King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 201

Exam II, 2011 (102)

Duration: 120 minutes

Name: ______________________________

Id: ______________________________

Section: ______________________________

Answer the questions in the space provided. You must show your work or explain your solution otherwise points may be deducted. If you make an unnecessary approximation in your solution to a problem, your answer will be judged on its accuracy. Points may be deducted for poor or inappropriate approximation.

1. Write clearly.
2. Show all your steps.
3. No credits will be given to wrong steps.
4. Calculators and mobile phones are NOT allowed in this exam.

Q#	**Marks**	**Maximum Marks**
1		**18**
2		**18**
3		**18**
4		**18**
5		**18**
6		**10**
Total		**100**

Problem 1. (18pts)

a) Find an equation in $\mathbf{x}, \mathbf{y}, \mathbf{z}$ for the plane that passes through the point $(\mathbf{1}, \mathbf{2}, -\mathbf{3})$ and is perpendicular to the line

$$\mathbf{x} = \mathbf{1} + \mathbf{2t}, \quad \mathbf{y} = \mathbf{2} + \mathbf{t}, \quad \mathbf{z} = -\mathbf{3} - \mathbf{5t}.$$

b) Find a set of parametric equations for the line that passes through $\mathbf{P}(-\mathbf{1}, \mathbf{2}, -\mathbf{3})$ and is perpendicular to the $\mathbf{xz}$−plane.

Problem 2. (18pts)

a) Find the distance from the point $\mathbf{P(2,5,0)}$ to the plane $\mathbf{x+z-7=0}$.

b) For each of the equations below, identify the graph as :

cylinder, hyperboloid of one sheet, cone, sphere, ellipsoid, hyperboloid of two sheets, elliptic paraboloid, hyperbolic paraboloid.

The graph of $\mathbf{2x^2-4x+y^2+2y-z^2+8=0}$ is a ______________

The graph of $\mathbf{x=z^2-y^2}$ is a______________

The graph of $\mathbf{z=4-y^2}$ is a______________

The graph of $\mathbf{y=x^2+z^2}$ is a______________

Problem 3. (18pts)

a) Find and sketch the domain of the function

$$\mathbf{f}(\mathbf{x},\mathbf{y}) = \ln(\mathbf{x}+\mathbf{y}-\mathbf{1})$$

b) Let $\mathbf{f}(\mathbf{x},\mathbf{y}) = \sqrt{\mathbf{16}-\mathbf{x}^2-\mathbf{y}^2}$. Draw a contour map of level curves $\mathbf{f}(\mathbf{x},\mathbf{y}) = \mathbf{k}$ with $\mathbf{k} = \mathbf{1}; \mathbf{2}; \mathbf{3}$. Label the level curves by the corresponding values of $\mathbf{k}$.

Problem 4. (18pts)

a) Consider the function

$$\mathbf{f}(\mathbf{x},\mathbf{y}) = \frac{\mathbf{7x^3y}}{\mathbf{2x^4+y^4}}.$$

Does the limit $\lim_{(\mathbf{x},\mathbf{y})\to(\mathbf{0},\mathbf{0})} \mathbf{f}(\mathbf{x},\mathbf{y})$ exist? Why or why not?

b) Let $\mathbf{f}$ be a function of two variables defined by

$$\mathbf{f}(\mathbf{x},\mathbf{y}) = \begin{cases} \frac{e^{\mathbf{x^2+y^2}}-\mathbf{1}}{\mathbf{x^2+y^2}} & \text{if } (\mathbf{x},\mathbf{y}) \neq (\mathbf{0},\mathbf{0}) \\ \mathbf{m} & \text{if } (\mathbf{x},\mathbf{y}) = (\mathbf{0},\mathbf{0}). \end{cases}$$

Find the value of $\mathbf{m}$ so that $\mathbf{f}$ is continuous at $(\mathbf{0},\mathbf{0})$.

Hint: you may use polar coordinates

Problem 5. (18pts)

a) If $\mathbf{f}(\mathbf{x},\mathbf{y}) = \mathbf{x}\mathbf{y}^2 + \sin(\mathbf{x}\mathbf{y})$, find $\dfrac{\partial \mathbf{f}}{\partial \mathbf{x}} + \dfrac{\partial^2 \mathbf{f}}{\partial \mathbf{x}\partial \mathbf{y}}$ at $(\mathbf{x},\mathbf{y}) = (\mathbf{0},\mathbf{1})$.

b) Find an equation of the tangent plane to the surface

$$\mathbf{z} = \sin^2(\mathbf{x}\mathbf{y})$$

at $(\mathbf{x},\mathbf{y}) = (\dfrac{\boldsymbol{\pi}}{\mathbf{4}},\mathbf{1})$.

Problem 6. (10pts)

Let $\mathbf{g(x,y) = ye^{x}}$. Estimate $\mathbf{g(0.1, 1.9)}$ using the linear approximation of $\mathbf{g(x,y)}$ at $\mathbf{(x,y) = (0,2)}$.