King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 101 Section 06 and 20 Quiz VI (Term 151)

Name : Solutions .... **ID #**.................................... Serial #: ....

1. **(5 points)** Verify that Rolle's Theorem is applicable to the function $f(x) = x - x^{1/3}$ on $[0, 1]$. Then find the value(s) of c promised by Rolle's Theorem.

- $f(x) = x - x^{1/3}$ is continuous on $[0,1]$
- $f'(x) = 1 - \frac{1}{3}x^{-2/3}$ is differentiable on $(0,1)$
- $f(0) = f(1) = 0$

$\Rightarrow$ all conditions of Rolle's Theorem are satisfied

$\Rightarrow$ Rolle's Theorem is applicable on $f \Rightarrow$ There is $c \in (0,1)$

such that $f'(c) = 0 \Rightarrow 1 - \frac{1}{3}x^{-2/2} = 1 - \frac{1}{3}c^{-2/3} = 0 \Rightarrow$

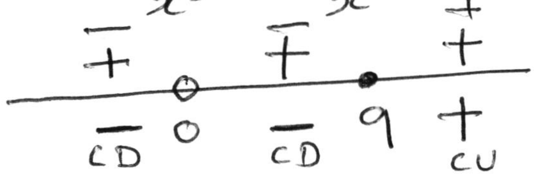
$c^{+2/3} = \frac{1}{3} \Rightarrow c^2 = \frac{1}{27} \Rightarrow c = \frac{-1}{3\sqrt{3}} \notin (0,1)$, or

$\boxed{c = \frac{1}{3\sqrt{3}} \in (0,1)}$ #.

2. **(5 points)** Given the curve $y = 1 + \dfrac{1}{x} - \dfrac{3}{x^2}$. Find the x-coordinates of all inflection points, and determine the concavity.

$y' = -\frac{1}{x^2} + \frac{6}{x^3} \Rightarrow y'' = \frac{2}{x^3} - \frac{18}{x^4} = \frac{2x - 18}{x^4}$

$\Rightarrow y'' = \frac{2(x-9)}{x^4}$



Notice that $0 \notin \operatorname{dom} y$.

From the sign graph of $y'' \Rightarrow$

- The curve is concave downward on $(-\infty, 0)$ and $(0, 9)$
- The curve is concave upward on $(9, \infty)$
- $x = 9$ is the x-coordinate of the only inflection point.

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3. **(7 points)** Given $f(x) = x^{1/3}(x+4)$ on $[-8, 8]$, find
a) All intervals of increasing or decreasing. b) All local extrema.
c) The absolute extrema values.

$f(x) = x^{4/3} + 4x^{1/3} \Rightarrow f'(x) = \frac{4}{3}x^{1/3} + \frac{4}{3}x^{-2/3} \Rightarrow$

$f'(x) = \frac{4}{3}x^{-2/3}(x+1) = \dfrac{4(x+1)}{3x^{2/3}} \Rightarrow$ the critical

numbers of f are $0 \in (-8, 8)$ since $f'(0)$ DNE

and $-1 \in (-8, 8)$ since $f'(-1) = 0 \Rightarrow$ The sign graph

of f': -8 [$-$ dec $\searrow$ -1 $\nearrow$ $+$ inc 0 $+$ inc] 8

a) f is increasing on $(-1, 0)$ and $(0, 8)$; decreasing on $(-8, -1)$

b) f has only one local min at $x = -1$ and no local max

c) $f(-8) = (-2)(-4) = 8$, $f(-1) = -3$, $f(0) = 0$, $f(8) = 24$

The absolute max value of $f = 24$ at $x = 8$

The absolute min value of $f = -3$ at $x = -1$ #

4. **(5 points)** Use the second derivative test to classify all local extrema of $f(x) = 2\cos x + \sqrt{3}\,x$ on $[0, 2\pi]$.

$f'(x) = -2\sin x + \sqrt{3} = 0$ when $\sin x = \frac{\sqrt{3}}{2} \Rightarrow$

The critical numbers of f on $[0, 2\pi]$ are $\frac{\pi}{3}$ and $\frac{2\pi}{3}$.

$f''(x) = -2\cos x \Rightarrow f''(\frac{\pi}{3}) = -1 < 0$ and $f''(\frac{2\pi}{3}) = 1 > 0$

Therefore, by the Second derivative test, we have

f has a local max at $\frac{\pi}{3}$ and a local min at $\frac{2\pi}{3}$.

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