1. Where is $f(x) = \dfrac{x^3+1}{x\sqrt[3]{x+2}}$ continuous?

(a) $(-\infty,-2)\cup(-2,0)\cup(0,\infty)$

(b) $(-2,\infty)$

(c) $(-\infty,0)\cup(0,\infty)$

(d) $(-\infty,-2)\cup(0,\infty)$

(e) $(-2,0)\cup(0,\infty)$

2. $\lim\limits_{x\to 3^-} \dfrac{x^3-9x}{|3-x|} =$

(a) -18

(b) 18

(c) Does not exist

(d) -6

(e) 9

3. $\lim_{t\to\infty} e^{\frac{t}{t+1}} \cos\left(\frac{1}{t^2}\right) =$

(a) e

(b) 1

(c) ∞

(d) 0

(e) Does not exist

4. The graph of the curve $y = \dfrac{x^3 + x^2}{x^2 - 1}$ has

(a) One vertical asymptote and one oblique asymptote.
(b) One vertical asymptote and one horizontal asymptote.
(c) Two vertical asymptotes and one oblique asymptote.
(d) Two vertical asymptotes and one horizontal asymptote.
(e) Two vertical asymptotes only.

5. An equation for the **normal line** to the curve $y = x - \frac{1}{x}$ at $x = 1$ is

(a) $x + 2y = 1$

(b) $x - 2y = 1$

(c) $2x + 4y = 1$

(d) $3x + 2y = 3$

(e) $4x - y = 4$

6. If $h(x) = f(\sqrt{x} \cdot g^2(x)), f'(4) = 2, g(1) = 2$ and $g'(1) = -1,$ then $h'(1) =$

(a) -4

(b) -2

(c) 2

(d) 8

(e) -8

7. If $f(x) = \tan^{-1}(e^{-2x})$, then $f'(x) =$

(a) $\dfrac{-2e^{2x}}{e^{4x}+1}$

(b) $\dfrac{-2e^{-2x}}{e^{4x^2}+1}$

(c) $\dfrac{1}{e^{-4x}+1}$

(d) $\dfrac{-e^{-2x}}{e^{-4x}+1}$

(e) $\dfrac{1}{e^{4x^2}+1}$

8. If

$$f(x) = \begin{cases} x^2 \sin(1/x^4) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0, \end{cases}$$

then $f'(0) =$ [**Hint:** Use the limit definition of the derivative at a point]

(a) 0

(b) 1

(c) -1

(d) 2

(e) Does not exist

9. If $y = \ln\left(\frac{\sqrt{x+4}-2}{\sqrt{x+4}+2}\right)$, then $\frac{dy}{dx} =$

(a) $\frac{2}{x\sqrt{x+4}}$

(b) $\frac{\sqrt{x+4}}{x-\sqrt{x+4}}$

(c) $\frac{4}{x}$

(d) $\frac{\sqrt{x+4}-4}{2x}$

(e) $\frac{\sqrt{x+4}}{2x}$

10. If $x^{y+2} = y^{x+2}$, then $y' =$

(a) $\frac{y(y+2)-xy\ln y}{x(x+2)-xy\ln x}$

(b) $\frac{(y+2)-x\ln y}{(x+2)-y\ln x}$

(c) $\frac{2+xy-\ln y}{2+xy-\ln x}$

(d) $\frac{x+2-\ln y}{y+2-\ln x}$

(e) $\frac{x+y+x\ln y}{x+y+y\ln x}$

11. A balloon is rising vertically above a level straight road at a constant rate of 2 ft/sec. Just when the balloon is 20 ft above the ground, a bicycle moving at a constant rate of 10 ft/sec passes under it. How fast is the distance between the balloon and the bicycle increasing 3 sec later?

$(a)\ \dfrac{352}{\sqrt{1576}} ft\ /\sec$

$(b)\ \dfrac{543}{\sqrt{123}} ft\ /\sec$

$(c)\ 0.5 ft\ /\sec$

$(d)\ 1.5 ft\ /\sec$

$(e)\ \dfrac{815}{\sqrt{4105}} ft\ /\sec$

12. $\lim_{x\to 1} \dfrac{x\ \tan^{-1}(x-1)}{\ln(2x)\cdot\sin(x-1)} =$

$(a)\ \dfrac{1}{\ln 2}$

$(b)\ -\ln 2$

$(c)\ 2\ln 2$

$(d)\ \dfrac{1}{2\ln 2}$

$(e)\ -3\ln 2$

13. Which one of the following statements is **FALSE** about the function $f(x) = 1 - |x - 3|, \ -1 \le x \le 4$

(a) It has no critical points.
(b) It has absolute maximum and absolute minimum values.
(c) It has one critical point.
(d) It is continuous on $[-1, 4]$.
(e) It is not differentiable on $(-1, 4)$.

14. The value(s) of c satisfying the conclusion of the Mean Value Theorem for the function $f(x) = x + \sqrt{x}$ on the interval $[0, 4]$ is (are)

(a) 1

(b) -1 and 1

(c) 2

(d) $\frac{1}{3}$ and 1

(e) $\frac{1}{2}$ and 1

15. The **sum** of the critical numbers of $f(x)=x^{2/3}(1-x^2)$ is

(a) 0

(b) 1

(c) $\frac{1}{2}$

(d) 2

(e) $-\frac{1}{3}$

16. If m and M are respectively the absolute minimum and absolute maximum values of $f(x)=e^{x^4-2x^2}$ on $[-1,1]$, then $m+M=$

(a) $\frac{e+1}{e}$

(b) $e+1$

(c) e

(d) $\frac{1}{e}$

(e) $\frac{e^2+1}{e}$

17. $\lim_{t\to 2^+}\left(\frac{1}{t-2}-\frac{1}{\ln(t-1)}\right)=$

$(a)\ -\frac{1}{2}$

$(b)\ 0$

$(c)\ \infty$

$(d)\ 2$

$(e)\ -\frac{1}{4}$

18. Which of the following statements is **TRUE** about the function $f(x)=x\ln x$?

(a) It is decreasing on $(0,e^{-1})$ and increasing on (e^{-1},∞).

(b) It is increasing on $(0,e^{-1})$.

(c) It is increasing on its domain.

(d) It has a local maximum at $x=e$.

(e) It has a local maximum at $x=e^{-1}$.

19. If $y = f(x)$ is a function such that

$$y'' = (x-1)^2 (x-2)^3 (-x-3),$$

then the graph of f

(a) is concave up on $(-3, 2)$.

(b) has three inflection points.

(c) is concave down on $(1, \infty)$.

(d) has one inflection point.

(e) is concave down on $(-\infty, 1)$.

20. $\lim_{x \to \infty} \left(1 + \dfrac{2}{3x}\right)^{6x} =$

(a) e^4

(b) e^3

(c) e^9

(d) e

(e) $e^{1/3}$

21. $\int \frac{-1+4\sqrt[3]{x}}{x^2}dx =$

$(a)\ \frac{1}{x}-\frac{6}{\sqrt[3]{x^2}}+C$

$(b)\ -\ln\left|x^2\right|-\frac{6}{\sqrt[3]{x^2}}+C$

$(c)\ -\frac{1}{x}-\frac{3}{2}x^{-3/2}+C$

$(d)\ \ln\left|x\right|+6x^{-5/3}+C$

$(e)\ \frac{1}{x}+4x^{-2/3}+C$

22. Newton's Method is used to approximate the real root of the equation

$$2x-\cos(\pi x)-2=0.$$

If $x_1=1$, then $x_2=$

$(a)\ \frac{1}{2}$

$(b)\ 1$

$(c)\ \frac{3}{2}$

$(d)\ 0$

$(e)\ \frac{5}{2}$

23. A rectangle is to have its base on the $x-axis$ and its upper two vertices on the parabola $y=16-x^2$. The largest possible area of the rectangle is

(a) $\frac{256}{3\sqrt{3}}$

(b) $\frac{32}{\sqrt{3}}$

(c) $\frac{64}{5\sqrt{3}}$

(d) $\frac{100}{3\sqrt{5}}$

(e) $\frac{132}{25}$

24. If the motion of an object moving in a straight line is described as follows

$$a(t)=4-6t,\ v(0)=-1,\ s(0)=2$$

where $a(t),v(t),s(t)$ are respectively the acceleration, velocity, and the position of the object at time t, then $s(3)=$

(a) -10

(b) 12

(c) -16

(d) 18

(2) 30

25. Let $f(x) = x^2 + 1,\ x > 0$. The minimum value of the slope of the line passing through the origin and the point (x, y) on the graph of f is

(a) 2

(b) $\frac{1}{2}$

(c) -2

(d) $-\frac{1}{2}$

(e) 1

26. If $x^a + y^a = b$, then $y'' =$ $(a \neq 0)$

(a) $-\frac{(a-1)bx^{a-2}}{y^{2a-1}}$

(b) $-\frac{bx^{a-1}}{y^{a-1}}$

(c) $\frac{(a-1)bx^{a-2}}{y^{2a-2}}$

(d) $\frac{(a-1)y^{a-2}}{bx^{2a-1}}$

(e) $-\frac{(a-1)x^{a-1}}{by^{a-1}}$

27. Using **an upper sum** with three rectangles of equal width, the area under the graph of $f(x) = x^3$ between $x = 0$ and $x = 3$ is approximately equal to

(a) 36

(b) 9

(c) 31

(d) 12

(e) 22

28. $\sum_{k=1}^{n} \frac{2k+n}{n^2} =$

(a) $2 + \frac{1}{n}$

(b) $1 + \frac{2}{n}$

(c) $\frac{2+n}{n^2}$

(d) $n + 2$

(e) $\frac{n+3}{n}$