February 28, 2016

Quiz 1 (30 min) - No work = No marks

Exercise 1: (5 points)
Transform this into a definite integral on the interval [0, 2]

$$\lim_{n\to\infty}\sum_{k=1}^{n}\frac{2}{n}+\frac{3k}{n^2}$$

Exercise 2: (5 points)
Assume f(x) is and increasing function over [0, 4]. Also assume that: f(0) = 1, f(1) = 2, f(3) = 5 and f(4) = 8.
Which of the following must be true?

a) $26 \le \int_0^4 f(x)dx \le 33$
b) $7 \le \int_0^4 f(x)dx \le 9$
c) $10 \le \int_0^4 f(x)dx \le 20$
d) $21 \le \int_0^4 f(x)dx \le 25$
e) $1 \le \int_0^4 f(x)dx \le 6$

Exercise 3: (5 points)
Assume that $\int_0^x f(t)dt + 2\sin(x) = 4x$. Find f(π)

Exercise 4: (5 points)
Let f(x) = x + 2 be defined over [-1; 1]. Find Riemann sum by partitioning the interval into n equal-width subintervals, and using the **left end point** of each subinterval as a sample point.

ex 1

$$\lim_{n\to\infty}\sum_{k=1}^{n}\frac{2}{n}+\frac{3k}{n^2}$$

$$=\lim_{n\to\infty}\sum_{k=1}^{n}\frac{2}{n}\left(1+\frac{3}{2}\frac{k}{n}\right)$$

$$=\lim_{n\to\infty}\frac{2}{n}\sum_{k=1}^{n}\left(1+\frac{3}{4}k\frac{2}{n}\right)$$

$$=\boxed{\int_0^2\left(1+\frac{3}{4}x\right)dx}$$

ex 3

$$\int_0^x f(t)\,dt + 2\sin x = 4x$$

differenciate both sides;

$$\Rightarrow f(x) + 2\cos x = 4$$

$$\Rightarrow f(x) = 4 - 2\cos x$$

$$\Rightarrow \boxed{f(\pi) = 6}$$

ex 4

$$R_n(x) = \sum_{i=0}^{n-1} f(x_i)\,\Delta x$$

$$= \frac{2}{n}\sum_{i=0}^{n-1} f\left(-1 + i\frac{2}{n}\right)$$

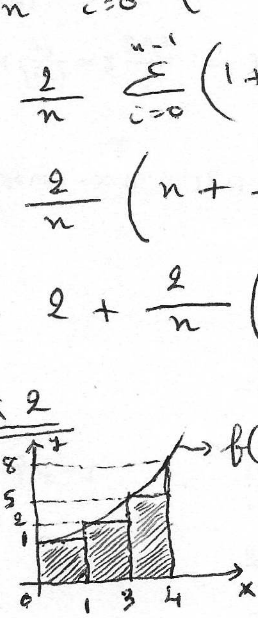
$$= \frac{2}{n}\sum_{i=0}^{n-1}\left(-1 + i\frac{2}{n} + 2\right)$$

$$= \frac{2}{n}\sum_{i=0}^{n-1}\left(1 + i\frac{2}{n}\right) = \frac{2}{n}\left(n + \frac{2}{n}\sum_{i=0}^{n-1} i\right)$$

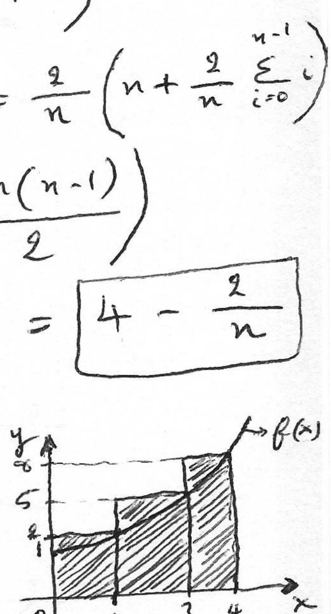
$$= \frac{2}{n}\left(n + \frac{2}{n}\,\frac{n(n-1)}{2}\right)$$

$$= 2 + \frac{2}{n}(n-1) = \boxed{4 - \frac{2}{n}}$$

ex 2



area = 1+4+5
= 10



area = 2+10+8
= 20

$$10 \le \int_0^4 f(x)\,dx \le 20$$