Math201.05, Quiz #1, Term 161

Name:

ID #: Solutions **Serial #:**

1. **[3 points]** Describe and sketch, with directions, the parametric curve given by

$$x = -\sqrt{t-1}, \qquad y = t+1, \qquad 1 \le t \le 10.$$

2. **[3 points]** Find the area of the surface generated by revolving the following parametric curve about the $x - axix$:

$$x = cos^3 t, \qquad y = sin^3 t, \qquad 0 \le t \le \pi/2.$$

3. **[4 points]** Find the area of the polar region that lies **inside both curves** $r = 1 + cos\theta$ and $r = 1 - cos\theta$.

Good luck,

Ibrahim Al-Rasasi

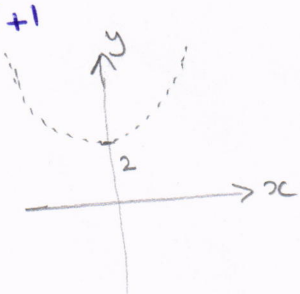
$\boxed{1}$. Convert to a Cartesian equation:

$x = -\sqrt{t-1} \Rightarrow \sqrt{t-1} = -x \Rightarrow t - 1 = x^2 \Rightarrow t = x^2 + 1$

$\Rightarrow y = t + 1 = x^2 + 1 + 1$

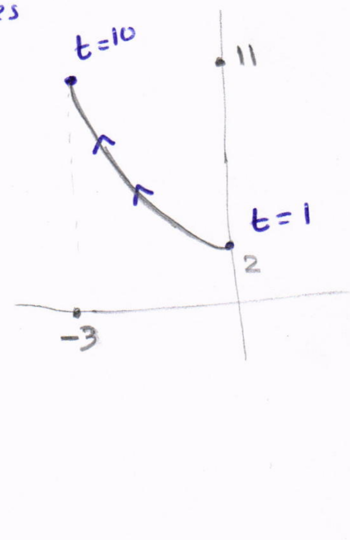
$\Rightarrow y = x^2 + 2$, a parabola

1.5 0.5



- For $1 \le t \le 10$, $x = -\sqrt{t-1} \le 0$ and $y = t + 1 > 0$.

So we choose the part of the parabola that lies in the 2nd quadrant.



- For the direction

$t = 1 \Rightarrow (x, y) = (0, 2)$, initial pt

$t = 10 \Rightarrow (x, y) = (-3, 11)$, terminal pt

$\boxed{2}$ $$S = \int_0^{\pi/2} 2\pi y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$ $\boxed{2}$

- $\frac{dx}{dt} = 3\cos^2 t\,(-\sin t)$; $\frac{dy}{dt} = 3\sin^2 t\,(\cos t)$
- $\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = 9\cos^4 t \sin^2 t + 9\sin^4 t \cos^2 t$

$$= 9\cos^2 t \sin^2 t\,\underbrace{(\cos^2 t + \sin^2 t)}_{=1}$$
$$= 9\cos^2 t \sin^2 t$$

- $\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = 3\,|\cos t \sin t|$

$$= 3\cos t \sin t \quad , \quad 0 \le t \le \frac{\pi}{2}$$

$$= 6\pi \int_0^{\pi/2} \sin^3 t \cdot \cos t \sin t\, dt$$
$$= 6\pi \int_0^{\pi/2} \sin^4 t \cdot \cos t\, dt \qquad : \quad u = \sin t \Rightarrow du = \cos t\, dt$$

- $t = 0 \Rightarrow u = 0$
- $t = \frac{\pi}{2} \Rightarrow u = 1$

$$= 6\pi \int_0^1 u^4\, du$$
$$= 6\pi \cdot \left.\frac{u^5}{5}\right]_0^1 = 6\pi\left(\frac{1}{5} - 0\right) = \frac{6\pi}{5}.$$

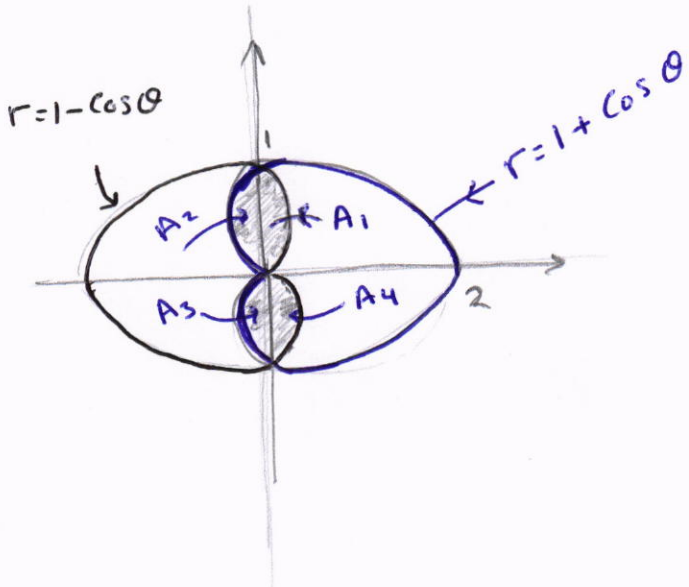
$\boxed{3}$ • points of intersection:

$1 + \cos\theta = 1 - \cos\theta$

$\Rightarrow 2\cos\theta = 0$

$\Rightarrow \cos\theta = 0$

$\Rightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2}$



• By Summetry, $A_1 = A_2 = A_3 = A_4$

- A_1 is the area of the region included by the curve $r = 1 - \cos\theta$ for $0 \leq \theta \leq \frac{\pi}{2}$

$$A_1 = \int_0^{\pi/2} \frac{1}{2}(1-\cos\theta)^2 \, d\theta$$

$$= \frac{1}{2}\int_0^{\pi/2} 1 - 2\cos\theta + \cos^2\theta \, d\theta$$

$$= \frac{1}{2}\int_0^{\pi/2} 1 - 2\cos\theta + \frac{1}{2} + \frac{1}{2}\cos(2\theta) \, d\theta$$

$$= \frac{1}{2}\int_0^{\pi/2} \frac{3}{2} - 2\cos\theta + \frac{1}{2}\cos(2\theta) \, d\theta$$

$$= \frac{1}{2}\left[\frac{3}{2}\theta - 2\sin\theta + \frac{1}{4}\sin(2\theta)\right]_0^{\pi/2}$$

$$= \frac{1}{2}\left[\left(\frac{3\pi}{4} - 2 + 0\right) - 0\right] = \frac{3\pi}{8} - 1$$

- The Total area $= 4A_1 = 4\left(\frac{3\pi}{8} - 1\right) = \frac{3\pi}{2} - 4.$