Math 132 03/14/2009

Quiz 2

Ex 1: a) Use the definition of the derivative to find $f'(x)$, where $f(x) = \sqrt{3x+1}$

b) Use the rules for differentiation to differentiate the function $g(x) = \dfrac{x+5}{x^{\frac{2}{3}}}$

Ex 2 Consider the function $h(x) = \sqrt{x} + \dfrac{2}{\sqrt{x}}$

a) Find the slope of the tangent line to the graph of h at $x = 1$

b) Find an equation of the tangent line to the graph of h at $x = 1$

Solutions

Ex 1: a) $\displaystyle \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\sqrt{3(x+h)+1} - \sqrt{3x+1}}{h}$

$$= \lim_{h \to 0} \frac{\left(\sqrt{3(x+h)+1} - \sqrt{3x+1}\right)\left(\sqrt{3(x+h)+1} + \sqrt{3x+1}\right)}{h\left(\sqrt{3(x+h)+1} + \sqrt{3x+1}\right)}$$

$$= \lim_{h \to 0} \frac{3(x+h)+1-(3x+1)}{h\left(\sqrt{3(x+h)+1} + \sqrt{3x+1}\right)} = \lim_{h \to 0} \frac{3x+3h+1-3x-1}{h\left(\sqrt{3(x+h)+1} + \sqrt{3x+1}\right)}$$

$$= \lim_{h \to 0} \frac{3}{\sqrt{3(x+h)+1} + \sqrt{3x+1}} = \frac{3}{2\sqrt{3x+1}}$$

Hence f is differentiable and $f'(x) = \dfrac{3}{2\sqrt{3x+1}}$

b) $g(x) = x^{-2/3}(x+5) = x^{-2/3}x + 5x^{-2/3} = x^{1/3} + 5x^{-2/3}$

Hence $g'(x) = \frac{1}{3} x^{\frac{1}{3}-1} + 5\left(-\frac{2}{3} x^{-\frac{2}{3}-1}\right)$

$= \frac{1}{3} x^{-\frac{2}{3}} - \frac{10}{3} x^{-\frac{5}{3}} = \frac{x-10}{3x^{5/3}}$

Ex2: a) $h(x) = x^{\frac{1}{2}} + 2x^{-\frac{1}{2}}$.

So $h'(x) = \frac{1}{2} x^{\frac{1}{2}-1} + 2\left(-\frac{1}{2} x^{-\frac{1}{2}-1}\right)$

$= \frac{1}{2} x^{-\frac{1}{2}} - x^{-\frac{3}{2}}$

The slope of the tangent line at $x=1$ is given by:

$$h'(1) = \frac{1}{2} - 1 = -\frac{1}{2}$$

b) An equation of the tangent line at $x=1$ is given by

$$y = h'(1)(x-1) + h(1)$$
$$= -\frac{1}{2}(x-1) + 3 = \frac{x+7}{2}.$$