KFUPM MATH201 Quiz#2 Time:20min.

Q1.Describe the set of all points in a space equidistant from the origin and the point (0, 2, 0) with single equation.

Let the points $P(x, y, z)$, *then*

the distance between P *and the origin*

$= $ *the distance between* P *and the ponit* $(0, 2, 0)$

$$\sqrt{(x-0)^2+(y-0)^2+(z-0)^2} = \sqrt{(x-0)^2+(y-2)^2+(z-0)^2}$$

$$\sqrt{x^2+y^2+z^2} = \sqrt{x^2+y^2-4y+4+z^2}$$

$$x^2+y^2+z^2 = x^2+y^2-4y+4+z^2$$

$$\rightarrow y = 1$$

Q2. Using the definition of the projection of **u** onto **v,** show by calculation that

$$(\mathrm{u} - \mathrm{proj_v u}) \cdot \mathrm{proj_v u} = 0.$$

$$\mathrm{proj_v u} = \frac{u \cdot v}{|v|^2} v \rightarrow \ (\mathrm{u} - \mathrm{proj_v u}) \cdot \mathrm{proj_v u} = \left(\mathrm{u} - \frac{u \cdot v}{|v|^2} v\right) \cdot \frac{u \cdot v}{|v|^2} v$$

$$= u \cdot \frac{u \cdot v}{|v|^2} v - \left(\frac{u \cdot v}{|v|^2} v\right) \cdot \left(\frac{u \cdot v}{|v|^2} v\right) = \left(\frac{u \cdot v}{|v|^2}\right)(u \cdot v) - \left(\frac{u \cdot v}{|v|^2}\right)^2 (v \cdot v)$$

$$= \frac{(u \cdot v)^2}{|v|^2} - \frac{(u \cdot v)^2}{|v|^4}|v|^2 = \frac{(u \cdot v)^2}{|v|^2} - \frac{(u \cdot v)^2}{|v|^2} = 0$$

Q3. Find a unit vector perpendicular to the plane of

$\mathrm{P}(1, -1, 2), \mathrm{Q}(2, 0, -1)$, and $\mathrm{R}(0, 2, 1)$.

$$\overrightarrow{\mathrm{PQ}} = \langle 1, 1, -3 \rangle, \ \ \overrightarrow{\mathrm{PR}} = \langle -1, 3, -1 \rangle$$

$$\overrightarrow{\mathrm{PQ}} \times \overrightarrow{\mathrm{PR}} = \begin{vmatrix} \mathrm{i} & \mathrm{j} & \mathrm{k} \\ 1 & 1 & -3 \\ -1 & 3 & -1 \end{vmatrix} = 8\mathrm{i} + 4\mathrm{j} + 4\mathrm{k}$$

$$\left|\overrightarrow{\mathrm{PQ}} \times \overrightarrow{\mathrm{PR}}\right| = \sqrt{64+16+16} = 4\sqrt{6}$$

$$\mathrm{u} = \frac{\overrightarrow{\mathrm{PQ}} \times \overrightarrow{\mathrm{PR}}}{\left|\overrightarrow{\mathrm{PQ}} \times \overrightarrow{\mathrm{PR}}\right|} = \frac{1}{\sqrt{6}}(2\mathrm{i} + \mathrm{j} + \mathrm{k})$$