected weekly demand, $E(X)$.

[3 marks]

$E(X) =$ _____.

(b) Find $E(X^2)$, the expected value of X^2 .

[3 marks]

$E(X^2) =$ _____

(c) Find $\sigma^2 = Var(X) = E(X^2) - [E(X)]^2$, the variance of X .

[1 marks]

$\sigma^2 =$ _____.

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Q.13.84McClave3.43. Daily sales records for a computer manufacturing firm show that it will sell 0, 1, or 2 mainframe computer systems with probabilities as listed:

Number of Sales	0	1	2
Probability	0.7	0.2	0.1

(a) Find the expected value, and variance for daily sales.

[1,1,1 marks]

(i) $E(X) =$ _____.

(ii) $E(X^2) =$ _____.

(iii) $\sigma^2 = Var(X) = E(X^2) - [E(X)]^2$
 $=$ _____.

(b) Find the CDF (Cumulative Distributive Function) $F(x) = P(X \leq x)$ of the random variable X .

[3 marks]

$$F(x) = \begin{cases} \text{-----} & \text{if } x < 0 \\ \text{-----} & \text{if } 0 \leq x < 1 \\ \text{-----} & \text{if } 1 \leq x < 2 \\ \text{-----} & \text{if } x \geq 2 \end{cases}$$

(c) Find $E(2X^2 + X + 3)$, the expected value of $2X^2 + X + 3$.

[1 marks]

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Q.14.20TB2. A tire manufacturer wants to determine the inner diameter of a certain grade of tire. Ideally, the diameter would be 570 *mm*. The data are as follows:

572, 572, 573, 568, 569, 575, 565, 570.

(a) Find the Sample Mean.

[1 marks]

(b) Find the Median.

[1 marks]

(c) Find the Sample Variance σ^2 .

[1 marks]

(d) Find the Range of the above data.

[1 marks]

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Q.16. 51DeGroot6. Two students A and B are both registered for the STAT 319 course. Suppose student A attends class 80 percent of the time and student B attends class 60 percent of the time, and suppose the absences of the two students are independent.

(a) What is the probability that both students A and B will be in class on a particular day?

[2 marks]

Probability : _____.

(b) What is the probability that at least one (minimum one) of the two students will be in class on a given day.

[2 marks]

Probability : _____.

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Q.17.287HB27Q.5. Two Coins are tossed and a Die is rolled.

(a) List the outcomes of the sample space S .

[1 marks]

[Hint: Examples of some outcome: HT5, TT6, TH5, etc.]

(b) Find the probability that exactly two heads and any even number appear in the above experiment.

[1 marks]

(c) Let E be the event that die has an even number and O be the event that the die has an odd number. Then Mark True or False:

[2 marks]

(I) The events E and O are independent. True or False (check one)

(II) The events E and O are mutually exclusive (disjoint). True or False (check one)

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Q18.434T131B12. (*College Selection and Family Income*).

A survey of 175 students resulted in the data shown in the table given below.

It shows the type of college the student attends and the income level of the student's family. Suppose a student in the survey is randomly selected.

<i>INCOME</i>	PRIVATE COLLEGE	<i>PUBLIC COLLEGE</i>	TOTAL
<i>HIGH INCOME</i>	14	11	25
<i>MIDDLE INCOME</i>	25	55	80
<i>LOW INCOME</i>	10	60	70
TOTAL	49	126	175

(a) Find the probability that the student attends a public college, given that the student comes from a middle-income family.

[1 marks]

Probability : _____.

(b) Find the probability that the student is from a high-income family, given that the student attends a private college.

[1 marks]

Probability : _____.

(c) If the student comes from a high-income family, find the probability that the student attends a private college.

[1 marks]

Probability : _____.

(d) Find the probability that the student attends a public college or comes from a low-income family.

[1 marks]

Probability : _____.

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Q19.387SM2. A box contains 12 light bulbs, of which 5 are defective.

All bulbs look alike and have equal probability of being chosen. Three light bulbs are selected and placed in a box.

(a) What is the probability that all three are defective?

[2 marks]

Probability : _____.

(b) What is the probability that exactly 2 are defective?

[2 marks]

Probability : _____.

(c) What is the probability that at least (Minimum) 2 are defective?

[2 marks]

Probability : _____.

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Q.20. 101Freund3.55. The total lifetime (*in years*) of five-year-old dogs of a certain breed is a random variable whose distribution function (cumulative distributive function) is given by

$$F(x) = \begin{cases} 0 & \text{for } x \leq 5 \\ 1 - \frac{25}{x^2} & \text{for } x > 5 \end{cases}$$

Find the probability $P(12 \leq X \leq 15)$ that such a five-year-old dog will live anywhere from 12 to 15 years.

[5 marks]

$$P(12 \leq X \leq 15) = \text{_____}.$$

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Q.21. 98Freund3.39. Let X be a continuous random variable and let $f(x)$ be a probability density function given by

$$f(x) = \begin{cases} = Kx & \text{for } 0 < x \leq 1 \\ = K & \text{for } 1 < x \leq 2 \\ = K(3-x) & \text{for } 2 < x \leq 3 \\ = 0 & \text{for elsewhere} \end{cases} .$$

Find the value of the constant K so that $f(x)$ is a probability density function.
[5 marks]

$K =$ _____.