King Fahd University of Petroleum and Minerals
Math & stat. Departement
Quiz (3)

Name	ID	SEC	Sr

Q1) Evaluate $\int \sin(\ln x)\,dx$, By parts.

Let $u = \sin(\ln x)$, $dv = dx$, so, $du = \cos(\ln x)\frac{1}{x}dx$, $v = x$.

$$\int \sin(\ln x)dx = x\sin(\ln x) - \underbrace{\int x\cos(\ln x)\frac{1}{x}dx}$$

by parts, let $u = \cos(\ln x)$, $dv = dx$
$du = -\sin(\ln x)\frac{dx}{x}$, $v = x$

$$\int \sin(\ln x)dx = x\sin(\ln x) - \left[\cos(\ln x)x + \int \sin(\ln x)dx\right] \Rightarrow$$

$$\int \sin(\ln x)dx = \frac{1}{2}\left(x\sin(\ln x) - x\cos(\ln x)\right) + C.$$

Q2) find all numbers b such that the average value of $f(x) = \sqrt{x}$ on $[0, b]$ is 6.

$$f_{ave} = \frac{1}{b-0}\int_0^b \sqrt{x}\,dx = 6 \Rightarrow \frac{1}{b}\cdot\frac{2}{3}x^{3/2}\Big|_0^b = 6$$

$$\Rightarrow \frac{2}{3b}b^{3/2} = 6 \Rightarrow \frac{2\sqrt{b}}{3} = 6 \Rightarrow \sqrt{b} = 9 \Rightarrow \boxed{b = 81}$$

Q3) Evaluate $\int \csc^3 x\,dx$ Hint (you can use $\int \csc x\,dx = \ln|\csc x - \cot x| + c$)

$$\int \csc^3 x\,dx = \int \csc^2 x \csc x\,dx = \int (1+\cot^2 x)\csc x\,dx$$

$$(*) = \int \csc x\,dx + \underbrace{\int \cot^2 x \csc x\,dx},$$

by parts,

Let $u = \cot x$, $dv = \cot x \csc x\,dx$

$du = -\csc^2 x\,dx$, $v = -\csc x$.

Now,

$$\int \cot^2 x \csc x\,dx = -\csc x\cot x - \int \csc^3 x\,dx$$

$\therefore$, From $(*)$

$$\int \csc^3 x\,dx = \int \csc x\,dx - \csc x \cot x - \int \csc^3 x\,dx$$

$$\Downarrow$$

$$2\int \csc^3 x\,dx = \ln|\csc x - \cot x| - \csc x \cot x$$

$$\Downarrow$$

$$\int \csc^3 x\,dx = \frac{1}{2}\ln|\csc x - \cot x| - \frac{\csc x \cot x}{2} + C$$

Q4) Evaluate $\int_{2/5}^{4/5} \frac{\sqrt{25x^2-4}}{x}\,dx$, Let $5x = 2\sec\theta$, $\boxed{0 \le \theta < \frac{\pi}{2} \text{ or } \pi \le \theta < \frac{3\pi}{2}}$

$\Rightarrow x = \frac{2}{5}\sec\theta \Rightarrow dx = \frac{2}{5}\tan\theta\sec\theta\,d\theta.$

Now,

$$\int_{2/5}^{4/5} \frac{\sqrt{25x^2-4}}{x}\,dx = \int_0^{\frac{\pi}{3}} \frac{\sqrt{4\sec^2\theta-4}}{\cancel{\frac{2}{5}}\cancel{\sec\theta}} \cdot \cancel{\frac{2}{5}}\cancel{\sec\theta}\tan\theta\,d\theta$$

$$= \int_0^{\pi/3} 2\tan^2\theta\,d\theta = 2\int_0^{\frac{\pi}{2}} (\sec^2\theta - 1)\,d\theta$$

$$= 2\left(\tan\theta - \theta\right)\Big|_0^{\frac{\pi}{3}} = 2\left[\sqrt{3} - \frac{\pi}{3}\right] = 2\sqrt{3} - 2\frac{\pi}{3}.$$

Q5) Evaluate $\int \sin^5(3t)\cos^{4/5}(3t)\,dt$

The power of $\sin(3t)$ is odd, so let $u = \cos 3t$

$\Rightarrow u = -3\sin 3t\,dt$

$$\int \sin^5(3t)\cos^{4/5}(3t)\,dt = \int \sin^4(3t)\cos^{4/5}(3t)\frac{du}{-3} \quad ,$$

$$= \int (1-u^2)^2 u^{4/5}\frac{du}{-3} = -\frac{1}{3}\int (1-2u^2+u^4)\,u^{4/5}\,du$$

$$= -\frac{1}{3}\int \left(u^{4/5} - 2u^{\frac{14}{5}} + u^{\frac{24}{5}}\right)du$$

$$= -\frac{1}{3}\left(\frac{5}{9}u^{9/5} - 2\cdot\frac{5}{19}u^{\frac{19}{5}} + \frac{5}{29}u^{\frac{29}{5}}\right) + C$$

$$= -\frac{1}{3}\left(\frac{5}{9}\cos^{9/5}(3t) - \frac{10}{19}\cos^{\frac{19}{5}}(3t) + \frac{5}{29}\cos^{\frac{29}{5}}(3t)\right) + C$$