Solution to Quiz #1

Question (10 points total)

Find the equation of the tangent line to the curve

$$x = 2\cos(t), y = \sin(2t)$$

at the point given by $t = \pi/2$. Find the points on the curve where the tangent line is horizontal.

Solution:

Since $x = 2\cos t$ and $y = \sin 2t$, we have

$$\frac{d}{dt}\sin(2t) = 2\cos(2t)$$

and

$$\frac{d}{dt}2\cos t = -2\sin t.$$

(2 points)

Thus

$$\frac{dy}{dx} = \frac{\cos(2t)}{-\sin(t)}$$

and so at $t = \frac{\pi}{2}$ the slope of the tangent line is 1. *(1 point)*

Moreover, when $t = \frac{\pi}{2}$, $x = 0$ and $y = 0$. *(1 point)* So, the equation of the tangent line is

$$y = x.$$

(1 point)

There will be a horizontal tangent when $\frac{dy}{dx} = 0$, that is when $\frac{dy}{dt} = 0$ and $\frac{dx}{dt} \neq 0$. *(1 point)*

If $\frac{dy}{dt} = 0$ then $2\cos(2t) = 0$ and so

$$2t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \text{ or } \frac{7\pi}{2}$$

that is

$$t = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4} \text{ or } \frac{7\pi}{4}.$$

(2 points)

Moreover,

$$-2\sin(\pi/4) = -\frac{2}{\sqrt{2}}; -2\sin((3\pi)/4) = -\frac{2}{\sqrt{2}}; -2\sin((5\pi)/4) = \sqrt{2}$$

and

$$-2\sin((7\pi)/4) = \sqrt{2}$$

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When $t = \frac{\pi}{4}$ then $x = 2\cos(\pi/4) = \frac{2}{\sqrt{2}} = \sqrt{2}$ and $y = \sin(\pi/2) = 1$.

When $t = \frac{3\pi}{4}$ then $x = 2\cos((3\pi)/4) = -\sqrt{2}$ and $y = \sin((3\pi)/2) = -1$.

When $t = \frac{5\pi}{4}$ then $x = 2\cos((5\pi)/4) = -\sqrt{2}$ and $y = \sin((5\pi)/2) = 1$.

When $t = \frac{7\pi}{4}$ then $x = 2\cos((7\pi)/4) = \frac{2}{\sqrt{2}}$ and $y = \sin((7\pi)/2) = -1$.

So, there are horizontal tangents at $(\sqrt{2}, 1)$, $(-\sqrt{2}, -1)$, $(-\sqrt{2}, 1)$ and $(\sqrt{2}, -1)$. *(2 points)*

Note: Points will be deducted for incomplete or incorrect answers. Points will also be deducted for not fully or properly showing your work.