MATH 102 - Quiz 3 Section number: Student ID:

Instructions: You are required to attempt all questions. Each is worth 10 points.

1. Evaluate $\displaystyle\int \frac{2x+1}{4x^2+12x-7}dx$

Solution:
$\frac{2x+1}{4x^2+12x-7} = \frac{2x+1}{4x^2+12x+9-16} = \frac{2x+1}{(2x+3)^2-4^2} = \frac{2x+1}{(2x-1)(2x+7)}$
$\frac{2x+1}{(2x-1)(2x+7)} = \frac{A}{2x-1} + \frac{B}{2x+7}$. This leads to:

$2 = 2A + 2B$
$1 = 7A - B$

Solving the above set of simultaneous equations and we end up with: $A = 1/4$ and $B = 3/4$

So, $\displaystyle\int \frac{2x+1}{4x^2+12x-7}dx = \frac{ln(2x-1)}{8} + \frac{3ln(2x+7)}{8}$

2. Is $\displaystyle\int_1^2 \frac{1}{2x-1}dx$ an improper integral? State why or why not.

Solution:
The asymptote for the integrand is at $x = 1/2$. For interval $[1, 2]$, the integrand has no infinite discontinuity. In addition, the interval is not infinite. Hence, it is not an improper integral.

3. Determine if $\displaystyle\int_{-\infty}^{\infty} \frac{x^2}{9+x^6}dx$ is convergent or divergent. Evaluate the integral if it is convergent.

Solution:
Let $u = x^3 \rightarrow du = 3x^2dx$

$\displaystyle\int \frac{x^2}{9+x^6}dx = \int \frac{du}{3(u^2+9)} = [\frac{\tan^{-1}(x/3)}{9}]_{-\infty}^{\infty} = \frac{\pi}{9}$ since the tangent function goes to $\pm\infty$ at $x = \pm\pi/2$ (hence convergent).