April 27,2013

QUIZ♯4 Math102-sec7.

Net Time Allowed: 25 minutes

Name: **ID ♯ :** **Serial ♯:**

Exercise1: (04pts)

Determine whether the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}+2^n}{3^n}$, is convergent or divergent. If it is convergent, find its sum.

• The series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{3^n} = \frac{1}{3}\sum_{n\geq 1}^{\infty}\left(-\frac{1}{3}\right)^{n-1}$ is Convergent as a geometric series (01) $\left(\begin{array}{l} a=+\frac{1}{3} \\ r=-\frac{1}{3}\end{array}\right)$

Thus $|r| = \frac{1}{3} < 1$ and its sum is: $S_1 = \frac{1}{3}\cdot\frac{1}{1+\frac{1}{3}} = \frac{1}{4}$ (0.5)

• The series $\sum_{n=1}^{\infty} \frac{2^n}{3^n} = \sum_{n\geq 1}^{\infty}\left(\frac{2}{3}\right)^n$ is Convergent geometric series (01) $\left(\begin{array}{l} a=\frac{2}{3} \\ r=\frac{2}{3}\end{array}\right)$

Thus $|r| = \frac{2}{3} < 1$ and its sum is: $S_2 = \frac{2/3}{1-2/3} = 2$ (0.5)

Hence $\sum_{n\geq 1}^{\infty} \frac{(-1)^{n-1}+2^n}{3^n}$ is Convergent (0.5) and has Sum: $S = S_1 + S_2 = \frac{9}{4}$. (0.5)

Exercise2: (06pts)

Determine whether the following series converges or diverges:

(03) a)- $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$; Consider $\int_2^{+\infty} \frac{dx}{x(\ln x)^2}$

The function $x \longmapsto \frac{1}{x(\ln x)^2}$ is positive, continuous and decreasing (0.5)

$\int_2^{+\infty} \frac{dx}{x(\ln x)^2} = \lim_{t\to+\infty} \int_2^t \frac{dx}{x(\ln x)^2} = \lim_{t\to+\infty}\left[-\frac{1}{\ln x}\right]_2^t = \lim_{t\to+\infty}\left[-\frac{1}{\ln t}+\frac{1}{\ln 2}\right] = \frac{1}{\ln 2}$ (0.5) (0.5) (0.5)

Thus $\int_2^{+\infty} \frac{dx}{x(\ln x)^2}$ Converges, (0.5) Hence $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ Converges too. (0.5)

(03) b)- $\sum_{n=1}^{\infty} n \sin \frac{1}{n}$; Let $a_n = n\sin(\frac{1}{n})$

We have $\lim_{n\to+\infty} a_n = \lim_{n\to\infty}\left(n \sin\frac{1}{n}\right) = \lim_{n\to\infty} \frac{\sin\frac{1}{n}}{\frac{1}{n}} = 1 \neq 0$ (0.5) (01)

Hence By the divergence Test, (0.5) The series diverges. (01)