Math 132 Quiz 5 Date 04/20/2011

Ex1: A cylindrical can, open at the top is to be made from a fixed amount of material K. If the volume is to be maximum, show that both the radius and height are equal to $\sqrt{\frac{K}{3\pi}}$

Ex2: Use differentials to approximate $\sqrt[3]{65.5}$

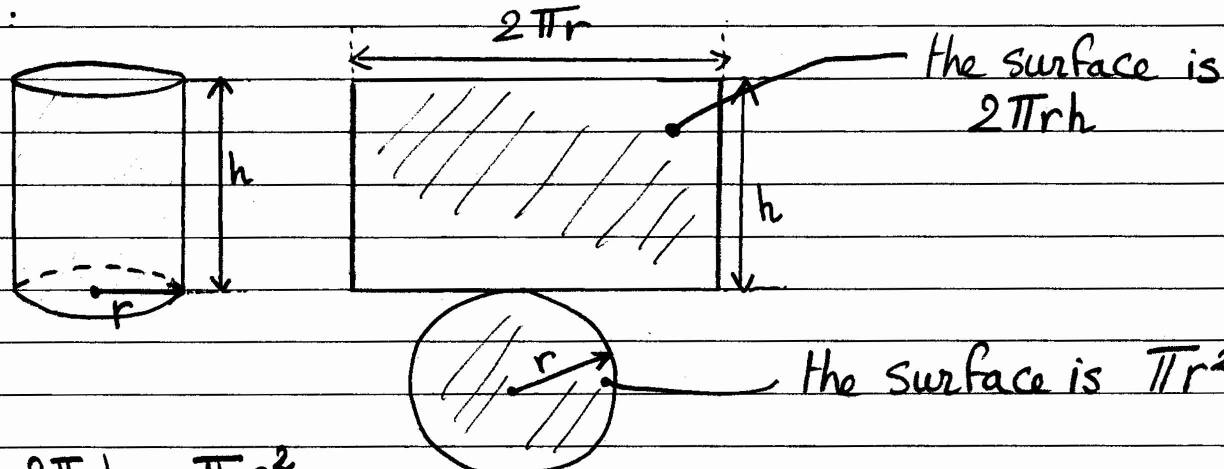
Ex3: Find

a) $I = \int \left(2\sqrt{x} - 3\sqrt[4]{x}\right) dx$

b) $J = \int v^{-2}\left(2v^4 + 3v^2 - 2v^{-3}\right) dv$

Solution

Ex1:



$$K = 2\pi rh + \pi r^2$$

$$= \pi r(2h + r), \text{ therefor } h = \frac{1}{2}\left(\frac{K}{\pi r} - r\right)$$

Now: $\text{Volume} = V(r) = \pi r^2 h = \pi r^2 \frac{1}{2}\left(\frac{K}{\pi r} - r\right) = \frac{\pi}{2}\left(\frac{Kr}{\pi} - r^3\right)$.

Thus $\frac{dV}{dr} = \frac{\pi}{2}\left(\frac{K}{\pi} - 3r^2\right)$. So $\frac{dV}{dr} = 0$ iff $3r^2 = \frac{K}{\pi}$

and so $\frac{dV}{dr}=0$ iff $r=\sqrt{\frac{K}{3\pi}}$. It is easy to see that this critical point is actually a maximum. When $r=\sqrt{\frac{K}{3\pi}}$, we have that

$$h=\frac{1}{2}\left(\frac{K}{\pi\sqrt{\frac{K}{3\pi}}}-\sqrt{\frac{K}{3\pi}}\right)=\frac{1}{2}\left(\frac{K-\pi\frac{K}{3\pi}}{\pi\sqrt{\frac{K}{3\pi}}}\right)$$

$$=\frac{1}{2}\left(\frac{K-\frac{K}{3}}{\pi\sqrt{\frac{K}{3\pi}}}\right)=\frac{K}{3\pi\sqrt{\frac{K}{3\pi}}}=\frac{K}{\sqrt{3\pi K}}=\sqrt{\frac{K}{3\pi}}$$

Ex2: Let $f(x)=\sqrt[3]{x}$. So $f'(x)=\frac{1}{3}x^{\frac{1}{3}-1}=\frac{1}{3x^{2/3}}$

$65.5=64+1.5$. So

$$f(65.5)\approx f(64)+f'(64)(1.5)$$

$$\approx\sqrt[3]{64}+\frac{1}{3(64)^{2/3}}\cdot\frac{3}{2}=4+\frac{3}{32}=\frac{131}{32}$$

Ex3:

a) $$I=\int\left(2x^{\frac{1}{2}}-3x^{\frac{1}{4}}\right)dx=2\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}-3\frac{x^{\frac{1}{4}+1}}{\frac{1}{4}+1}+C$$

$$=\frac{4}{3}x^{\frac{3}{2}}-\frac{12}{5}x^{\frac{5}{4}}+C$$

b) $$J=\int\left(2v^2+3-2v^{-5}\right)dv=\frac{2}{3}v^3+3v+\frac{1}{2}v^{-4}+C$$