1. An equation of the plane containing the lines given by

$$\frac{x-1}{-2} = y - 4 = z \quad \text{and} \quad \frac{x-2}{-3} = \frac{y-1}{4} = \frac{z-2}{-1}$$

is

(a) $x + y + z = 5$

(b) $-2x + y - z = 2$

(c) $-3x + 4y - z = 13$

(d) $-z - 4y + x = -15$

(e) $-2x - y + 2z = -6$

2. The distance between the planes $x - y + 2z = 4$ and $3x - 3y + 6z = 10$ is equal to

(a) $\dfrac{2}{3\sqrt{6}}$

(b) $\dfrac{2}{\sqrt{6}}$

(c) $\dfrac{2}{3}$

(d) $\dfrac{5}{2\sqrt{6}}$

(e) $\dfrac{3}{2\sqrt{6}}$

3. The equation

$$4z^2 - 4x^2 - y^2 = 0$$

represents

(a) An elliptic cone

(b) An elliptic paraboloid

(c) A hyperbolic paraboloid

(d) A hyperboloid of one sheet

(e) A hyperboloid of two sheets

4. The domain of the function

$$f(x, y) = \arcsin(x + y)$$

is

(a) An intersection of two (half) planes

(b) A line

(c) An intersection of two lines

(d) The whole xy-plane

(e) An intersection of a line and a plane

5. The range of the function

$$f(x,y) = \frac{2 + \sqrt{4 - x^2 - y^2}}{e^{x^2+y^2}}$$

is

(a) $[2e^{-4}, 4]$

(b) $[e^{-4}, 2]$

(c) $[0, \infty)$

(d) $[0, 4]$

(e) $(-\infty, \infty)$

6. If $L = \lim\limits_{(x,y)\to(0,0)} \dfrac{1 - e^{x^2+y^2}}{x^2 + y^2}$, then

(a) $L = -1$

(b) L does not exist

(c) $L = 0$

(d) $L = e$

(e) $L = 1 - e$

7. $\dfrac{\partial^{100}}{\partial x^{95}\,\partial y^2\,\partial x^3}\,(ye^x x + \cos x)$ is equal to

(a) 0

(b) $100e^x - \sin x$

(c) $\dfrac{1}{100!}\,e^x - 95\cos x$

(d) $\dfrac{x}{95!}\,e^x - 2\sin x$

(e) $\dfrac{1}{95!} + \dfrac{1}{2!} + \dfrac{1}{3!}$

8. Let $f(x, y)$ be a differentiable function such that $f(1, 1) = 3$, $f_x(1, 1) = 2$, and $f_y(1, 1) = -1$. The best estimate for $f(1.1, 0.9)$ is

(a) 3.3

(b) 2

(c) 4.9

(d) 2.01

(e) 3.9

9. Let $z = x\sin(xy)$, and suppose that $x = e^{t^2}$, $y = 2t$. The value of $\left.\dfrac{dz}{dt}\right|_{t=0}$ is

(a) 2

(b) 0

(c) -1

(d) 4

(e) e

10. The critical points of the function

$$f(x, y) = y^3 + 3x^2 - 6xy - 15y + 6x$$

are

(a) $(2, 3)$ and $(-2, -1)$

(b) $(2, -3)$ and $(-2, 1)$

(c) $(-2, -3)$ and $(-2, 1)$

(d) $(-4, -3)$ and $(0, 1)$

(e) $(-2, -3)$ and $(2, -1)$

11. The function $f(x,y) = y^2x - x + \frac{1}{y}$ has critical points $C = \left(\frac{1}{2}, 1\right)$ and $D = \left(-\frac{1}{2}, -1\right)$. We have

(a) both C and D are saddle points for f

(b) f has a local minimum at C and a local maximum at D

(c) f has a local maximum at C and a local maximum at D

(d) C is a saddle point for f and f has a local maximum at D

(e) not enough information to determine the nature of the critical points C and D

12. Find the equation of the tangent plane to the surface given by

$$f(x,y) = \arctan\left(\frac{y}{x}\right)$$

at the point $\left(1, 1, \frac{\pi}{4}\right)$

(a) $x - y + 2z = \frac{\pi}{2}$

(b) $\frac{\pi}{4}x - y + 2z = \frac{5\pi}{4} - 1$

(c) $2x - 2y + z = \frac{\pi}{4}$

(d) $4z = x - y + \pi$

(e) $x - \frac{1}{2}y + 4z - \frac{1}{2} = \pi$

13. A normal vector to the level curve

$$\frac{x}{x^2+y^2}=\frac{1}{2}$$

at the point $(1,1)$ is

(a) $\vec{n}=-\frac{1}{2}\vec{j}$

(b) $\vec{n}=\frac{1}{2}\vec{i}+\vec{j}$

(c) $\vec{n}=\frac{1}{2}\vec{i}$

(d) $\vec{n}=\frac{1}{2}\vec{i}-\frac{1}{2}\vec{j}$

(e) $\vec{n}=2\vec{i}-\vec{j}$

14. Find the equation of the tangent plane to the level surface given by

$x^2+4y^2=169-9z^2$ at the point $(3,2,4)$.

(a) $3x+8y+36z=169$

(b) $3x+2y+4z=29$

(c) $9x+2y+3z=43$

(d) $x-y+9z=37$

(e) $8x+3y-4z=14$

15. If the derivative of $f(x, y)$ at $(1, 2)$ in the direction of $\langle 1, 1\rangle$ is equal to $2\sqrt{2}$ and the derivative of $f(x, y)$ at $(1, 2)$ in the direction of $\langle 0, -2\rangle$ is 3, then the derivative of f at $(1, 2)$ in the direction of $\langle -1, -2\rangle$ is

(a) $\dfrac{-1}{\sqrt{5}}$

(b) $-\dfrac{7}{\sqrt{5}}$

(c) $-4\sqrt{5}$

(d) $\dfrac{\sqrt{2}}{\sqrt{5}}$

(e) $-\dfrac{\sqrt{3}}{\sqrt{5}}$

16. The direction in which the function $f(x, y) = xye^{xy}$ increases most rapidly at $(1, 1)$ is given by the vector

(a) $\vec{u} = \langle 1, 1\rangle$

(b) $\vec{u} = \langle 2e, 0\rangle$

(c) $\vec{u} = \langle 1, 2e\rangle$

(d) $\vec{u} = \langle e, -1\rangle$

(e) $\vec{u} = \langle -1, -e\rangle$

17. The maximum rate of change of $f(x, y) = \sin(xy)$ at the point $(1, 0)$ is

(a) 1

(b) $\sin 2$

(c) $\sin^2 1$

(d) $\sqrt{\sin 1}$

(e) $\sqrt{\cos 1}$

18. The differential of the function

$$f(x, t) = e^{5t} \sin(3x) \text{ at } (x, t)$$

is

(a) $df(x, t) = 5e^{5t} \sin(3x)dt + 3e^{5t} \cos(3x)dx$

(b) $df(x, t) = \langle 5e^{5t} \sin(3x), 3e^{5t} \cos(3x)\rangle$

(c) $df(x, t) = \sqrt{[5e^{5t} \sin(3x)]^2 + [3e^{5t} \cos(3x)]^2}$

(d) $df(x, t) = 5xe^{5t} \sin(3x)dt + 3te^{5t} \cos(3x)dx$

(e) $df(x, t) = e^{5x} \sin 3x - e^{5t} \sin 3t$

19. If $f(x,y) = \int_y^x \cos(t^8)dt$, then $f_x\left(\sqrt[8]{\pi}, \frac{3\pi}{4}\right)$ is equal to

(a) -1

(b) $\sqrt[8]{\pi} - \frac{3\pi}{4}$

(c) π^8

(d) $8\pi^8$

(e) $\int_{\frac{3\pi}{4}}^{\sqrt[8]{\pi}} \cos(t^8)dt$

20. Which of the following statements is **FALSE** about the function f of (x,y) given by

$$f(x,y) = \begin{cases} \dfrac{x^2y^2}{x^4+y^4} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

(a) f is continuous at $(0,0)$

(b) $f(x,0)$ is continuous at $x=0$

(c) $f(0,y)$ is continuous at $y=0$

(d) $(-1,1)$ is in the domain of f

(e) $x^2y^2 = x^4+y^4$ is a level curve of f