

✓ 1. The graph of $f(x) = \frac{x^4 + x^2}{(x^2 + 9)(x + 1)}$ has

- ✓ (a) one slant asymptote and one vertical asymptote
- ✗ (b) two slant asymptotes and two vertical asymptotes
- ✗ (c) one horizontal asymptote and two vertical asymptotes
- ✓ (d) one slant asymptote and three vertical asymptotes
- ✗ (e) one horizontal, one vertical, and one slant asymptote

$(x^2 + 9)(x + 1) = 0$
 $x = -1, x^2 = -9$ rejected

one vertical

$$\lim_{x \rightarrow \infty} \frac{x^4 + x^2}{(x^2 + 9)(x + 1)}$$

$$= \lim_{x \rightarrow \infty} \frac{x^4}{x^3} = x = \infty$$

no H.A.

slant Asym.

$$f = \frac{4}{9} \text{ with } d^{\circ} 4 = d^{\circ} 9 + 1$$

✓ 2. The function $f(x) = \frac{\ln(x - 1)}{2 - \sqrt{x}}$ is continuous on

- ✓ (a) $(1, 4) \cup (4, +\infty)$
- (b) $[0, +\infty)$
- (c) $(\sqrt{2}, +\infty)$
- (d) $(2, +\infty)$
- (e) $(1, +\infty)$

$\ln(x - 1) : x > 1$

$2 - \sqrt{x} : x \geq 0, x \neq 4$

$D = (1, 4) \cup (4, \infty)$



3. Using Newton's Method to find a root of the equation

$$3x - \sin(2\pi x) = 1$$

starting with $x_1 = \frac{1}{2}$ we find that $x_2 =$

(a) $\frac{\pi + 1}{2\pi + 3}$

(b) $2\pi - 1$

(c) $\frac{1}{4\pi + 6}$

(d) $\frac{1}{2\pi + 1}$

(e) $\frac{1}{2}$

$$f(x) = 3x - \sin(2\pi x) - 1$$

$$f\left(\frac{1}{2}\right) = \frac{3}{2} - 0 - 1 = \frac{1}{2}$$

$$f'(x) = 3 - 2\pi \cos 2\pi x$$

$$f'\left(\frac{1}{2}\right) = 3 - 2\pi \cos \pi$$

$$= 3 + 2\pi$$

$$x_2 = \frac{1}{2} - \frac{\frac{1}{2}}{3 + 2\pi} = \frac{2(\pi + 1)}{2(3 + 2\pi)}$$

not included

not included

4. $\lim_{x \rightarrow 5^+} \frac{4-x}{(x-5)^3} = \lim_{5^+} \frac{4-5}{(5-5)^3} = \frac{-1}{0^+} = -\infty$

(a) $\frac{4}{5}$

(b) -1

(c) $\frac{1}{2}$

(d) $+\infty$

(e) $-\infty$

5. The number of the inflection points of the graph of $y = \frac{1}{56}x^8 - \frac{1}{30}x^6 + 80$ is

(a) 4

(b) 6

(c) 3

(d) 2

(e) 1

$$y' = \frac{1}{7}x^7 - \frac{1}{5}x^5$$

$$y'' = x^6 - x^4 = 0$$

$$x^4(x^2 - 1) = 0$$

$$x = 0, x = 1, x = -1$$

x	-1	0	1
F''	+	-	+
f	U	n	U
	IP		IP

6. The slope of the tangent line to the curve $\cos(xy^2) = y^3 - x + \frac{\pi}{2} - 1$ at

$(\frac{\pi}{2}, 1)$ is

(a) $-\frac{1}{3}$ (b) $\frac{1}{3}$

(c) 1

(d) 0

(e) -1

$$-\sin(xy^2) \cdot (y^2 + x \cdot 2yy') = 3y^2y' - 1$$

$$-\sin(xy^2)y^2 + -\sin(xy^2) \cdot 2xyy' = 3y^2y' - 1$$

$$1 - \sin(xy^2)y^2 = 3y^2y' + \sin(xy^2) \cdot 2xyy'$$

$$y' = \frac{y^2 \sin(xy^2) - 1}{-2xy \sin(xy^2) - 3y^2}$$

$$y' = \frac{(1) \sin(\frac{\pi}{2}) - 1}{-2(\frac{\pi}{2}) \sin(\frac{\pi}{2}) - 3} = \frac{0}{-\pi - 3} = 0$$

7. The sum of the critical numbers of the function $f(x) = \sqrt[3]{x^2 - x}$ is

D: ~~all~~ all real number

(a) 1 $f'(x) = \frac{1}{3} (x^2 - x)^{-\frac{2}{3}} (2x - 1) = 0$

(b) $\frac{3}{2}$ $x = \frac{1}{2}, x(x-1) = 0$

(c) $-\frac{1}{2}$ $x = 0, x = 1$

thsum = $1 + \frac{1}{2} + 0 = \frac{3}{2}$

(d) $\frac{1}{2}$

(e) 2

8. All values of x where the tangent line to the graph of $y = \tan^2 x$ is horizontal are given by

$$f'(x) = 2 \tan x (1 + \tan^2 x) = 0$$

(a) $n\pi$, n is integer

$\tan x = 0$ \rightarrow $\tan^2 x = -1$
 $x = n\pi$ rejected

(b) $\frac{2n-1}{2}\pi$, n is integer

(c) $\frac{n}{2}\pi$, n is integer

(d) $(2n-1)\pi$, n is integer

(e) $\left(n + \frac{1}{2}\right)\pi$, n is integer

9. If g is a differentiable function and $f(x) = [g(x^2)]^2$, then $f'(x) =$

(a) $2g'(x^2)$

(b) $4xg'(x^2)$

(c) $4xg(x^2)g'(x^2)$

(d) $4x^3g'(x^4)$

(e) $2g(x^2)$

$$f'(x) = 2g(x^2) \cdot g'(x^2) \cdot 2x$$

$$= 4xg(x^2)g'(x^2)$$

10. If $f(t) = te^t \sin t$, then $f'(t) =$

(a) $e^t \cos t + t \sin t$

(b) $te^t \sin t - te^t \cos t$

(c) $e^t \cos t$

(d) $te^t \cos t + (t+1)e^t \sin t$

(e) $te^t \cos t + te^t \sin t$

$$f'(t) = (e^t + te^t) \sin t + te^t \cdot \cos t$$

$$= (1+t)e^t \sin t + te^t \cos t$$

11. An equation of the tangent line to the curve $y = x^{(2^x)}$ at the point (1, 1) is

(a) $y = \frac{1}{2}x + \frac{1}{2}$

(b) $y = 3x - 2$

(c) $y = \frac{1}{3}x + \frac{2}{3}$

(d) $y = 2x - 1$

(e) $y = \frac{x}{x}$

So $|y'(1)| = 2$

$y = x^{2^x}$
 $\ln y = 2^x \ln x$

$\frac{y'}{y} = 2 \ln x + \frac{2^x}{x}$

$y' = y(2 \ln x + 2)$

at (1, 1) $y' = 1(2 \ln 1 + 2) = 2 = \text{slop}$

$y = x^{2^x}$

$\ln y = 2^x \ln x$

$\frac{y'}{y} = (2^x)' \ln x + 2^x (\ln x)'$

$y' = (\ln 2) 2^x \ln x + \frac{2^x}{x}$

$y' = x^{2^x} \left[(\ln 2) 2^x \ln x + \frac{2^x}{x} \right]$

If $f''(x) = -3x^{-2}$, $f'(3) = 2$, $f(1) = -1$, then $f(e) =$

$f'(x) = -3x^{-1} + C_1 = \frac{3}{x} + C_1$

$f'(3) = -1 + C_1 = 2$

$C_1 = 3$

$f(x) = 3 \ln x + x + C_2$

$f(1) = 3 \ln 1 + 1 + C_2 = -1$

$C_2 = -2$

$f(e) = 3 \ln e + e - 2$

$= e + 1$

not included

(a) $e - 3$

(b) $\frac{3}{e} + 1$

(c) $\frac{-3}{e^2}$

(d) 0

(e) $e + 1$

13. If $f(x) = e^{1-2x}$, then $f^{(n)}(x) =$

(a) $(-2)^n e^{1-2x}$

(b) e^{1-2x}

(c) $2^n e^{1-2x}$

(d) $(-1)^n e^{1-2x}$

(e) $(1-2x)^n e^{1-2x}$

$$f' = -2 e^{1-2x}$$

$$f'' = 4 e^{1-2x}$$

$$f''' = -8 e^{1-2x}$$

$$f^{(n)} = (-2)^n e^{1-2x}$$

14. Sand is being dumped from a truck at a rate of $0.5 \text{ ft}^3/\text{min}$ to form a pile in the shape of a cone whose height is always equal to the diameter of its base. When the pile is 2 ft high, the height of the pile is increasing at a rate of

[The volume of a cone is $V = \frac{1}{3}\pi r^2 h$]

$$V = \frac{1}{3} \pi \left(\frac{h}{2}\right)^2 h$$

$$= \frac{1}{3} \pi \frac{h^3}{4}$$

$$\frac{dV}{dt} = \frac{1}{3} \pi \cdot \frac{1}{4} \cdot 3h^2 \cdot \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{4}{\pi h^2} \frac{dV}{dt}$$

$$\frac{dh}{dt} = \frac{0.5}{\frac{1}{3} \pi} \cdot \frac{1}{4} = \frac{0.5}{\pi} = \frac{1}{2\pi} \text{ ft/min}$$

(a) $\frac{1}{2\pi} \text{ ft/min}$

(b) $\frac{\pi}{2} \text{ ft/min}$

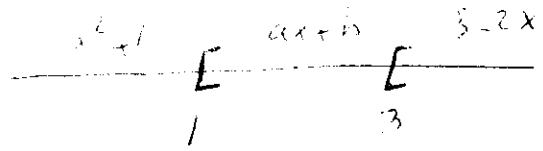
(c) $\frac{2}{\pi} \text{ ft/min}$

(d) $2\pi \text{ ft/min}$

(e) $\frac{1}{2} \text{ ft/min}$

✓ 15. If

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x < 1 \\ ax + b & \text{if } 1 \leq x < 3 \\ 3 - 2x & \text{if } x \geq 3 \end{cases}$$



is continuous on $(-\infty, +\infty)$, then $f(2) =$

- (a) 5
- (b) $\frac{7}{2}$
- (c) $\frac{1}{2}$
- (d) 2
- (e) $-\frac{5}{2}$

$\lim_{x \rightarrow 1^-} f(x) = f(1)$
 $x^2 + 1 = ax + b$
 $b = 2 - a$
 $\lim_{x \rightarrow 3^-} f(x) = f(3)$
 $3a + b = 3 - 6$
 $b = -3 - 3a$

$\lim_{x \rightarrow 1^+} f(x) = f(1)$
 $2 = a + b$
 $\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^+} f(x) = f(3)$
 $3a + b = -3$
 $\Rightarrow \begin{cases} a = -\frac{5}{2} \\ b = \frac{9}{2} \end{cases}$
 $f(2) = -\frac{5}{2}(2) + \frac{9}{2} = -\frac{1}{2}$

$\Rightarrow 2 - a = -3 - 3a$
 $a = -\frac{5}{2}, b = 2 + \frac{5}{2} = \frac{9}{2}$
 $\Rightarrow f(2) = 2\left(-\frac{5}{2}\right) + \frac{9}{2} = -\frac{1}{2}$

✓ 16. If $x^6 + y^6 = 1$, then $y'' =$ implicitly

- (a) $\frac{5x^4}{y^{11}}$
- (b) $\frac{x^5}{y^5}$
- (c) $\frac{10x^4}{y^{10}}$
- (d) $\frac{x^6 + 1}{y^5}$
- (e) $\frac{-1}{y^6}$

$6x^5 + 6y^5 y' = 0$
 $y' = -\frac{x^5}{y^5}$
 $y'' = \frac{-5x^4 y^5 + x^5 (5y^4 y')}{y^{10}}$
 $= \frac{-5x^4 y^5 + x^5 (5y^4 y')}{y^{10}}$
 $= \frac{-5x^4 y^5 + x^5 (5y^4 (-\frac{x^5}{y^5}))}{y^{10}}$
 $= \frac{-5x^4 y^5 - 5x^{10} y^{-1}}{y^{10}}$
 $= \frac{-5x^4 (y^6 + x^6)}{y^{10}}$
 $= \frac{-5x^4}{y''}$

17. Let $f(x) = e^x + \sin x$. Using the linear approximation of f at $a = 0$, we find that $f(0.1) \approx$

$$f'(x) = e^x + \cos x$$

- (a) 1.2
- (b) 1
- (c) 2.2
- (d) 2
- (e) 1.5

$$f(x) \approx_0 f(a) + f'(a)(x-a)$$

$$= e^0 + \sin 0 + (e^0 + \cos(0))(x-0)$$

$$= 1 + 0 + (1+1)(x)$$

$$f(0.1) = 1 + 2 \times 0.1 = 1.2$$

$$f(x) \approx_0 f(0) + f'(0)(x-0) = f(0) + f'(0)x$$

$$f(0) = 1, \quad f'(0) = 2$$

$$f(x) \approx_0 1 + 2x$$

$$f(0.1) \approx 1.2$$

18. Which one of the following statements is TRUE?

not included

- (a) if $f'(x) = g'(x)$ for all x , then $f(x) = g(x)$ for all x . ~~X~~
- (b) If f is continuous at a , then f is differentiable at a . ~~X~~
- (c) If $f(x) = \pi^4$, then $f'(x) = 4\pi^3$. ~~X~~
- (d) if $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at c .
- (e) If f has an absolute minimum value at c , then $f'(c) = 0$. ~~X~~

✓ 19. If $\tanh x = -\frac{2}{3}$, then $\cosh x =$

(a) $\pm \frac{3}{\sqrt{5}}$

(b) $\frac{3}{\sqrt{5}}$

(c) $-\frac{1}{\sqrt{5}}$

(d) $\pm \frac{1}{\sqrt{5}}$

(e) $-\frac{3}{\sqrt{5}}$

$$\begin{aligned} \operatorname{sech}^2 x &= 1 - \tanh^2 x \\ &= 1 - \left(-\frac{2}{3}\right)^2 \\ &= 1 - \frac{4}{9} \\ &= \frac{5}{9} \end{aligned}$$

$$\cosh x = \frac{1}{\operatorname{sech} x} = \frac{1}{\pm \sqrt{\frac{5}{9}}} = \pm \frac{3}{\sqrt{5}}$$

always positive

✓ 20. If $G(x) = \frac{1 + \sinh x}{1 + \cosh x}$, then $G(0) + G'(0) =$

(a) $1/4$

(b) 0

(c) $3/4$

(d) 2

(e) 1

$$G'(x) = \frac{\cosh x + \cosh^2 x - \sinh x - \sinh^2 x}{(1 + \cosh x)^2}$$

$$\begin{aligned} G(0) + G'(0) &= \frac{1 + e^0 - e^0}{1 + e^0 + e^0} + \frac{1 + 1 - 0 - 0}{(1 + 1)^2} \\ &= \frac{1}{2} + \frac{1}{2} = 1 \end{aligned}$$

21. ~~not included~~

If the point (x, y) lying on the line $y + 3x = 3$ is the closest point to the origin, then $x + 2y =$

- (a) $\frac{6}{5}$
- (b) $\frac{3}{5}$
- (c) $\frac{3}{2}$**
- (d) 2
- (e) 3

$$y = -3x + 3$$

$$d^2 = x^2 + y^2$$

$$= x^2 + (-3x + 3)^2 = x^2 + 9x^2 - 18x + 9$$

$$0 = 10x^2 - 18x + 9$$

$$0 = 20x - 18 \Rightarrow x = \frac{9}{10}$$

$$y = -3 \cdot \frac{9}{10} + 3 = \frac{3}{10}$$

$$\Rightarrow x + 2y = \frac{9}{10} + \frac{6}{10} = \frac{15}{10} = \frac{3}{2}$$

✓

22. The graph of $f(x) = \sqrt[3]{x}(2-x)$

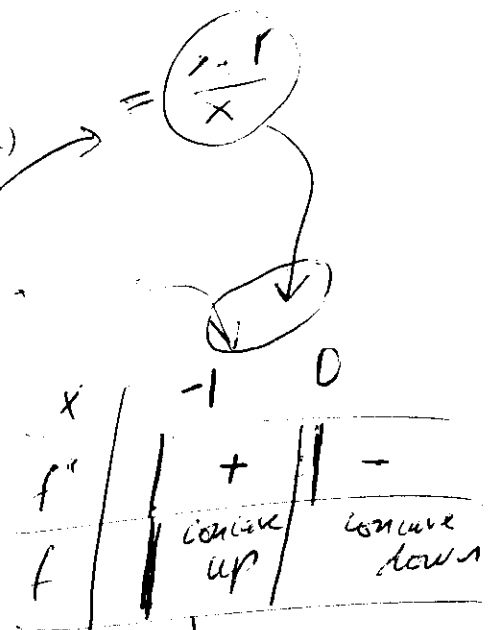
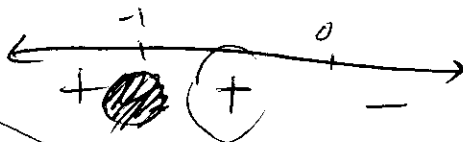
- (a) is concave down on the interval $(-\infty, 0)$
- (b) is concave up on the intervals $(-\infty, -1)$ and $(0, 1)$
- (c) has one inflection point only
- (d) has an inflection point at $x = 1$
- (e) is concave up on the interval $(-1, 0)$**

$$f'(x) = \frac{2}{3}x^{-\frac{2}{3}} - \frac{4}{3}x^{\frac{1}{3}}$$

$$f''(x) = \frac{-4}{9}x^{-\frac{5}{3}} - \frac{4}{9}x^{-\frac{2}{3}} = 0$$

$$x^{-\frac{2}{3}}(x^{-1} + 1) = 0$$

$$x = 0 \quad ; \quad x = -1$$



23. If an equation of the tangent line to the curve $y = e^x$ that is parallel to the line $x - 4y = 1$ is given by $y = ax + b$, then $4(b - a) =$

- (a) $\ln 4$
- (b) 1
- (c) 0
- (d) $1 + \ln 4$
- (e) $2 - \ln 4$

$-4y = 1 - x$
 $y = \frac{1-x}{-4}$
 $y = \frac{x}{4} - \frac{1}{4}$

$(e^x)' = \frac{1}{4} \leftarrow \text{slope} \Rightarrow x = \ln \frac{1}{4}$
 $y = \frac{1}{4}$

$4b = 1 - \ln \frac{1}{4}$
 $4a = 1$
 $4b - 4a = -\ln \frac{1}{4} = \ln 4$

$y - y_0 = \frac{1}{4}(x - x_0)$
 $y - \frac{1}{4} = \frac{1}{4}(x - \ln \frac{1}{4})$
 $y = \frac{1}{4}x - \frac{1}{4} \ln \frac{1}{4} + \frac{1}{4}$

$\Rightarrow 4(b-a) = 4(\frac{1}{4} \ln \frac{1}{4} + \frac{1}{4} - \frac{1}{4}) = -\ln \frac{1}{4}$

24. $\lim_{x \rightarrow 0^+} (1 - \tan^{-1}(2x))^{1/x} =$

- (a) e
- (b) $-e$
- (c) e^{-2}
- (d) e^{-1}
- (e) \sqrt{e}

$(1 - \tan^{-1}(0))^{\frac{1}{0}}$

$(1 - 0)^{\infty} = 1^{\infty}$ indetermination

$\Rightarrow \lim_{x \rightarrow 0^+} e^{\frac{1}{x} \ln(1 - \tan^{-1}(2x))}$

$= e^{\lim_{x \rightarrow 0^+} \frac{\ln(1 - \tan^{-1}(2x))}{x}}$

$= e^{\lim_{x \rightarrow 0^+} \frac{\frac{-2}{1+4x^2}}{1 - \tan^{-1}(2x)}}$

$= e^{\frac{-2}{2}} = e^{-1}$

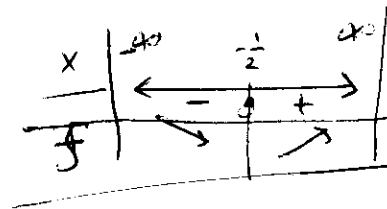
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✓ 25. The function $f(x) = xe^{2x}$

- (a) is increasing on $(-1, +\infty)$
- (b) is increasing on $(-\infty, -\frac{1}{2})$
- (c) has a local maximum at $x = -\frac{1}{2}$
- (d) has a local minimum at $x = -1$
- (e) is increasing on $(-\frac{1}{2}, +\infty)$

$$f'(x) = e^{2x} + 2xe^{2x} = e^{2x}(1+2x) = 0$$

$$1 + 2x = 0 \Rightarrow x = -\frac{1}{2}$$



✓ 26. If M and m are respectively the absolute maximum and absolute minimum values of $f(x) = x + 2 \cos x$ on $[0, \frac{\pi}{3}]$, then $3M - \sqrt{3}m =$

- (a) $\pi + 3\sqrt{3}$
- (b) $\frac{\pi}{3} + 2$
- (c) $2\pi + \sqrt{3}$
- (d) 2
- (e) $\frac{\pi}{2} + \sqrt{3}$

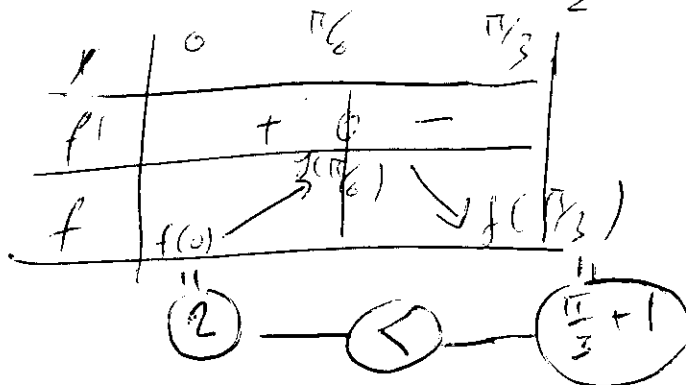
$$f' = 1 - 2 \sin x = 0$$

$$\sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}$$

$$f(0) = 2 \text{ (min)}$$

$$f\left(\frac{\pi}{6}\right) = \frac{\pi}{6} + 2 \cos \frac{\pi}{6} = \frac{\pi}{6} + \sqrt{3} \text{ (max)}$$

$$\Rightarrow 3M - \sqrt{3}m = 3\left(\frac{\pi}{6} + \sqrt{3}\right) - (2\sqrt{3}) = \frac{\pi}{2} + \sqrt{3}$$



$$M = f\left(\frac{\pi}{6}\right)$$

$$m = f\left(\frac{\pi}{3}\right)$$

27. The value(s) of c satisfying the conclusion of the Mean Value Theorem for $f(x) = \frac{x}{x+2}$ on $[1, 4]$ is(are)

- (a) 4
- (b) $-2 \pm 3\sqrt{2}$
- (c) $-2 - 3\sqrt{2}$
- (d) $-2 + 3\sqrt{2}$
- (e) 1, 2

$f(x)$ is cont
 $f(x)$ is diff

$$\frac{f(4) - f(1)}{4 - 1} = f'(c)$$

$$= \frac{\frac{4}{4+2} - \frac{1}{1+2}}{3} = \frac{\frac{4}{6} - \frac{1}{3}}{3}$$

$$= \frac{\frac{2}{6}}{3} = \frac{2}{18} = \frac{1}{9}$$

$$\Rightarrow \frac{1}{9} = \frac{x+2 - x}{(x+2)^2} = \frac{2}{x^2 + 4x + 4}$$

$$x^2 + 4x + 4 - 18 = 0$$

$$x^2 + 4x - 14 = 0$$

~~$x = -2 - 3\sqrt{2}$~~ $x = -2 + 3\sqrt{2}$

28. $\lim_{x \rightarrow 0} \frac{\cos(mx) - \cos(nx)}{x^2} = \frac{0}{0}$ (m and n are constants)

- (a) does not exist

$$= \lim_{x \rightarrow 0} \frac{-\sin(mx)m + \sin(nx)n}{2x} \stackrel{\frac{0}{0}}{=} \frac{-\cos(mx)m^2 + \cos(nx)n^2}{2}$$

- (b) 1

$$= \frac{-(1)m^2 + (1)n^2}{2} = \frac{-m^2 + n^2}{2}$$

- (c) $\frac{1}{2}(n^2 - m^2)$

- (d) 0

- (e) $n^2 + m^2$

L'Hospital twice