

Dept of Mathematics and Statistics  
King Fahd University of Petroleum & Minerals

AS482: Actuarial Contingencies II  
Dr. Mohammad H. Omar  
Major 2 Exam Term 161 FORM A  
Monday Dec 12 2016  
6.00pm-7.20pm

Name \_\_\_\_\_ ID#: \_\_\_\_\_ Serial #: \_\_\_\_\_

**Instructions.**

1. Please turn off your cell phones and place them under your chair. Any student caught with mobile phones on during the exam will be considered under the **cheating rules** of the University.
2. If you need to leave the room, please do so quietly so not to disturb others taking the test. No two person can leave the room at the same time. No extra time will be provided for the time missed outside the classroom.
3. Only materials provided by the instructor can be present on the table during the exam.
4. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.
5. Use the blank portions of each page for your work. Extra blank pages can be provided if necessary. If you use an extra page, indicate clearly what problem you are working on.
6. Only answers supported by work will be considered. Unsupported guesses will not be graded.
7. While every attempt is made to avoid defective questions, sometimes they do occur. In the rare event that you believe a question is defective, the instructor cannot give you any guidance beyond these instructions.
8. Mobile calculators, I-pad, or communicable devices are disallowed. Use regular scientific calculators or financail calculators only. Write important steps to arrive at the solution of the following problems.

The test is 80 minutes, GOOD LUCK, and you may begin now!

Question	Total Marks	Marks Obtained	Comments
1	5		
2	5		
3	6		
4	4+5=9		
5	10		
6	1+4=5		
Total	40		

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1. (5 points) Consider the joint density function of  $T_x$  and  $T_y$  given by

$$f_{x,y}(t_x, t_y) = \frac{4}{(1 + t_x + 2t_y)^3}$$

for  $t_x > 0$  and  $t_y > 0$ . Clearly,  $T_x$  and  $T_y$  are not independent. Find an expression for  ${}_n p_{xy}$ .

2. (5 points) A discrete **unit** benefit contingent contract is issued to the *last survivor* status  $(\overline{xx})$ , where the two future lifetime random variables  $T_x$  are **independent** of each other. The contract is funded by *discrete* net annual premiums, which are *reduced by 30% after* the first failure. Under the equivalence principle, find the value of the initial *net annual premium* given the values  $A_x = 0.40$ ,  $A_{xx} = 0.05$  and  $\ddot{a}_x = 10.00$ .

3. (6 points) The APV for a last survivor whole life insurance on  $(\overline{xy})$ , with **unit benefit** paid at the instant of failure of the status, was calculated assuming independent future lifetimes for  $(x)$  and  $(y)$  with constant hazard rate 0.06 for each. It is now discovered that although the hazard rate of 0.06 is correct, the two lifetimes are **not independent** since each includes a *common shock* hazard factor with constant force 0.02. The force of interest used in the calculation is  $\delta = 0.04$ .

Calculate the **increase in the APV** that results from the recognition of the *common shock* element.

4. (4+5=9 points) You are given

i) the following *spot rates* of interest per year.

$y_1$	$y_2$	$y_3$	$y_4$	$y_5$	$y_6$	$y_7$	$y_8$	$y_9$	$y_{10}$
0.032	0.035	0.038	0.041	0.043	0.045	0.046	0.047	0.048	0.048

ii) the survival model follows *Makeham's law* with  $A = 0.0001$ ,  $B = 0.00035$  and  $c = 1.075$ . (Note: *Makeham's law* states that  $\mu(x) = A + Bc^x$ )

iii) Premium are paid for the first 5 years only

iv) 6-year deferred 4-year insurance issued to a life aged 80 with sum insured \$100000 is payable at the *end of the year of death*

(a) Calculate the discount function  $v(t)$  for  $t = 1, 2, \dots, 10$ .

(b) Calculate the net level annual **premium** for this insurance policy using :

(1) a *level interest rate* of 4.8% per year effective, and,

(2) the *spot rate* of interest given by the table in (i) above.

5. (10 points) .Calculate all forward rates that can be inferred from the annual coupon bearing bond yield rates in the following table.

Maturity (in years)	Annual Yield Rates for Coupon bearing Bonds(%)
1	2.0
2	4.0
3	6.0
4	8.0

6. (1+4=5 points) For a fully discrete 3-year term life insurance on (50) you are given:

- (i) The death benefit is 5000.
- (ii) An extract from a mortality table

$x$	50	51	52
$q_x$	0.005	0.006	0.007

- (iii) The rate of interest is based on the yield curve at  $t = 0$ .

You are also given the following information based on the yield curve at  $t = 0$  :

$t$	0	1	2
Annual forward rate of interest	0.030	0.032	0.035

Calculate the second moment of the present value of the death benefit random variable.

- (A) 392,000
- (B) 406,000
- (C) 419,000
- (D) 432,000
- (E) 446,000

Work Shown (4 points):

Hence the answer is (\_\_\_)

**END OF TEST PAPER**