King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Final Exam 101 in Finite Mathematics 131

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Formulas

Compound Interest:

$P :=$ Principal = Present Value
$S :=$ Compound Amount = Future Value
$r :=$ nominal rate = annual rate
$r_e :=$ effective rate
$n :=$ number of periods per year
$t :=$ number of years

$$S = P(1+r_e)^t = \begin{cases} P(1+\frac{r}{n})^{nt} & \text{if compounded } n \text{ times per year,} \\ Pe^{rt} & \text{if compounded continuously.} \end{cases}$$

$$1+r_e = \begin{cases} (1+\frac{r}{n})^{n} & \text{if compounded } n \text{ times per year,} \\ e^{r} & \text{if compounded continuously.} \end{cases}$$

Annuities:

$R :=$ payment per period
$A :=$ Present Value
$S :=$ Amount = Future Value
$r :=$ rate per period
$n :=$ number of periods

	Ordinary Annuity	Annuity Due
Pressent Value:	$A = R\dfrac{1-(1+r)^{-n}}{r}$	$A = R\left[\dfrac{1-(1+r)^{1-n}}{r}+1\right]$
Future Value:	$S = R\dfrac{(1+r)^{n}-1}{r}$	$S = R\left[\dfrac{(1+r)^{n+1}-1}{r}-1\right]$

Counting/Probability:

$${}_nP_r = \frac{n!}{(n-r)!} \qquad {}_nC_r = \binom{n}{r} = \frac{n!}{(n-r)!r!} \qquad P(A\cup B) = P(A)+P(B)-P(A\cap B) \qquad P(E\mid F) = \frac{P(E\cap F)}{P(F)}$$

For a Discrete Random Variable X and $f(x) := P(X=x)$, the mean and variance of X are:
$\mu = E(X) = \sum_{\text{all } x} x f(x)$ and $\sigma^2 = \text{Var}(X) = \sum_{\text{all } x}(x-\mu)^2 f(x) = \sum_{\text{all } x} x^2 f(x) - \mu^2 = E(X^2)-(E(X))^2$.

For a Binomial Random Variable X with probability p of Success in any of the n trails and $q := 1-p$
$P(X=x) = {}_nC_x\, p^x q^{n-x}$ for $x = 0,1,\dots,n$. In this case $\mu = E(X) = np$ and $\sigma^2 = \text{Var}(X) = npq$.

If a Continuous Random Variable X is normally distributed with mean μ and standard deviation σ, then $Z = \dfrac{X-\mu}{\sigma}$ has standard normal distribution.

Statistics:

	Sample	Number	Mean	Sample Standard Deviation
One by one	$x_1, x_2, \dots, x_n$	n = number of sample points	$\bar{x} := \dfrac{\sum_{\text{all i}} x_i}{n}$	$s := \sqrt{\dfrac{\sum_{\text{all i}} (x_i - \bar{x})^2}{n-1}} = \sqrt{\dfrac{\sum_{\text{all i}} x_i^2 - n\bar{x}^2}{n-1}}$
Grouped	each value or middle point x_i occurs f_i times	$n = \sum_{\text{all i}} f_i$	$\bar{x} := \dfrac{\sum_{\text{all i}} x_i f_i}{n}$	$s := \sqrt{\dfrac{\sum_{\text{all i}} (x_i - \bar{x})^2 f_i}{n-1}} = \sqrt{\dfrac{\sum_{\text{all i}} x_i^2 f_i - n\bar{x}^2}{n-1}}$

For the Population Standard Deviation σ replace each $n-1$ with n.

Q1 The incomes of industrial workers in a certain region are normally distributed with a mean of \$ 12500 and a standard deviation of \$ 1000. Find the probability that a randomly selected worker has an income between \$ 11000 and \$ 16000. If Z is a standard normal random variable then this probability can be written as:

(A) $P(Z \le 3.5) - P(-1.5 \le Z)$

(B) $P(Z \le 3.5) - P(-2.5 \le Z)$

(C) $P(Z \le 2.5) - P(-1.5 \le Z)$

(D) $P(Z \le 3.5) - P(Z \le -1.5)$ ←

(E) $P(Z \le 3.5) - P(Z \le -2.5)$

Q2 There are 3 stocks, A, B, and C. On a given day there is a 0.3 chance that stock A will go up in price, a 0.5 chance that stock B will go up in price, and a 0.2 chance that stock C will go up in price. Assume that the stocks behave independently of one another. Find the probability that exactly one of the stocks will go up in price.

(A) 0.64

(B) 0.33

(C) 0.47 ←

(D) 0.85

(E) 0.24

Q3 Television station call letters consist of either 3 or 4 letters and must begin with either K or with W. If there are no other restrictions and all 26 letters of the alphabet are allowed, how many possible call letters are there?

(A) 28800

(B) 1352

(C) 35152

(D) 475255

(E) 36504 ←

Q4 The following table gives the success record of a certain weather forecaster in predicting rain over a 700-day period.

	Predicted Rain	Predicted No Rain	Total
It rained	180	120	300
It did not rain	190	210	400
Total	370	330	700

One day from this period is selected at random. Find the probability that it rained on the selected day, given that the forecaster predicted rain.

(A) $\frac{18}{37}$ ← (B) $\frac{3}{5}$ (C) $\frac{4}{11}$ (D) $\frac{9}{35}$ (E) $\frac{37}{70}$

Q5 Find the dual approach to minimizing $Z = x_1 + 3x_2$ subject to

$$\begin{cases} x_1 - 2x_2 & \geq & 4 \\ 3x_1 + x_2 & \geq & 1 \\ x_1, \quad x_2 & \geq & 0 \end{cases}$$

(A) Maximize $W = y_1 + 3y_2$ subject to $y_1 + 3y_2 \geq 4$, $-2y_1 + y_2 \geq 1$, $y_1, y_2 \geq 0$.

(B) Maximize $W = y_1 + 3y_2$ subject to $y_1 - 2y_2 \leq 4$, $3y_1 + y_2 \leq 1$, $y_1, y_2 \geq 0$.

(C) Maximize $W = 4y_1 + y_2$ subject to $y_1 - 2y_2 \geq 1$, $3y_1 + y_2 \geq 3$, $y_1, y_2 \geq 0$.

(D) Maximize $W = 4y_1 + y_2$ subject to $y_1 + 3y_2 \leq 1$, $-2y_1 + y_2 \leq 3$, $y_1, y_2 \geq 0$. ←

(E) Maximize $W = 4y_1 + y_2$ subject to $y_1 + 3y_2 \geq 1$, $-2y_1 + y_2 \geq 3$, $y_1, y_2 \geq 0$.

Q6 A debt of \$ 2000 due in one year is to be repaid by a payment due two years from now and a final payment of \$ 1000 three years from now. If interest is at the rate of 4 % compounded annually, then the payment due in two years is

(A) \$ 1000.00

(B) \$ 1118.46 ←

(C) \$ 1155.43

(D) \$ 1191.00

(E) \$ 1203.14

Q7 Find the maximum value of $Z = x_1 - 2x_2 + 3x_3$ subject to

$$\begin{cases} 2x_1 + x_2 + 2x_3 & \leq & 10 \\ x_1 - x_2 + x_3 & \leq & 8 \\ x_1,\ x_2,\ x_3 & \geq & 0 \end{cases}$$

(A) 0

(B) 5

(C) 10

(D) 15 ←

(E) 20

Q8 A charity sells 1000 raffle tickets for $ 1 each. There is one grand prize of $ 250, two prizes of $ 75 each, and five prizes of $ 10 each. The following table gives the probability distribution for the net winnings X of someone who buys a single ticket. Find the expected winnings.

Possible net winning x	249	74	9	-1
Probability $\Pr(X = x)$	0.001	0.002	0.005	0.992

(A) – 0.55 ←

(B) 0.442

(C) 0.55

(D) – 0.442

(E) – 0.2431

Q9 Suppose that 10 % of the patients who have a certain disease die from it. If 5 patients have the disease, what is the probability that no more than 2 patients will die from it? (Hint: Binomial Distribution)

(A) 0.59049

(B) 0.99144 ←

(C) 0.32805

(D) 0.07290

(E) 0.85000

Q10 In a field study different shopkeepers were asked about the selling price of a certain product. The collected data is displayed below. Find the corresponding sample standard deviation.

Price in $	Frequency f_i	Middle Point x_i				
20 – 24	170					
25 – 29	230					
30 – 34	100					
35 – 39	35					
Total	535					

(A) 5.52

(B) 3.21

(C) 4.38 ←

(D) 9.42

(E) 7.94

Q11 Suppose $ 500 is initially placed in a savings account that earns interest at the rate of 8 % compounded semiannually. Thereafter, $ 500 is deposited in the account at the end of every six months for five years. The value of the account at the end of five years is

(A) $ 6743.18 ←

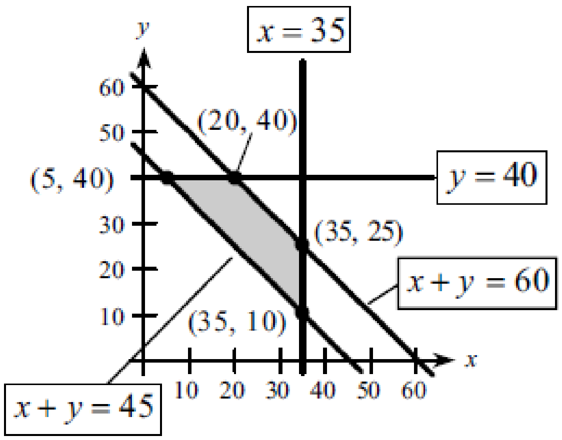
(B) $ 4555.45

(C) $ 6003.05

(D) $ 4055.45

(E) $ 6799.78

Q12 The region indicated in the diagram



is described by: (A) ←

(A) $\begin{cases} x+y \le 60 \\ x+y \ge 45 \\ x \le 35 \\ y \le 40 \end{cases}$ (B) $\begin{cases} x+y \le 45 \\ x \le 35 \\ y \le 40 \\ x, y \ge 40 \end{cases}$ (C) $\begin{cases} x+y \ge 60 \\ x+y \le 45 \\ x \le 35 \\ y \le 40 \end{cases}$ (D) $\begin{cases} x+y \le 60 \\ x+y \ge 45 \\ x \ge 35 \\ y \ge 40 \end{cases}$ (E) $\begin{cases} x+y \le 60 \\ x+y \ge 45 \\ x \ge 35 \\ y \le 40 \end{cases}$

Q13 A health clinic dietician is planning a meal consisting of three foods whose ingredients are summarized as follows:

	One Unit of Food I	One Unit of Food II	One Unit of Food III
Units of Protein	10	15	20
Units of Carbohydrates	1	2	1
Units of Iron	4	8	1
Calories	80	120	100

The dietician wishes to determine the number of units of each food to use to create a balanced meal containing at least 40 units of protein, at least 6 units of carbohydrates, and at least 12 units of iron, with as few calories C as possible. Let x, y, and z be the numbers of units of Food I, Food II and Food III, respectively. Then the dietician has to minimize $C = 80x + 120y + 100z$ subject to the constraints: (E) ←

(A) $\begin{cases} 10x+15y+20z \le 40 \\ 2x+y+z \le 6 \\ 8x+4y+z \le 12 \\ x \ge 0, \ y \ge 0, \ z \ge 0 \end{cases}$ (B) $\begin{cases} 10x+15y+20z \le 40 \\ x+2y+z \le 6 \\ 4x+8y+z \le 12 \\ x \ge 0, \ y \ge 0, \ z \ge 0 \end{cases}$ (C) $\begin{cases} 10x+15y+20z \ge 40 \\ x+2y+z \ge 12 \\ 4x+8y+z \ge 6 \\ x \ge 0, \ y \ge 0, \ z \ge 0 \end{cases}$

(D) $\begin{cases} 20x+15y+10z \le 40 \\ x+2y+z \le 6 \\ 4x+8y+z \le 12 \\ x \ge 0, \ y \ge 0, \ z \ge 0 \end{cases}$ (E) $\begin{cases} 10x+15y+20z \ge 40 \\ x+2y+z \ge 6 \\ 4x+8y+z \ge 12 \\ x \ge 0, \ y \ge 0, \ z \ge 0 \end{cases}$

Q14 A cookie company makes three kinds of cookies – *oatmeal raisin*, *chocolate chip*, and *shortbread* – packaged in *small*, *medium*, and *large* boxes. The small box contains 1 dozen oatmeal raisin and 1 dozen chocolate chip; the medium box has 2 dozen oatmeal raisin, 1 dozen chocolate chip, and 1 dozen shortbread; the large box contains 2 dozen oatmeal raisin, 2 dozen chocolate chip, and 3 dozen shortbread. If you require exactly 15 dozen oatmeal raisin, 10 dozen chocolate chip, and 11 dozen shortbread cookies, how many of each box size should you buy? Let x, y, and z be the number of small, medium and large boxes, respectively. Then the system of equations to solve the problem is: (C) ←

(A) $\begin{cases} x + y = 15 \\ 2x + y + z = 10 \\ 2x + 2y + 3z = 11 \end{cases}$ (B) $\begin{cases} x + 2y + 2z = 15 \\ y + 2z = 10 \\ x + y + 3z = 11 \end{cases}$ (C) $\begin{cases} x + 2y + 2z = 15 \\ x + y + 2z = 10 \\ y + 3z = 11 \end{cases}$

(D) $\begin{cases} x + 2y + 2z = 15 \\ x + y + 2z = 11 \\ 2y + 3z = 10 \end{cases}$ (E) $\begin{cases} x + 2y + 2z = 15 \\ 2x + y + 2z = 10 \\ y + 3z = 11 \end{cases}$

Q15 A teacher gives evening classes for adults. He wrote down the age of each participant in the following list: 91, 64, 23, 40, 32, 24, 50, 18, 34, 24.
Which of the following statements about this set of data is **false**?

(A) The median is 33.

(B) The mode is 24.

(C) The range is 73.

(D) The sample standard deviation is 22. ←

(E) The mean is 40.

Q16 Suppose a fair coin is tossed 100 times, use the normal approximation to estimate the probability of getting at least 60 heads. If Z is a standard normal variable this approach leads to:

(A) $P(Z \le 1.9)$

(B) $1 - P(Z \le 2.3)$

(C) $1 - P(Z \le 1.9)$ ←

(D) $P(Z \le 2.3)$

(E) $1 - P(Z \le 2)$

Q17 From a group of 10 men and 10 women, two people are randomly selected to form a committee. If X is the number of women on the committee, then $P(X = 2) =$

(A) $\frac{7}{380}$

(B) $\frac{9}{38}$ ←

(C) $\frac{1}{5}$

(D) $\frac{17}{190}$

(E) $\frac{17}{38}$

Q18 A farmer has 200 yards of fencing with which to enclose a rectangular field. One side of the field can make use of a fence that already exists. What is the maximum area that can be enclosed?

(A) 3000 square yards

(B) 4000 square yards

(C) 5000 square yards ←

(D) 8000 square yards

(E) 6000 square yards

Q19 A manufacturer sells his product \$ 23 per unit, selling all he produces. His fixed cost is \$ 20250 and his variable cost per unit is \$ 18.50. The level of production at which the manufacturer breaks even is

(A) 5500 units

(B) 6500 units

(C) 3500 units

(D) 4000 units

(E) 4500 units ←

Q20 A couple has 10 children. What is the probability that they have at least 2 boys? Take into account that boys are slightly more likely than girls. The odds are 13:12, that is, 13 out of every 25 children are male.

(A) At most 1 %.

(B) More than 1 % but not more than 35 %.

(C) More than 35 % but not more than 65 %.

(D) More than 65 % but not more than 99 %.

(E) More than 99 %. ←

Q21 How many ways are there to assign the grades A, B, C and D to the 17 students in a class? Assume that 2 have to get an A, 5 should get a B, 6 a C and 4 a D.

(A) 479,001,600

(B) 85,765,680 ←

(C) 144

(D) 245,260

(E) 78

Q22 Two dice are rolled. The probability that both show a six given that at least one shows a six is equal to:

(A) $\frac{2}{11}$

(B) $\frac{1}{6}$

(C) $\frac{1}{3}$

(D) $\frac{1}{11}$ ←

(E) $\frac{1}{12}$

Q23 A purse contains 5 coins of 50 halala, 3 coins of 25 halala and 2 coins of 10 halala. Two coins are randomly drawn in succession without replacement. What is the probability that the second coin is smaller than the first one?

(A) $\frac{15}{90}$

(B) $\frac{21}{90}$

(C) $\frac{25}{90}$

(D) $\frac{31}{90}$ ←

(E) $\frac{66}{90}$

Q24 From your math book with 448 pages 4 pages have been ripped out. Now you have to live with a 444 pages book. How many such "subbooks" are possible?

(A) ${}_{448}C_4$ ←

(B) ${}_{448}P_4$

(C) 448^4

(D) $448+447+446+445$

(E) 4×448

Q25 A system of equations has the following augmented coefficient matrix: $\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 4 & 3 & 3 \\ 0 & 0 & 2 & 4 & 4 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right]$

Which of the following statements about the solutions of the system of equations is true?

(A) There are no solutions. ←

(B) There is exactly one solution.

(C) There are exactly two solutions.

(D) There are infinitely many solutions that can be described with just one parameter.

(E) There are infinitely many solutions that require at least two parameters.

Q26 Dr. Studentcare produces solution manuals for future math exams. The costs to the book shop inside KFUPM are 150 SR per copy. As a convenience to the shop, Dr. Studentcare will print a tag price on each manual. What tag price should he print so that the shop may reduce this tag price by 10 % and still make a profit of 20 % on the costs?

(A) 180 SR

(B) 195 SR

(C) 200 SR ←

(D) 165 SR

(E) 214.29 SR

Q27 A system of equations has the following augmented coefficient matrix: $\left[\begin{array}{ccccc|c} 1 & 2 & 0 & 3 & 0 & 5 \\ 0 & 0 & 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$

Then all possible solutions of the system of equations can be described with parameters r, s, t as follows: ←(C)

(A) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 3 \\ 0 \\ 4 \end{bmatrix}$ (B) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 3 \\ 0 \\ 4 \end{bmatrix} + r\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + s\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} -3 \\ 0 \\ -2 \\ 1 \\ 0 \end{bmatrix}$ (C) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 3 \\ 0 \\ 4 \end{bmatrix} + r\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + s\begin{bmatrix} -3 \\ 0 \\ -2 \\ 1 \\ 0 \end{bmatrix}$ (D) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 3 \\ 0 \\ 4 \end{bmatrix} + r\begin{bmatrix} -5 \\ 1 \\ -2 \\ 1 \\ 0 \end{bmatrix}$

(E) None of the above.

Q28 Suppose the cost to produce 100 units of a product is SR 80 and the cost to produce 350 units is SR 140. If costs depend linearly on the production level, what are the costs to produce 5000 units?

(A) SR 4000

(B) SR 3524

(C) SR 2000

(D) SR 1256 ←

(E) SR 826

Q29 Solve the system:
$$x-3y^2=2$$
$$-2y=\sqrt{x+4y-2}$$

(A) There is no solution.

(B) There is only one solution (x_1, y_1), it fulfils $x_1 - y_1 = 2$. ←

(C) There is only one solution (x_1, y_1), it fulfils $x_1 - y_1 = 46$.

(D) There are two solutions (x_1, y_1) and (x_2, y_2), they fulfil $x_1 + x_2 + y_1 + y_2 = 56$.

(E) There are two solutions (x_1, y_1) and (x_2, y_2), they fulfil $x_1 + x_2 + y_1 + y_2 = 10$.

Q30 A trust fund for a newborn is being set up by a single payment so that after 20 years the child will receive 100,000 SR. If the fund earns interest at the rate of 10 % compounded semiannually, how much money should be paid in the fund now?

(A) 14864.36 SR

(B) 70399.89 SR

(C) 14204.57 SR ←

(D) 13533,53 SR

(E) 69314.72 SR

Q31 The cost for publishing one copy of a certain newspaper is 3.00 SR. The revenue from dealers is 2.80 SR per copy. The advertising revenue is 10 % of the revenue received from dealers for all copies sold beyond 10,000. What is the least number of copies that must be sold so as to have a profit?

(A) 25,501

(B) 50,001

(C) 10,770

(D) 70,001

(E) 35,001 ←

Q32 An investment doubles in value every year (every 365 days), based on a certain interest rate which is compounded continuously. How long does it take till the investment reaches 10 times its original value?

(A) about 986 days

(B) about 723 days

(C) about 1616 days

(D) about 1212 days ←

(E) about 1168 days

Q33 An urn contains 1 blue, 2 yellow, 3 green and 4 red marbles. One marble is randomly drawn and then put back into the urn. Now, again a marble is randomly drawn. What is the probability to get a marble of the same color as in the first trial?

(A) 0.333

(B) 0.300 ←

(C) 0.500

(D) 0.111

(E) 0.667

Q34 For a family with 3 children, examine the events $E = \{\text{at least one child of each gender}\}$ and $F = \{\text{at most one girl}\}$. Assuming that a child of either gender is equally likely, which of the following statements is true?

(A) E and F are complementary events.

(B) E and F are equally likely.

(C) E and F are mutually exclusive.

(D) E and F are independent. ←

(E) $P(E \cap F) = 0.25$.

Q35 Use the graphic method to maximize the function $Z = 2x_1 + x_2$ subject to the constrains

$$\begin{cases} 2x_1 + x_2 \le 4 \\ x_1 + 2x_2 \le 4 \\ x_1 + x_2 \ge 2 \\ x_1,\ x_2 \ge 0 \end{cases}$$

The maximum is obtained at the following points:

(A) $(x_1, x_2) = (2, 0)$

(B) $(x_1, x_2) = \left(\frac{4}{3}, \frac{4}{3}\right)$ and $(x_1, x_2) = (0, 4)$ and all points in between.

(C) $(x_1, x_2) = (4, 0)$

(D) $(x_1, x_2) = \left(\frac{4}{3}, \frac{4}{3}\right)$

(E) $(x_1, x_2) = \left(\frac{4}{3}, \frac{4}{3}\right)$ and $(x_1, x_2) = (2, 0)$ and all points in between. ←

Q36 A builder makes a certain type of concrete by mixing together 1 part Portland cement, 3 parts sand, and 5 parts crusted stone (by volume). If 54 cubic meters of concrete are needed, how many cubic meters sand are needed?

(A) 6

(B) 18 ←

(C) 30

(D) 9

(E) 15