KFUPM Term (122) Name Solutions Serial#__________

MATH 102-01-02 Quiz # 3 ID#________________ Section_____

Version (2)

1) (6-points) Show that the series $\sum_{n=0}^{\infty} \frac{1}{n^2+7n+12}$ is convergent and find its sum. (Hint: You may use telescoping sums).

$$a_n = \frac{1}{n^2+7n+12} = \frac{1}{(n+3)(n+4)} = \frac{1}{n+3} - \frac{1}{n+4}, \quad n \geq 0$$

$$S_n = a_1 + a_2 + \cdots + a_n = \left(\frac{1}{3} - \cancel{\frac{1}{4}}\right) + \left(\cancel{\frac{1}{4}} - \cancel{\frac{1}{5}}\right) + \left(\cancel{\frac{1}{5}} - \cancel{\frac{1}{6}}\right) + \cdots + \cdots + \left(\cancel{\frac{1}{n+2}} - \cancel{\frac{1}{n+3}}\right) + \left(\cancel{\frac{1}{n+3}} - \frac{1}{n+4}\right)$$

$= \frac{1}{3} - \frac{1}{n+4} =$ the nth term of the sequence of partial sums of the given series.

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty}\left(\frac{1}{3} - \frac{1}{n+4}\right) = 1/3$$

Conclusion The given series converges and has the sum $\frac{1}{3}$.

2) (4-points) Test the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{32n^2+14n}}$ for convergence or divergence.

$a_n = \frac{1}{\sqrt[5]{32n^2+14n}}$ and let $b_n = \frac{1}{n^{2/5}} = \frac{1}{\sqrt[5]{n^2}}$

and use Limit Comparison test:

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\sqrt[5]{n^2}}{\sqrt[5]{32n^2+14}} = \lim_{n\to\infty} \frac{1}{\sqrt[5]{32+\frac{4}{n^2}}}$$

$= \frac{1}{2} > 0$ and since $\sum \frac{1}{n^{2/5}}$ is a divergent p-series ($p = \frac{2}{5}$) $\Longrightarrow$ the given series diverges by the limit comparison test.

3) (6-points) Find the set of all x for which the series $\sum_{n=0}^{\infty}(-1)^n(3x+2)^{n+1}$ converges. Then find its sum for there values of x.

The first few terms of the series: $(3x+2) - (3x+2)^2 + (3x+2)^3 - \dots$

which show that it is a geometric series with:

first term $= a = 3x+2$ and common ratio $r = -(3x+2)$.

$\Rightarrow$ It converges for all $|r| = |3x+2| < 1 \Longrightarrow$

$-1 < 3x+2 < 1 \Longrightarrow -3 < 3x < -1 \Longrightarrow \boxed{-1 < x < -1/3}$

The sum $= \dfrac{a}{1-r}$, $|r|<1$

$$\Rightarrow \text{The Sum} = \frac{3x+2}{1+3x+2} = \frac{3x+2}{3x+3}, \quad -1 < x < -1/3$$

4) (4-points) Test the series $\sum_{n=2}^{\infty}\frac{1}{10+7^{n-1}}$ for convergence or divergence.

$$\frac{1}{10+7^{n-1}} < \frac{1}{7^{n-1}}$$

And since $\displaystyle\sum_{n=2}^{\infty}\frac{1}{7^{n-1}} = \frac{1}{7} + \frac{1}{7^2} + \frac{1}{7^3} + \dots$ is

a convergent geometric series $\left(|r| = \frac{1}{7} < 1\right)$

$\Rightarrow$ the given series converges by the direct comparison test.

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MATH 102-01-02 Quiz # 5 ID#________________ Section_____

1) (6-points) Show that the series $\sum_{n=0}^{\infty} \frac{1}{n^2+5n+6}$ is convergent and find its sum. (Hint: You may use telescoping sums).

$$a_n = \frac{1}{n^2+5n+6} = \frac{1}{(n+2)(n+3)} = \frac{1}{n+2} - \frac{1}{n+3} \quad ; n \geq 0$$

$$\Rightarrow S_n = a_1 + a_2 + \cdots + a_n = \left(\frac{1}{2} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{4}}\right) + \left(\cancel{\frac{1}{4}} - \cancel{\frac{1}{5}}\right) + \cdots + \left(\cancel{\frac{1}{n+1}} - \cancel{\frac{1}{n+2}}\right) + \left(\cancel{\frac{1}{n+2}} - \frac{1}{n+3}\right)$$

$= \frac{1}{2} - \frac{1}{n+3} =$ the nth term of the sequence of partial sums of the given series.

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty}\left(\frac{1}{2} - \frac{1}{n+3}\right) = \frac{1}{2}$$

Conclusion: The given series converges and has the sum $\frac{1}{2}$.

2) (4-points) Test the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{8n^2+16n}}$ for convergence or divergence.

$a_n = \frac{1}{\sqrt[3]{8n^2+16n}}$ and let $b_n = \frac{1}{n^{2/3}} = \frac{1}{\sqrt[3]{n^2}}$

and use the limit comparison test

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\sqrt[3]{n^2}}{\sqrt[3]{8n^2+16n}} = \lim_{n\to\infty} \frac{1}{\sqrt[3]{8+\frac{16}{n}}} = \frac{1}{2} > 0$$

and since $\sum \frac{1}{n^{2/3}}$ is a divergent p-series ($p = \frac{2}{3}$)

$\Rightarrow$ the given series converges by the limit comparison test.

3) (6-points) Find the set of all x for which the series $\sum_{n=0}^{\infty}(-1)^n(2x+3)^{n+1}$ converges. Then find its sum for there values of x.

The first few terms of the series: $(2x+3)-(2x+3)^2+(2x+3)^3 - \cdots$

which show that it in a geometric series with:

first term $= a = 2x+3$ and common ratio $r = -(2x+3)$

$\Rightarrow$ it converges for all $|r| = |2x+3| < 1 \Rightarrow$

$-1 < 2x+3 < 1 \Rightarrow -4 < 2x < -2 \Rightarrow \boxed{-2 < x < -1}$

The sum $= \dfrac{a}{1-r}$, $|r| < 1$

The sum $= \dfrac{2x+3}{1+2x+3} = \dfrac{2x+3}{2x+4}$, $-2 < x < -1$

4) (4-points) Test the series $\sum_{n=2}^{\infty}\frac{1}{2+5^{n-1}}$ for convergence or divergence.

$$\frac{1}{2+5^{n-1}} < \frac{1}{5^{n-1}}$$

And since $\displaystyle\sum_{n=2}^{\infty}\frac{1}{5^{n-1}} = \frac{1}{5}+\frac{1}{5^2}+\frac{1}{5^3}+\cdots$

is a convergent geometric series with $|r| = \frac{1}{5} < 1$

$\Rightarrow$ the given series converges by the direct comparison test.