Math 101 Quiz 1 Date 15/09/2012

Ex1: Evaluate $\lim\limits_{x \to 4} \dfrac{4+3x-x^2}{\sqrt{x}-2}$

Ex2: Let $\varepsilon = 1$, $L = 3$, $x_0 = 10$ and $f(x) = \sqrt{19-x}$. Find a $\delta > 0$ such that $|f(x) - L| < \varepsilon$ for $0 < |x - x_0| < \delta$.

Ex3: Evaluate $\lim\limits_{x \to 0} \dfrac{x + x\cos 3x}{\sin 2x \cos 4x}$

Solution:

1) $$\lim_{x \to 4} \frac{4+3x-x^2}{\sqrt{x}-2} = \lim_{x \to 4} \frac{(4-x)(1+x)}{\sqrt{x}-2} = \lim_{x \to 4} \frac{(4-x)(1+x)(\sqrt{x}+2)}{(\sqrt{x}-2)(\sqrt{x}+2)}$$

$$= \lim_{x \to 4} \frac{(4-x)(1+x)(\sqrt{x}+2)}{x-4} = \lim_{x \to 4} -(1+x)(\sqrt{x}+2) = -20$$

2) $|\sqrt{19-x} - 3| < 1$ iff $-1 < \sqrt{19-x} - 3 < 1$ iff $2 < \sqrt{19-x} < 4$

iff $4 < 19 - x < 16$ iff $-4 > x - 19 > -16$ iff $3 < x < 15$

Therefore $|\sqrt{19-x} - 3| < 1$ iff $x \in (3, 15)$. Hence δ can be any positive real number such that the interval $(10-\delta, 10+\delta)$ is included in the interval. Now it is easy to see that:

$$(10-\delta, 10+\delta) \subseteq (3, 15) \text{ iff } 0 < \delta < 5$$

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Ex3: $\lim\limits_{x\to 0} \dfrac{x + x\cos 3x}{\sin 2x . \cos 4x} = \lim\limits_{x\to 0} \left(\dfrac{x}{\sin 2x} . \dfrac{1+\cos 3x}{\cos 4x} \right)$

$= \lim\limits_{x\to 0} \left(\dfrac{2x}{\sin 2x} . \dfrac{1}{2} . \dfrac{1+\cos 3x}{\cos 4x} \right)$

$= \dfrac{1}{2} \lim\limits_{x\to 0} \left(\dfrac{1}{\frac{\sin 2x}{2x}} . \dfrac{1+\cos 3x}{\cos 4x} \right).$

Since $\lim\limits_{x\to 0} \dfrac{\sin 2x}{2x} = 1$ and $\lim\limits_{x\to 0} \dfrac{1+\cos 3x}{\cos 4x} = 2$, we have that:

$\lim\limits_{x\to 0} \dfrac{x + x\cos 3x}{\sin 2x . \cos 4x} = \dfrac{1}{2} . 1 . 2 = 1.$