

Ex. Let $f(x) = e^{2x} + 2e^{-x}$.

a) Find the open intervals on which f is increasing and decreasing

b) Find any local or absolute extremum.

Solution: The domain of f is \mathbb{R} .

For every x in \mathbb{R} , we have: $f'(x) = 2e^{2x} - 2e^{-x} = 2(e^{3x} - 1)e^{-x}$

So $f'(x) = 0$ iff $e^{3x} = 1$ iff $x = 0$. Thus f has only one critical point $x = 0$

x	$-\infty$	0	$+\infty$
$f'(x)$	$-$	0	$+$
$f(x)$	$+\infty$	3	$+\infty$

$\lim_{x \rightarrow +\infty} f(x) = +\infty$, $\lim_{x \rightarrow -\infty} f(x) = +\infty$

So f has no absolute maximum.

a) f is increasing on $(0, +\infty)$

f is decreasing on $(-\infty, 0)$

b) 0 is the absolute minimum, and the absolute minimum value is $f(0) = 3$.