Math 132 Quiz 6 08/15/2009

Ex1: (a) Sketch the region bounded by $y^2 = x$ and $y = x - 2$

(b) Find the y-coordinates of the intersection points of the curves $y^2 = x$ and $y = x - 2$.

(c) Evaluate the area of the region bounded by $y^2 = x$ and $y = x - 2$.

Ex2: Use integration by parts to find

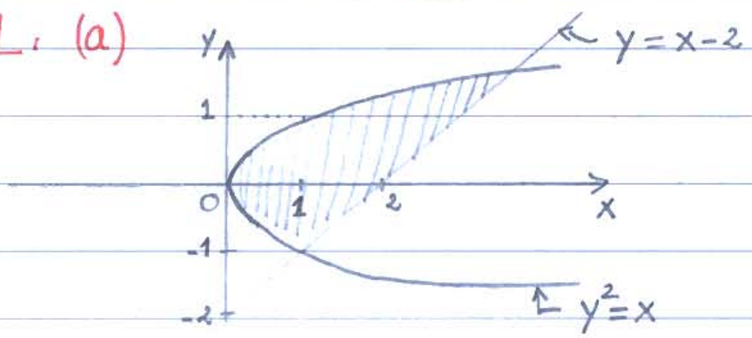
(a) $I_1 = \int \frac{\ln x}{x^2}\, dx$

(b) $I_2 = \int x^2 e^x\, dx$

Ex3: Find the integral $I = \int \sin^2 x \cos^3 x\, dx$.

Solutions

Ex1: (a)



(b) $y^2 = x$ and $x = y + 2$, then: $y^2 = y + 2$ or $y^2 - y - 2 = 0$ which implies $y = -1$ or $y = 2$.
Therefore the y-coordinates of the intersection points are -1 and 2. In fact the intersection points are $(+1, -1)$ and $(4, 2)$.

(c) $\text{Area} = \int_{-1}^{2} \left[(y+2) - y^2\right] dy = 2y + \frac{y^2}{2} - \frac{y^3}{3} \Big|_{-1}^{2} = \frac{9}{2}$

Ex2: (a) Let $u'(x) = \frac{1}{x^2} \Rightarrow u(x) = -\frac{1}{x}$

and let $v(x) = \ln x \Rightarrow v'(x) = \frac{1}{x}$. Thus

$$I_1 = \int u'(x)\,v(x)\,dx = u(x)v(x) - \int u(x)\,v'(x)\,dx$$

$$= -\frac{\ln x}{x} + \int \frac{1}{x^2}\,dx = -\frac{\ln x}{x} - \frac{1}{x} + c$$

$$= -\frac{1}{x}(1+\ln x) + c = -\frac{1+\ln x}{x} + c.$$

(b) Let $u(x) = x^2 \Rightarrow u'(x) = 2x$ and let $v'(x) = e^x \Rightarrow v(x) = e^x$.

Then: $$I_2 = \int u(x)\,v'(x)\,dx = u(x)v(x) - \int u'(x)\,v(x)\,dx$$

$$= x^2 e^x - 2\underbrace{\int x e^x\,dx}_{J}.$$

We let $J = \int x e^x\,dx$.

Again we use integration by parts to evaluate J:

let $u(x) = x \Rightarrow u'(x) = 1$ and let $v'(x) = e^x$ so $v(x) = e^x$.

Therefore $J = xe^x - \int e^x dx = xe^x - e^x = (x-1)e^x + c$

Hence: $I_2 = x^2 e^x - 2(x-1)e^x + c = (x^2 - 2x + 2)e^x + c$

Ex3. $I = \int \sin^2 x \cos^2 x \cos x\,dx = \int \sin^2 x\,(1-\sin^2 x) \cos^2 x\,dx$

Let $u = \sin x$. Then $du = \cos x\,dx$. So

$$I = \int u^2(1-u^2)\,du = \int (u^2 - u^4)\,du$$

$$= \frac{u^3}{3} - \frac{u^5}{5} + c = \frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + c$$