Math 132 Quiz 6 Date 22/10/2011

Ex1: Find the absolute extrema of $f(x) = x^4 - 9x^2 + 2$ on the interval $[-1, 3]$.

Ex2 Let $f(x) = \dfrac{\ln x}{2x}$

a) Find $f''(x)$

b) Determine concavity and the points of inflection

Solution:

Ex1: •) Since f is a polynomial, it is continuous on the closed interval $[-1, 3]$.

•) $f'(x) = 4x^3 - 18x = 2x(2x^2 - 9)$. So $f'(x) = 0$ if and only if $x = 0$ or $x = \pm 3/\sqrt{2}$. Thus f has only two critical points inside the interval $[-1, 3]$. $\left(-\frac{3}{\sqrt{2}} \notin [-1, 3]\right)$.

•) $f(0) = 2$, $f(-1) = -6$, $f(3) = 2$, $f\left(\frac{3}{\sqrt{2}}\right) = \frac{81}{16} - \frac{81}{2} + 2$

Therefore: -1 is the absolute minimum

$\frac{3}{\sqrt{2}}$ is the absolute maxima

Ex2

a) $f'(x) = \frac{1}{2}\left[\frac{1 - \ln x}{x^2}\right]$, $f''(x) = \frac{1}{2}\left[\frac{-x - 2x(1 - \ln x)}{x^4}\right]$

Thus:

$$f''(x) = \frac{1}{2}\left[\frac{2\ln x - 3}{x^3}\right]$$

b) $f''(x) = 0$ iff $\ln x = 3/2$ iff $x = e^{3/2}$

x	0		$e^{3/2}$		$+\infty$
$f''(x)$	‖	$-$	0	$+$	

f is concave up on $(e^{3/2}, +\infty)$
f is concave down on $(0, e^{3/2})$
$e^{3/2}$ is an inflection point