Quiz #3

Question (10 points total)

Show that

$$\lim_{(x,y)\to(0,0)} \frac{x^2 \sin^2 y}{x^2 + 2y^2} = 0.$$

Solution:

We will use the 'squeeze theorem'.

First note that, since $2y^2 \geq 0$, we have $x^2 \leq x^2 + 2y^2$ and so

$$0 \leq \frac{x^2}{x^2 + 2y^2} \leq 1$$

thus we have

$$0 \leq \frac{x^2 \sin^2 y}{x^2 + 2y^2} \leq \sin^2 y.$$

Finally, since $\sin^2 y \to 0$ as $(x, y) \to (0, 0)$ we can invoke the 'squeeze theorem' to say

$$\lim_{(x,y)\to(0,0)} \frac{x^2 \sin^2 y}{x^2 + 2y^2} = 0.$$

Note: Points will be deducted for incomplete or incorrect answers. Points will also be deducted for not fully or properly showing your work.