1. The graph of $f(x) = \dfrac{x^4 + x^2}{(x^2+9)(x+1)}$ has

(a) one slant asymptote and one vertical asymptote

(b) two slant asymptotes and two vertical asymptotes

(c) one horizontal asymptote and two vertical asymptotes

(d) one slant asymptote and three vertical asymptotes

(e) one horizontal, one vertical, and one slant asymptote

$(x^2+9)(x+1) = 0$

$x = -1$, $x^2 = -9$ rejected

one vertical

$\lim_{\infty} \dfrac{x^4+x^2}{(x^2+9)(x+1)} = \lim_{\infty} \dfrac{x^4}{x^3} = x = \infty$

no H.A.

Slant Asym

$f = \frac{4}{3}$ with $\deg = \deg + 1$

2. The function $f(x) = \dfrac{\ln(x-1)}{2-\sqrt{x}}$ is continuous on

(a) $(1,4) \cup (4,+\infty)$

(b) $[0,+\infty)$

(c) $(\sqrt{2},+\infty)$

(d) $(2,+\infty)$

(e) $(1,+\infty)$

$\ln(x-1)$: $x > 1$

$2-\sqrt{x}$: $x \geq 0$, $x \neq 4$

$D = (1,4) \cup (4,\infty)$

0 1 4

not included

3. Using Newton's Method to find a root of the equation

$$3x - \sin(2\pi x) = 1$$

starting with $x_1 = \frac{1}{2}$, we find that $x_2 =$

not included

$f(x) = 3x - \sin(2\pi x) - 1$

$f(\frac{1}{2}) = \frac{3}{2} - 0 - 1 = \frac{1}{2}$

$f'(x) = 3 - 2\pi \cos 2\pi x$

$f'(\frac{1}{2}) = 3 - 2\pi \cos\pi$

$= 3 + 2\pi$

$x_2 = \frac{1}{2} - \frac{\frac{1}{2}}{3+2\pi} = \frac{2(\pi+1)}{2(3+2\pi)}$

(a) $\frac{\pi+1}{2\pi+3}$

(b) $2\pi - 1$

(c) $\frac{1}{4\pi+6}$

(d) $\frac{1}{2\pi+1}$

(e) $\frac{1}{2}$

4. $\lim_{x\to5^+} \frac{4-x}{(x-5)^3}$ $= \lim_{5^+} \frac{4-5}{(5-5)^3} = \frac{-1}{0^+} = -\infty$

(a) $\frac{4}{5}$

(b) -1

(c) $\frac{1}{2}$

(d) $+\infty$

(e) $-\infty$

5. The number of the inflection points of the graph of $y = \frac{1}{56}x^8 - \frac{1}{30}x^6 + 80$ is

$y' = \frac{1}{7}x^7 - \frac{1}{5}x^5$

$y'' = x^6 - x^4 = 0$

$x^4(x^2-1) = 0$

$x = 0,\ x = 1,\ x = -1$

(a) 4

(b) 6

(c) 3

(d) 2

(e) 1

x		-1		0		1	
F''	+		-		-		+
f	∪		∩		∩		∪
		IP				IP	

6. The slope of the tangent line to the curve $\cos(xy^2) = y^3 - x + \frac{\pi}{2} - 1$ at $\left(\frac{\pi}{2}, 1\right)$ is

$-\sin(xy^2) \cdot (y^2 + x\,2yy') = 3y^2y' - 1$

$-\sin(xy^2)y^2 + -\sin(xy^2)x\,2yy' = 3y^2y' - 1$

$1 - \sin(xy^2)y^2 = 3y^2y' + \sin(xy^2)x\,2yy'$

$y' = \dfrac{y^2\sin(xy^2) - 1}{-2xy\sin(xy^2) - 3y^2}$

$y' = \dfrac{(1)\sin(\frac{\pi}{2}) - 1}{-2(\frac{\pi}{2})\sin(\frac{\pi}{2}) - 3} = \dfrac{0}{-\pi - 3} = 0$

(a) $-\frac{1}{3}$

(b) $\frac{1}{3}$

(c) 1

(d) 0

(e) -1

7. The sum of the critical numbers of the function $f(x) = \sqrt[3]{x^2 - x}$ is

D: [redacted] all real number

(a) 1

$f'(x) = \frac{1}{3}(x^2 - x)^{\frac{-2}{3}}(2x-1) = 0$

(b) $\frac{3}{2}$

$x = \frac{1}{2}$, $x(x-1) = 0$

$x = 0$, $x = 1$

(c) $-\frac{1}{2}$

thsum = $1 + \frac{1}{2} + 0 = \frac{3}{2}$

(d) $\frac{1}{2}$

(e) 2

8. **All values** of x where the tangent line to the graph of $y = \tan^2 x$ is horizontal are given by

$f'(x) = 2\tan x\,(1 + \tan^2 x) = 0$

(a) $n\pi$, n is integer

$\tan x = 0$, $\tan^2 x = -1$

$x = n\pi$ rejected

(b) $\frac{2n-1}{2}\pi$, n is integer

(c) $\frac{n}{2}\pi$, n is integer

(d) $(2n-1)\pi$, n is integer

(e) $\left(n + \frac{1}{2}\right)\pi$, n is integer

9. If g is a differentiable function and $f(x) = [g(x^2)]^2$, then $f'(x) =$

$f'(x) = 2\,g(x^2) \cdot g'(x^2) \cdot 2x$

$= 4x\,g(x^2)\,g'(x^2)$

(a) $2g'(x^2)$

(b) $4xg'(x^2)$

(c) $4xg(x^2)g'(x^2)$

(d) $4x^3g'(x^4)$

(e) $2g(x^2)$

10. If $f(t) = te^t \sin t$, then $f'(t) =$

$f'(t) = (e^t + te^t)\sin t + te^t \cdot \cos t$

$= (1+t)e^t \sin t + te^t \cos t$

(a) $e^t \cos t + t \sin t$

(b) $te^t \sin t - te^t \cos t$

(c) $e^t \cos t$

(d) $te^t \cos t + (t+1)e^t \sin t$

(e) $te^t \cos t + te^t \sin t$

X

11. An equation of the tangent line to the curve $y = x^{(2^x)}$ at the point $(1,1)$ is

$y = x^{2x}$

$\ln y = 2x \ln x$

$\frac{y'}{y} = 2\ln x + \frac{2x}{x}$

$y' = y(2\ln x + 2)$

at $(1,1)$ $y' = 1(2\ln 1 + 2) = 2 =$ slop

(a) $y = \frac{1}{2}x + \frac{1}{2}$

(b) $y = 3x - 2$

(c) $y = \frac{1}{3}x + \frac{2}{3}$

(d) $y = 2x - 1$

(e) $y = x$

so $\boxed{y'(1,1) = 2}$

$y = x^{2^x}$

$\ln y = 2^x \ln x$

$\frac{y'}{y} = (2^x)' \ln x + 2^x (\ln x)'$

$= (\ln 2) 2^x \ln x + \frac{2^x}{x}$

$\boxed{y' = x^{2^x}\left[(\ln 2) 2^x \ln x + \frac{2^x}{x}\right]}$

12. If $f''(x) = -3x^{-2}$, $f'(3) = 2$, $f(1) = -1$, then $f(e) =$

not included

$f'(x) = -3x^{-1} + C_1 = \frac{3}{x} + C_1$

$f'(3) = -1 + C_1 = 2$

$C = 1$

$f(x) = 3\ln x + x + C_2$

$f(1) = 3\ln 1 + 1 + C_2 = -1$

$C_2 = -2$

$f(e) = 3\ln e + e - 2$

$= e + 1$

(a) $e - 3$

(b) $\frac{3}{e} + 1$

(c) $\frac{-3}{e^2}$

(d) 0

(e) $e + 1$

13. If $f(x) = e^{1-2x}$, then $f^{(n)}(x) =$

(a) $(-2)^n e^{1-2x}$

(b) e^{1-2x}

(c) $2^n e^{1-2x}$

(d) $(-1)^n e^{1-2x}$

(e) $(1-2x)^n e^{1-2x}$

$f' = -2e^{1-2x}$

$f'' = 4e^{1-2x}$

$f''' = -8e^{1-2x}$

$f^{n} = (-2)^n e^{1-2x}$

14. Sand is being dumped from a truck at a rate of 0.5 ft^3/min to form a pile in the shape of a cone whose height is always equal to the diameter of its base. When the pile is 2 ft high, the height of the pile is increasing at a rate of [The volume of a cone is $V = \frac{1}{3}\pi r^2 h$]

(a) $\frac{1}{2\pi}$ ft/min

(b) $\frac{\pi}{2}$ ft/min

(c) $\frac{2}{\pi}$ ft/min

(d) 2π ft/min

(e) $\frac{1}{2}$ ft/min

$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$

$= \frac{1}{3}\pi \frac{h^3}{4}$

$\frac{dV}{dt} = \frac{1}{3}\pi \cdot \frac{1}{4} \cdot 3h^2 \cdot \frac{dh}{dt} \Rightarrow \boxed{\frac{dh}{dt} = \frac{4}{\pi h^2}\frac{dV}{dt}}$

$\frac{dh}{dt} = \frac{0.5 \cdot \cancel{12}}{\cancel{12}\pi} = \frac{0.5}{\pi} = \frac{1}{2\pi}$ ft/min

15. If

$$f(x) = \begin{cases} x^2+1 & \text{if } x<1 \\ ax+b & \text{if } 1 \le x < 3 \\ 3-2x & \text{if } x \ge 3 \end{cases}$$

is continuous on $(-\infty,+\infty)$, then $f(2) =$

(a) 5

(b) $\frac{7}{2}$

(c) $-\frac{1}{2}$

(d) 2

(e) $-\frac{5}{2}$

$\lim_{1} f(x) = f(1)$

$x^2+1 = ax+b$

$b = 2-a$

$\lim_{3} f(x) = f(3)$

$3a+b = 3-6$

$b = -3-3a$

$\Rightarrow 2-a = -3-3a$

$a = \frac{-5}{2}, \quad b = 2+\frac{5}{2} = \frac{9}{2}$

$\Rightarrow f(2) = 2\left(\frac{-5}{2}\right)+\frac{9}{2} = -\frac{1}{2}$

$\lim_{1^-} f(x) = \lim_{1^+} f(x) = f(1)$

$\boxed{2 = a+b}$

$\lim_{3^-} f(x) = \lim_{3^+} f(x) = f(3)$

$\boxed{3a+b = -3}$

$\Rightarrow a = \frac{-5}{2}, \quad b = \frac{9}{2}$

$f(2) = \frac{-5}{2}(2) + \frac{9}{2} = -\frac{1}{2}$

16. If $x^6+y^6=1$, then $y'' =$

implicitly

(a) $-\frac{5x^4}{y^{11}}$

(b) $-\frac{x^5}{y^5}$

(c) $\frac{10x^4}{y^{10}}$

(d) $\frac{x^6+1}{y^5}$

(e) $\frac{-1}{y^6}$

$6x^5 + 6y^5y' = 0$

$y' = \frac{-\cancel{6}x^5}{\cancel{6}y^5}$

$y'' = \frac{-5x^4y^5 + x^5\,5y^4y'}{y^{10}}$

$= \frac{-5x^4y^5 - 5x^5 y^4 \frac{x^5}{y^5}}{y^{10}}$

$= \frac{\frac{-5x^4y^6 - 5x^{10}}{y}}{y^{10}} = \frac{-5x^4(y^6+x^6)}{y^{11}}$

$= 1$

$= \frac{-5x^4}{y^{11}}$

X 17. Let $f(x) = e^x + \sin x$. Using the linear approximation of f at $a = 0$, we find that $f(0.1) \approx$

$f'(x) = e^x + \cos x$

(a) 1.2

(b) 1

(c) 2.2

(d) 2

(e) 1.5

$f(x) \underset{0}{\approx} f(a) + f'(a)(x-a)$

$= e^0 + \sin 0 + (e^0 + \cos(0))(x-0)$

$= 1 + 0 + (1+1)(x)$

$f(0.1) = 1 + \cancel{2} = 1.2$

$f(x) \underset{0}{\approx} f(0) + f'(0)(x-0) = \boxed{f(0) + f'(0)x}$

$\boxed{f(0) = 1}$, $\boxed{f'(0) = 2}$

$\boxed{f(x) \underset{0}{\approx} 1 + 2x} \Rightarrow f(0.1) \approx 1.2$

X 18. Which one of the following statements is **TRUE**?

not included

(a) if $f'(x) = g'(x)$ for all x, then $f(x) = g(x)$ for all x. X

(b) If f is continuous at a, then f is differentiable at a. X

(c) If $f(x) = \pi^4$, then $f'(x) = 4\pi^3$. X

(d) if $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at c.

(e) If f has an absolute minimum value at c, then $f'(c) = 0$. X

19. If $\tanh x = -\frac{2}{3}$, then $\cosh x =$

(a) $\pm\frac{3}{\sqrt{5}}$

(b) $\frac{3}{\sqrt{5}}$

(c) $-\frac{1}{\sqrt{5}}$

(d) $\pm\frac{1}{\sqrt{5}}$

(e) $-\frac{3}{\sqrt{5}}$

$$\operatorname{sech}^2 x = 1 - \tanh^2 x = 1 - \left(\frac{-2}{3}\right)^2 = 1 - \frac{4}{9} = \frac{5}{9}$$

$$\cosh x = \frac{1}{\operatorname{sech} x} = \frac{1}{+\sqrt{\frac{5}{9}}} = \frac{3}{+\sqrt{5}}.$$

always positive

20. If $G(x) = \frac{1+\sinh x}{1+\cosh x}$, then $G(0) + G'(0) =$

(a) 1/4

(b) 0

(c) 3/4

(d) 2

(e) 1

$$G'(x) = \frac{\cosh x + \cosh^2 x - \sinh x - \sinh^2 x}{(1+\cosh x)^2}$$

$$G(0) + G'(0) = \frac{1 + \frac{e^0 - e^0}{2}}{1 + \frac{e^0 + e^{-0}}{2}} + \frac{1+1-0-0}{(1+1)^2}$$

$$= \frac{1}{2} + \frac{1}{2} = 1$$

~~21.~~ If the point (x, y) lying on the line $y + 3x = 3$ is **the closest** point to the origin, then $x + 2y =$

not included

$y = -3x + 3$

$d^2 = x^2 + y^2$

$= x^2 + (-3x+3)^2 = x^2 + 9x^2 - 18x + 9$

$0 = 10x^2 - 18x + 9$

$0 = 20x - 18 \Rightarrow x = \frac{9}{10}$

$y = -3 \cdot \frac{9}{10} + 3 = \frac{3}{10}$

$\Rightarrow x + 2y = \frac{9}{10} + \frac{6}{10} = \frac{15}{10} = \frac{3}{2}$

(a) $\frac{6}{5}$

(b) $\frac{3}{5}$

(c) $\frac{3}{2}$

(d) 2

(e) 3

22. The graph of $f(x) = \sqrt[3]{x}\,(2 - x)$

(a) is concave down on the interval $(-\infty, 0)$

(b) is concave up on the intervals $(-\infty, -1)$ and $(0, 1)$

(c) has one inflection point only

(d) has an inflection point at $x = 1$

(e) is concave up on the interval $(-1, 0)$

$f'(x) = \frac{2}{3}x^{-\frac{2}{3}} - \frac{4}{3}x^{\frac{1}{3}}$

$f''(x) = \frac{-4}{9}x^{\frac{-5}{3}} - \frac{4}{9}x^{\frac{-2}{3}} = 0$

$\frac{-4}{9}x^{\frac{-2}{3}}\left(x^{-1} + 1\right) = 0$

$= \frac{1+x}{x}$

$x = 0 \;,\; x = -1$

x	-1	0
f''	$+$	$-$
f	concave up	concave down

-1 0

$+$ $+$ $-$

23. If an equation of the tangent line to the curve $y = e^x$ that is parallel to the line $x - 4y = 1$ is given by $y = ax + b$, then $4(b - a) =$

(a) $\ln 4$

(b) 1

(c) 0

(d) $1 + \ln 4$

(e) $2 - \ln 4$

$-4y = 1 - x$

$y = \frac{1-x}{-4}$

$\boxed{y = \frac{x}{4} - \frac{1}{4}}$

$(e^x)' = \frac{1}{4} \leftarrow$ slope $\Rightarrow$ $x = \ln \frac{1}{4}$, $y = \frac{1}{4}$

$y - y_0 = \frac{1}{4}(x - x_0)$

$y - \frac{1}{4} = \frac{1}{4}\left(x - \ln\frac{1}{4}\right)$

$y = \frac{1}{4}x - \frac{1}{4}\ln\frac{1}{4} + \frac{1}{4}$ b

a

$\Rightarrow 4(b-a) = 4\left(\frac{-1}{4}\ln\frac{1}{4} + \cancel{\frac{1}{4}} - \cancel{\frac{1}{4}}\right) = -\ln\frac{1}{4}$

$= -(\ln 1 - \ln 4)$

$= 0 + \ln 4$

$a = \frac{1}{4}$

$\Rightarrow y = \frac{1}{4}x + b$

$\frac{1}{4} = \frac{1}{4}\ln\frac{1}{4} + b$

$4b = 1 - \ln\frac{1}{4}$

$4a = 1$

$4b - 4a = -\ln\frac{1}{4}$

$= \ln 4$

24. $\lim_{x \to 0^+} (1 - \tan^{-1}(2x))^{1/x} =$

(a) e

(b) $-e$

(c) e^{-2}

(d) e^{-1}

(e) $\sqrt{e}$

$(1 - \tan^{-1}(0))^{\frac{1}{0}}$

$(1-0)^{\infty} = 1^{\infty}$ indetermenation

$\Rightarrow \lim_{0^+} e^{\frac{1}{x}\ln(1 - \tan^{-1}(2x))}$

$= e^{\lim_{0^+} \frac{\ln(1 - \tan^{-1} 2x)}{x}}$ $\left(\frac{0}{0}\right)$ L'Hospital

$= e^{\lim \frac{\left(\frac{-2}{1+4x^2}\right)}{\frac{1 - \tan^{-1}(2x)}{1}}} = e^{\frac{-2}{1}} = \boxed{e^{-2}}$

25. The function $f(x) = xe^{2x}$

(a) is increasing on $(-1, +\infty)$

(b) is increasing on $\left(-\infty, -\frac{1}{2}\right)$

(c) has a local maximum at $x = -\frac{1}{2}$

(d) has a local minimum at $x = -1$

(e) is increasing on $\left(-\frac{1}{2}, +\infty\right)$

$f'(x) = e^{2x} + 2xe^{2x}$

$= e^{2x}(1+2x) = 0$

$1 + 2x = 0$

$x = -\frac{1}{2}$

x	$-\infty$		$-\frac{1}{2}$		∞
f'		$-$	0	$+$	
f		$\searrow$		$\nearrow$	

26. If M and m are respectively the absolute maximum and absolute minimum values of $f(x) = x + 2\cos x$ on $\left[0, \frac{\pi}{3}\right]$, then $3M - \sqrt{3}m =$

(a) $\pi + 3\sqrt{3}$

(b) $\frac{\pi}{3} + 2$

(c) $2\pi + \sqrt{3}$

(d) 2

(e) $\frac{\pi}{2} + \sqrt{3}$

$f' = 1 - 2\sin x = 0$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}$

$f(0) = 2$ (min)

$f\left(\frac{\pi}{6}\right) = \frac{\pi}{6} + 2\cos\frac{\pi}{6} = \frac{\pi}{6} + \sqrt{3}$ (max)

$\Rightarrow 3M - \sqrt{3}m = 3\left(\frac{\pi}{6} + \sqrt{3}\right) - (2\sqrt{3})$

$= \frac{\pi}{2} + \sqrt{3}$

x	0		$\frac{\pi}{6}$		$\frac{\pi}{3}$
f'		$+$	0	$-$	
f	$f(0)$	$\nearrow$	$f\left(\frac{\pi}{6}\right)$	$\searrow$	$f\left(\frac{\pi}{3}\right)$

$2 < \frac{\pi}{3} + 1$

$M = f\left(\frac{\pi}{6}\right)$

$m = f(0)$

27. The value(s) of c satisfying the conclusion of the Mean Value Theorem for $f(x) = \frac{x}{x+2}$ on $[1, 4]$ is(are)

$f(x)$ is cont
$f(x)$ is diff

$\frac{f(4) - f(1)}{4-1} = f'(c)$

$= \frac{\frac{4}{4+2} - \frac{1}{1+2}}{3} = \frac{\frac{4}{6} - \frac{1}{3}}{3}$

$= \frac{\frac{2}{6}}{3} = \frac{2}{18} = \frac{1}{9}$

$\Rightarrow \frac{1}{9} = \frac{x+2-x}{(x+2)^2} = \frac{2}{x^2+4x+4}$

$x^2+4x+4-18=0$

$x^2+4x-14=0$

$\cancel{x=-2-3\sqrt{2}}$, $x=-2+3\sqrt{2}$

(a) 4

(b) $-2 \pm 3\sqrt{2}$

(c) $-2 - 3\sqrt{2}$

(d) $-2 + 3\sqrt{2}$

(e) 1, 2

28. $\lim_{x \to 0} \frac{\cos(mx) - \cos(nx)}{x^2} \overset{0/0}{=}$ (m and n are constants)

$= \lim_{0} \frac{-\sin(mx)m + \sin(nx)n}{2x} \overset{0/0}{=} \frac{-\cos(mx)m^2 + \cos(nx)n^2}{2}$

$= \frac{-(1)m^2 + (1)n^2}{2} = \frac{-m^2+n^2}{2}$

L'Hospital twice

(a) does not exist

(b) 1

(c) $\frac{1}{2}(n^2 - m^2)$

(d) 0

(e) $n^2 + m^2$