Math 132 Quiz 1 Date 17/09/2011

Ex1: Find the following limits:

a) $\lim_{t \to 0} \left(\frac{1}{t} - \frac{1}{t^2+t} \right)$; b) $\lim_{x \to 1} \frac{x^3-1}{x^2-1}$

c) $\lim_{x \to 0^-} \frac{1}{x} - \frac{1}{|x|}$

Ex2: Find the constant c for which the function

$$f(x) = \begin{cases} cx+1 & \text{if } x \leq 3 \\ cx^2-1 & \text{if } x > 3 \end{cases}$$ is continuous at 3.

Solution

Ex1: a) $\lim_{t \to 0} \left(\frac{1}{t} - \frac{1}{t^2+t} \right) = \lim_{t \to \infty} \frac{t^2+t-t}{t(t^2+t)} = \lim_{t \to 0} \frac{t^2}{t(t^2+t)}$

$= \lim_{t \to 0} \frac{1}{t+1} = 1$

b) $\lim_{x \to 1} \frac{x^3-1}{x^2-1} = \lim_{x \to 1} \frac{(x-1)(x^2+x+1)}{(x-1)(x+1)} = \lim_{x \to 1} \frac{x^2+x+1}{x+1} = \frac{3}{2}$

c) When $x \to 0^-$, x is less than 0 i.e $x < 0$ So $|x| = -x$

Thus $\lim_{x \to 0^-} \frac{1}{x} - \frac{1}{|x|} = \lim_{x \to 0^-} \frac{1}{x} - \frac{1}{-x} = \lim_{x \to 0^-} \frac{2}{x} = -\infty$

Ex2: $\lim_{x \to 3^-} f(x) = 3c+1$, $\lim_{x \to 3^+} f(x) = 9c-1$

$\lim_{x \to 3} f(x)$ exists if and only if $\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x)$

iff $3c + 1 = 9c - 1$

iff $6c = 2$ iff $c = \frac{2}{6} = \frac{1}{3}$.

Therefore when $c = \frac{1}{3}$, $\lim_{x \to 3} f(x)$ exists and $\lim_{x \to 3} f(x) = 2 = f(3)$, which implies that f is Continuous at 3.