

Q	MM	V1	V2	V3	V4
1	a	e	a	a	c
2	a	e	a	c	e
3	a	a	d	a	c
4	a	b	c	d	b
5	a	b	e	c	e
6	a	c	c	b	b
7	a	c	e	b	a
8	a	d	a	b	e
9	a	b	a	e	c
10	a	b	a	e	d
11	a	d	d	c	b
12	a	e	b	a	a
13	a	e	b	c	e
14	a	e	b	c	d
15	a	b	b	e	d
16	a	c	d	a	c
17	a	a	e	c	a
18	a	b	c	a	c
19	a	e	e	c	b
20	a	d	a	b	d

King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 102
Exam I
Term 132
Sunday 02/03/2014
Net Time Allowed: 120 minutes

MASTER VERSION

- Detailed Solution
- Solution Key (last page) [Answer Key]

1. Suppose g is an integrable function. If $\int_2^{-2} g(t) dt = 3$ and $\int_5^{-2} g(t) dt = 4$, then $\int_2^5 g(t) dt =$

(a) -1

(b) 1

(c) 7

(d) -7

(e) 3

$$\begin{aligned} \int_{-2}^5 g(t) dt &= \int_{-2}^2 g(t) dt + \int_2^5 g(t) dt \\ -4 &= -3 + \int_2^5 g(t) dt \\ \Rightarrow \int_2^5 g(t) dt &= -4 + 3 = -1 \end{aligned}$$

2. If $\int \frac{1}{\sqrt{x} e^{\sqrt{x}}} \sec^2(e^{-\sqrt{x}} + 1) dx = \int c \sec^2 u du$, then $cu =$

(a) $-2(e^{-\sqrt{x}} + 1)$

(b) $e^{-\sqrt{x}} + 1$

(c) $\frac{1}{\sqrt{x} e^{\sqrt{x}}} (e^{-\sqrt{x}} + 1)$

(d) $-\frac{1}{2}(e^{-\sqrt{x}} + 1)$

(e) $\frac{-2}{\sqrt{x} e^{\sqrt{x}}} (e^{-\sqrt{x}} + 1)$

$$u = e^{-\sqrt{x}} + 1 \Rightarrow du = \left(e^{-\sqrt{x}} \cdot \frac{-1}{2\sqrt{x}} \right) dx$$

$$\Rightarrow -2 du = \frac{1}{\sqrt{x} e^{\sqrt{x}}} dx$$

$$\text{So } \int \frac{1}{\sqrt{x} e^{\sqrt{x}}} \sec^2(e^{-\sqrt{x}} + 1) dx$$

$$= \int -2 \sec^2 u du$$

Thus $c = -2$ and so

$$cu = -2(e^{-\sqrt{x}} + 1)$$

$$3. \int_0^3 8\sqrt{9-x^2} dx = 8 \int_0^3 \sqrt{9-x^2} dx$$

$$(a) 18\pi \quad = 8 \cdot \frac{1}{4} \pi (3)^2$$

$$(b) 8\pi \quad = 18\pi$$

$$(c) 2\pi$$

$$(d) 36\pi$$

$$(e) 16\pi$$

$$4. \int 14r^4(1-r^5)^6 dr = \quad \text{Let } u = 1-r^5. \text{ Then } du = -5r^4 dr$$

$$(a) -\frac{2}{5}(1-r^5)^7 + C$$

$$(b) \frac{1}{10}(1-r^5)^7 + C$$

$$(c) \frac{-2}{(1-r^5)^7} + C$$

$$(d) -\frac{1}{7}(1-r^5)^7 + C$$

$$(e) \frac{2}{7}(1-r^5)^6 + C$$

$$\int 14r^4(1-r^5)^6 dr = 14 \int u^6 \cdot \frac{-1}{5} du$$

$$= -\frac{14}{5} \int u^6 du$$

$$= -\frac{14}{5} \cdot \frac{u^7}{7} + C$$

$$= -\frac{2}{5} u^7 + C$$

$$= -\frac{2}{5} (1-r^5)^7 + C$$

5. Let b be a real number such that $0 < b < 2$. If the average value of $f(x) = \cos(2x)$ on the interval $[0, b]$ is **zero**, then $b =$

(a) $\frac{\pi}{2}$

(b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$

(d) $\frac{\pi}{5}$

(e) $\frac{\pi}{6}$

$$f_{\text{ave}} = \frac{1}{b-0} \int_0^b \cos(2x) dx$$

$$0 = \frac{1}{b} \cdot \left. \frac{1}{2} \sin(2x) \right|_0^b$$

$$0 = \frac{1}{b} \cdot \left(\frac{1}{2} \sin(2b) - 0 \right)$$

$$(c) \quad 0 = \frac{1}{2b} \sin(2b)$$

$$(d) \quad \Rightarrow \sin(2b) = 0$$

$$\Rightarrow 2b = n\pi, \quad n \text{ is integer}$$

$$(e) \quad \Rightarrow b = \frac{n\pi}{2}$$

Since $0 < b < 2$, we must take $n=1$. So $b = \frac{\pi}{2}$

6. Let f be an **odd** and continuous function.

If $\int_0^4 f(x) dx = 6$, then $\int_0^2 f(-2x) dx =$

(a) -3

(b) -6

(c) 6

(d) -12

(e) 12

Let $u = -2x$. Then $du = -2dx$.

$$\int_0^2 f(-2x) dx = -\frac{1}{2} \int_0^{-4} f(u) du$$

$$= \frac{1}{2} \int_{-4}^0 f(u) du$$

$$= \frac{1}{2} \cdot (-6)$$

$$= -3$$

Since f is odd

$$\begin{aligned}
 7. \quad \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \sin |2x| dx &= \int_{-\frac{\pi}{6}}^0 \sin(-2x) dx + \int_0^{\frac{\pi}{6}} \sin(2x) dx \\
 &= - \int_{-\frac{\pi}{6}}^0 \sin(2x) dx + \int_0^{\frac{\pi}{6}} \sin(2x) dx \\
 &= - \left(-\frac{1}{2} \cos(2x) \right)_{-\frac{\pi}{6}}^0 + \left(-\frac{1}{2} \cos(2x) \right)_0^{\frac{\pi}{6}} \\
 &= - \left[-\frac{1}{2} + \frac{1}{4} \right] + \left(\frac{1}{2} + \frac{1}{2} \right) \\
 &= \frac{1}{4} + 1 \\
 &= \frac{5}{4}
 \end{aligned}$$

(a) $\frac{5}{4}$
 (b) $\frac{3}{4}$
 (c) $1 - \frac{\sqrt{3}}{4}$
 (d) $\frac{\sqrt{3}}{4}$
 (e) $1 + \frac{1}{\sqrt{2}}$

8. The position function of a particle moving in a straight line is given by

$$s(t) = \frac{1}{2}t^2 - t, \quad t \geq 0$$

The **total distance** traveled by the particle over the time interval $0 \leq t \leq 2$ is

$$\begin{aligned}
 &v(t) = t - 1 \\
 (a) \quad 1 & \quad D = \int_0^2 |v(t)| dt \\
 (b) \quad \frac{1}{3} & \quad = \int_0^2 |t-1| dt \\
 (c) \quad 3 & \quad = \int_0^1 (1-t) dt + \int_1^2 (t-1) dt \\
 (d) \quad \frac{5}{3} & \quad = \left[t - \frac{1}{2}t^2 \right]_0^1 + \left[\frac{1}{2}t^2 - t \right]_1^2 \\
 (e) \quad 5 & \quad = \left(1 - \frac{1}{2} \right) + \left(0 - \left(-\frac{1}{2} \right) \right) \\
 & \quad = \frac{1}{2} + \frac{1}{2} = 1
 \end{aligned}$$

9. Let $y = f(x)$ be a curve that has a positive derivative and passes through the point $(1, 1)$. If its length L , from $x = 1$ to $x = 4$, is given by

$$L = \int_1^4 \sqrt{1 + \frac{16}{9\sqrt{x}}} dx, \quad \left(\frac{dy}{dx}\right)^2 = \frac{16}{9\sqrt{x}}$$

then $f(2^{4/3}) =$

(a) $\frac{25}{9}$

(b) $\frac{5}{9}$

(c) $\frac{3}{4}$

(d) $\frac{16}{9}$

(e) $\frac{8}{3}$

$$\Rightarrow \frac{dy}{dx} = \frac{4}{3} \cdot \frac{1}{\sqrt[4]{x}} \quad (\text{derivative is +})$$

$$\Rightarrow y = \frac{4}{3} \cdot \frac{4}{3} x^{3/4} + C$$

$$\Rightarrow 1 = \frac{16}{9} (1)^{3/4} + C \Rightarrow C = 1 - \frac{16}{9} = -\frac{7}{9}$$

$$\text{So } f(x) = \frac{16}{9} x^{3/4} - \frac{7}{9}$$

$$\Rightarrow f(2^{4/3}) = \frac{16}{9} \cdot 2 - \frac{7}{9} = \frac{32-7}{9} = \frac{25}{9}$$

10. The **slope** of the tangent line to the curve

$$y = \int_2^{\sec x} \frac{\sqrt{t^2 - 1}}{t} dt, \quad 0 < x < \frac{\pi}{2}$$

at $x = \frac{\pi}{3}$ is

$$y' = \frac{\sqrt{\sec^2 x - 1}}{\sec x} \cdot \frac{d}{dx} [\sec x]$$

(a) 3

(b) 2

(c) $\frac{1}{2}$

(d) $2\sqrt{3}$

(e) $\frac{\sqrt{3}}{4}$

$$= \frac{\tan x}{\sec x} \cdot \sec x \tan x$$

$$= \tan^2 x$$

$$\text{slope} = y' \Big|_{x=\frac{\pi}{3}} = \tan^2\left(\frac{\pi}{3}\right) = (\sqrt{3})^2 = 3$$

11. The area of the region enclosed by the curves $y = \sqrt{x}$ and $y = \frac{1}{8}x^2$ is

$$\sqrt{x} = \frac{1}{8}x^2 \Rightarrow x = \frac{1}{64}x^4 \Rightarrow 64x = x^4$$

$$\Rightarrow x^4 - 64x = 0 \Rightarrow x(x^3 - 64) = 0 \Rightarrow x = 0 \text{ or } x = 4$$

(a) $\frac{8}{3}$

(b) $\frac{7}{4}$

(c) $\frac{9}{5}$

(d) $\frac{5}{2}$

(e) $\frac{11}{6}$

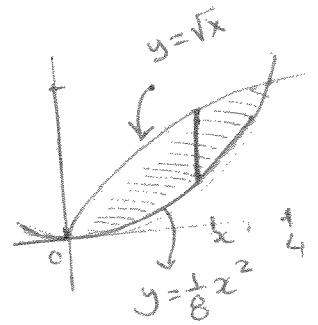
$$A = \int_0^4 \left(\sqrt{x} - \frac{1}{8}x^2 \right) dx$$

$$= \left[\frac{2}{3}x^{3/2} - \frac{1}{24}x^3 \right]_0^4$$

$$= \frac{2}{3} \cdot 8 - \frac{1}{24} \cdot 64$$

$$= \frac{16}{3} - \frac{8}{3}$$

$$= \frac{8}{3}$$



Let $u = x - 4$. Then $du = dx$

12. $\int \frac{x}{(x-4)^3} dx = \int \frac{u+4}{u^3} du = \int u^{-2} + 4u^{-3} du$

$$= \frac{u^{-1}}{-1} + 4 \cdot \frac{u^{-2}}{-2} + C$$

(a) $\frac{-1}{x-4} - \frac{2}{(x-4)^2} + C$

(b) $\ln|x-4| - \frac{4}{x-4} + C$

(c) $2 \ln|x-4| - \frac{1}{(x-4)^2} + C$

(d) $\frac{1}{(x-4)^2} + \frac{1}{(x-4)^3} + C$

(e) $\frac{-1}{x-4} + \frac{2}{(x-4)^4} + C$

13. The curve $y = \sqrt{4x - x^2}$, $1 \leq x \leq 4$, is revolved about the x -axis. The **area of the generated surface** is equal to

$$y' = \frac{4 - 2x}{2\sqrt{4x - x^2}} = \frac{2 - x}{\sqrt{4x - x^2}}$$

(a) 12π

(b) 6π

(c) 3π

(d) $4\pi - 2$

(e) 2π

$$\begin{aligned} S &= \int_1^4 2\pi y \sqrt{1 + (y')^2} dx \\ &= \int_1^4 2\pi \cdot \sqrt{4x - x^2} \cdot \sqrt{1 + \frac{(2-x)^2}{4x - x^2}} dx \\ &= \int_1^4 2\pi \sqrt{(4x - x^2) + (2-x)^2} dx \\ &= \int_1^4 2\pi \sqrt{4x - x^2 + 4 - 4x + x^2} dx \\ &= \int_1^4 2\pi \cdot 2 dx = 4\pi \int_1^4 dx = 12\pi \end{aligned}$$

14. The **volume** of the solid generated by revolving the region bounded by the curves $y^2 = -x$ and $x - y + 2 = 0$ about the line $y = 1$ is given by the integral

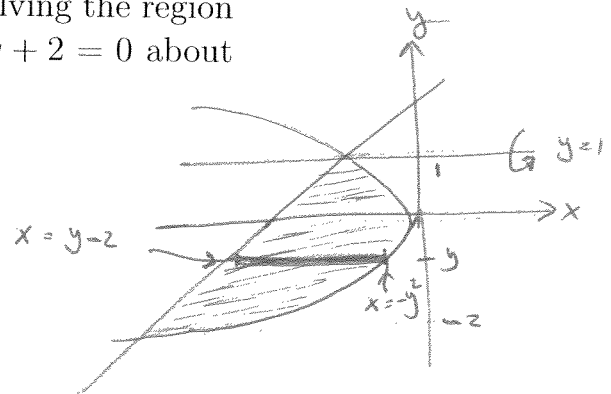
(a) $\int_{-2}^1 2\pi(y^3 - 3y + 2) dy$

(b) $\int_{-4}^0 2\pi(y^3 + y^2 + 3y - 2) dy$

(c) $\int_{-2}^1 2\pi(-y^3 + 3y - 4) dy$

(d) $\int_{-4}^0 2\pi(-y^3 + y^2 - 3y + 2) dy$

(e) $\int_{-2}^1 2\pi(y^3 + 2y^2 - 3y - 2) dy$



By the Shell method,

$$\begin{aligned} V &= \int_{-2}^1 2\pi \cdot (1-y) \cdot (-y^2 - (y-2)) \cdot dy \\ &= \int_{-2}^1 2\pi (1-y) (-y^2 - y + 2) dy \end{aligned}$$

$$\begin{aligned} &= \int_{-2}^1 2\pi (-y^2 - y + 2 + y^3 + y^2 - 2y) dy \\ &= \int_{-2}^1 2\pi (y^3 - 3y + 2) dy \end{aligned}$$

15. The
- length**
- of the curve

$$y = (1 - x^{2/3})^{3/2}, \quad \frac{1}{2} \leq x \leq 1$$

is

(a) $\frac{3}{2} \left(1 - \frac{1}{\sqrt[3]{4}}\right)$

(b) $\frac{2}{3} - \sqrt[3]{2}$

(c) 6

(d) $3\sqrt[3]{4} - 1$

(e) $6 - \frac{2}{\sqrt[3]{4}}$

$$\begin{aligned} \cdot y' &= \frac{3}{2} (1 - x^{2/3})^{1/2} \cdot \frac{2}{3} x^{-1/3} \\ \cdot (y')^2 &= (1 - x^{2/3}) \cdot x^{-2/3} \\ &= x^{-2/3} - 1 \\ \cdot 1 + (y')^2 &= x^{-2/3} \\ \cdot \sqrt{1 + (y')^2} &= x^{-1/3} \end{aligned}$$

$$\begin{aligned} L &= \int_{1/2}^1 \sqrt{1 + (y')^2} dx \\ &= \int_{1/2}^1 x^{-1/3} dx \\ &= \frac{3}{2} x^{2/3} \Big|_{1/2}^1 \\ &= \frac{3}{2} \left(1 - \frac{1}{\sqrt[3]{4}}\right) \end{aligned}$$

16. The
- volume**
- of the solid generated by revolving the shaded region about the
- y
- axis is

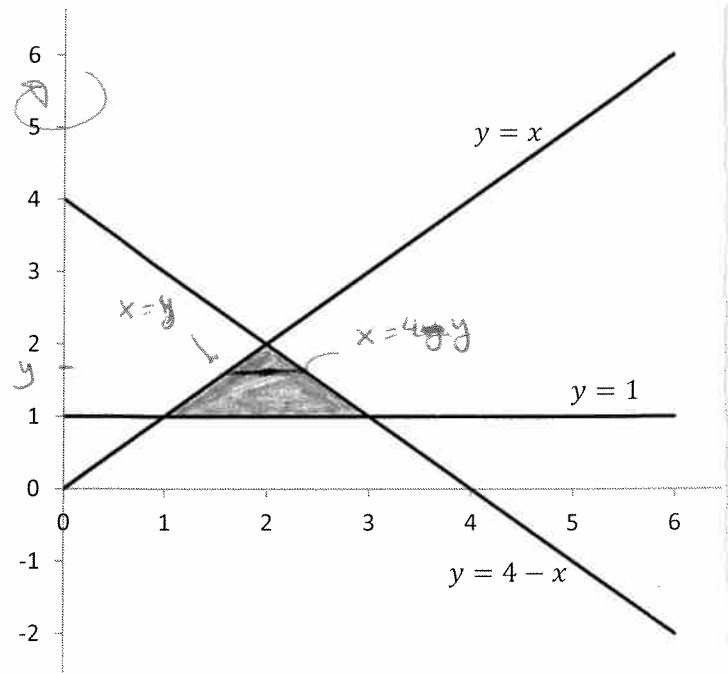
(a) 4π

(b) $\frac{8\pi}{3}$

(c) $\frac{3\pi}{4}$

(d) 3π

(e) $\frac{7\pi}{3}$



By the washer method:

$$V = \pi \int_1^2 (4-y)^2 - y^2 dy$$

$$= \pi \int_1^2 (16 - 8y) dy = \pi \cdot [16y - 4y^2]_1^2 = \pi \cdot [(32 - 16) - (16 - 4)] = \pi \cdot (16 - 12) = 4\pi$$

17. The base of a solid is bounded by the curves $y = e^x$, $y = 0$, $x = 0$, and $x = 1$. If the cross sections of the solid, perpendicular to the x -axis, are **semicircles**, then the **volume** of the solid is

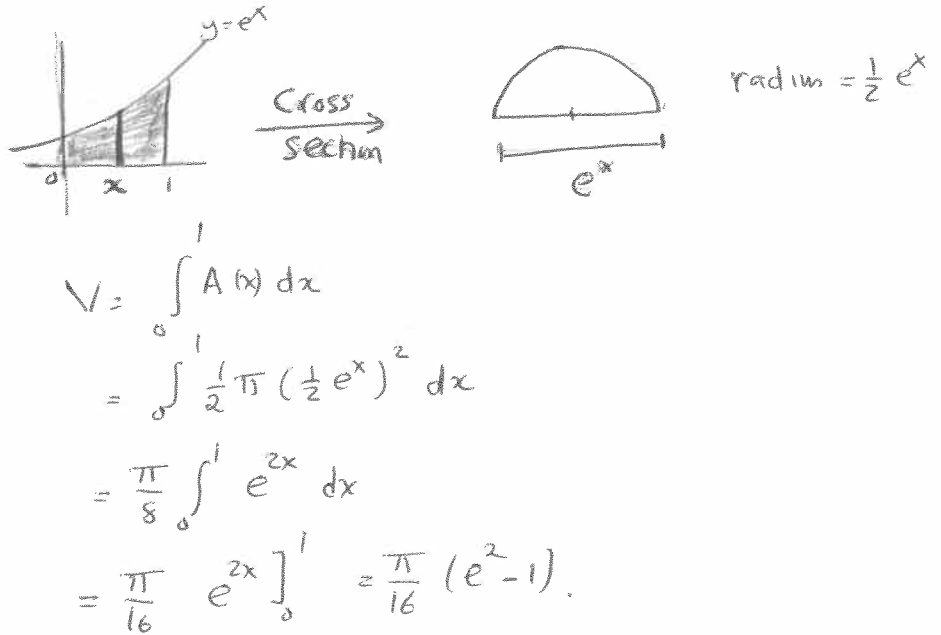
(a) $\frac{\pi(e^2 - 1)}{16}$

(b) $\frac{\pi(e^2 - 1)}{8}$

(c) $\pi(e^4 - 1)$

(d) $\frac{\pi(e - 1)}{2}$

(e) $\frac{\pi(e^2 - 2)}{8}$



18. If $F(x) = \int_1^x \frac{\sin t}{t} dt$, $x > 0$, then $\int_1^5 \frac{\sin(2t)}{t} dt =$

(a) $F(10) - F(2)$

(b) $F(5)$

(c) $\frac{1}{2} F(5)$

(d) $F(10) - F(5)$

(e) $2F(5) - F(2)$

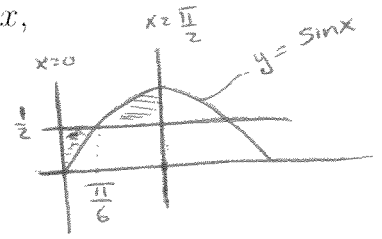
Let $u = 2t$. Then $du = 2 dt$.

$$\int_1^5 \frac{\sin(2t)}{t} dt = \int_2^{10} \frac{\sin u}{\frac{u}{2}} du$$

$$= F(u) \Big|_2^{10}$$

$$= F(10) - F(2)$$

19. The area of the region ^{between} enclosed by the curves $y = \sin x$, $y = \frac{1}{2}$, $x = 0$ and $x = \frac{\pi}{2}$ is ~~for~~ $0 \leq x \leq \frac{\pi}{2}$ is



(a) $\sqrt{3} - 1 - \frac{\pi}{12}$

(b) $\sqrt{3} - \frac{\pi}{3}$

(c) $\sqrt{3} + 1 - \frac{\pi}{12}$

(d) $3\sqrt{3} - \frac{\pi}{2}$

(e) $\sqrt{3} - 2 + \frac{\pi}{6}$

$$\begin{aligned}
 A &= \int_0^{\pi/6} \left(\frac{1}{2} - \sin x\right) dx + \int_{\pi/6}^{\pi/2} (\sin x - \frac{1}{2}) dx \\
 &= \left[\frac{1}{2}x + \cos x\right]_0^{\pi/6} + \left[-\cos x - \frac{1}{2}x\right]_{\pi/6}^{\pi/2} \\
 &= \left(\frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1\right) + \left(-0 - \frac{\pi}{4}\right) - \left(-\frac{\sqrt{3}}{2} - \frac{\pi}{12}\right) \\
 &= \frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1 - \frac{\pi}{4} + \frac{\sqrt{3}}{2} + \frac{\pi}{12} \\
 &= \sqrt{3} - 1 + \frac{1-3+\pi}{12} \\
 &= \sqrt{3} - 1 - \frac{\pi}{12}
 \end{aligned}$$

20. $\int \frac{x-3}{3+x^2} dx = \int \frac{x}{3+x^2} - \frac{3}{3+x^2} dx = \frac{1}{2} \ln(3+x^2) - 3 \cdot \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C$
 $= \frac{1}{2} \ln(3+x^2) - \sqrt{3} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C.$

(a) $\frac{1}{2} \ln(x^2 + 3) - \sqrt{3} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C$

(b) $\frac{1}{2} \ln(x^2 + 3) - \frac{1}{3} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C$

(c) $\frac{1}{2} \ln(x^2 + 3) - \tan^{-1}\left(\frac{x}{3}\right) + C$

(d) $\ln(x^2 + 3) - \sqrt{3} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C$

(e) $\ln(x^2 + 3) - \frac{1}{\sqrt{3}} \tan^{-1}(x\sqrt{3}) + C$