Math 201 Quiz 1 04/02/2012

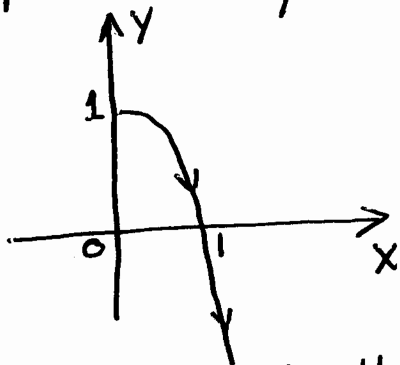
**Ex1**: Let $x=\sqrt{t}$, $y=1-t$

a) Find the Cartesian equation

b) Sketch the curve

**Solution**: a) Since $x=\sqrt{t}$, we have that $x \geqslant 0$ and $t=x^2$. Thus $y=1-x^2$ with $x \geqslant 0$.

b) $y=1-x^2$, $x \geqslant 0$ represents the right branch of the parabola $y=1-x^2$ with vertex $(0,1)$




**Ex2**: Describe the motion of the particle with:

$$x = 5\sin t, \quad y = 2\cos t \quad \text{as } -\pi \leqslant t \leqslant 5\pi$$

**Solution**: $\frac{x}{5} = \sin t$, $\frac{y}{2} = \cos t$. Thus $\left(\frac{x}{5}\right)^2 + \left(\frac{y}{2}\right)^2 = \cos^2 t + \sin^2 t = 1$, which is the equation of an ellipse. Therefore the particle is describing an ellipse. It is moving around this ellipse 3 times clockwise, starting and ending at the same point $(0,-2)$

**Ex3**: Find an equation of the tangent line to the curve

$$x = \cos t + \cos 2t, \quad y = \sin t + \sin 2t, \quad \text{when } t = \frac{\pi}{2}.$$

**Solution**: $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\cos t + 2\cos 2t}{-\sin t - 2\sin 2t}$. The slope is then given by: $\left.\frac{dy}{dx}\right|_{t=\frac{\pi}{2}} = \frac{-2}{-1} = 2$

When $t = \frac{\pi}{2}$, $x = -1$ and $y = 1$. Hence an equation of the tangent line to the curve at the point $(-1, 1)$ is:

$$y = 2(x+1) + 1 = 2x + 3.$$