Math 102 Quiz 5 12/03/2013

Evaluate the following integrals:

a) $I = \int e^{\sqrt{3s+9}}\, ds$

b) $J = \int e^{x} \sec^3(e^{x})\, dx$

c) $K = \int x \sin^2 x\, dx$

Solution:

a) Let $u = \sqrt{3s+9}$. Then $du = \frac{3}{2}\frac{ds}{\sqrt{3s+9}} = \frac{3}{2u}\, ds$. Thus

$ds = \frac{2}{3} u\, du$. Now the integral I becomes:

$I = \frac{2}{3}\int u e^{u}\, du$. Using an integration by parts, one can easily rewrite I as: $I = \frac{2}{3}\left[u e^{u} - \int e^{u}\, du\right] = \frac{2}{3}(u-1)e^{u} + C$

$= \frac{2}{3}\left(\sqrt{3s+9} - 1\right) e^{\sqrt{3s+9}} + C$

b) Let $u = e^{x}$, so that $du = e^{x} dx$. Thus $J = \int \sec^3 u\, du$

$= \int \sec^2 u . \sec u\, du = \int (\tan^2 u + 1) \sec u\, du$

$= \underbrace{\int \tan^2 u \sec u\, du}_{\mathrm{I}} + \underbrace{\int \sec u\, du}_{\mathrm{II}}.$

$\mathrm{II} = \int \sec u\, du = \ln|\sec u + \tan u| + C.$

$\mathrm{I} = \int \tan^2 u \sec u\, du = \int \tan u . (\tan u \sec u)\, du = \tan u \sec u - \int \sec^3 u\, du$

$= \tan u \sec u - J$

Therefore: $2J = \tan u \sec u + \ln|\sec u + \tan u| + C$. Hence

$$J = \frac{1}{2}\left[\tan e^x \sec e^x + \ln\left|\sec e^x + \tan e^x\right|\right] + C$$

c) $K = \int x \sin^2 x \, dx = \int x \frac{1-\cos 2x}{2} dx = \frac{1}{2}\int (x - x\cos 2x) dx$

$$= \frac{1}{2}\left[\frac{x^2}{2} - \int x \cos 2x \, dx\right]$$

$$= \frac{1}{2}\left[\frac{x^2}{2} - \left(\frac{x}{2}\sin 2x - \frac{1}{2}\int \sin 2x \, dx\right)\right]$$

$$= \frac{1}{4}\left[x^2 - x\sin 2x - \frac{1}{2}\cos 2x\right] + C$$