King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 102 - 12 Quiz II (Term 162)

Name : Solutions ID #.................................... Serial #: ....

1. (5 points) Evaluate $\int \frac{4 + 12x}{(3 - 4x - 6x^2)^{3/2}}\, dx = I$

Let $u = 3 - 4x - 6x^2 \Rightarrow du = (-4 - 12x)\,dx$

$\Rightarrow -du = (4 + 12x)\,dx$

$I = -\int u^{-3/2}\, du = -\frac{u^{-1/2}}{-1/2} + c$

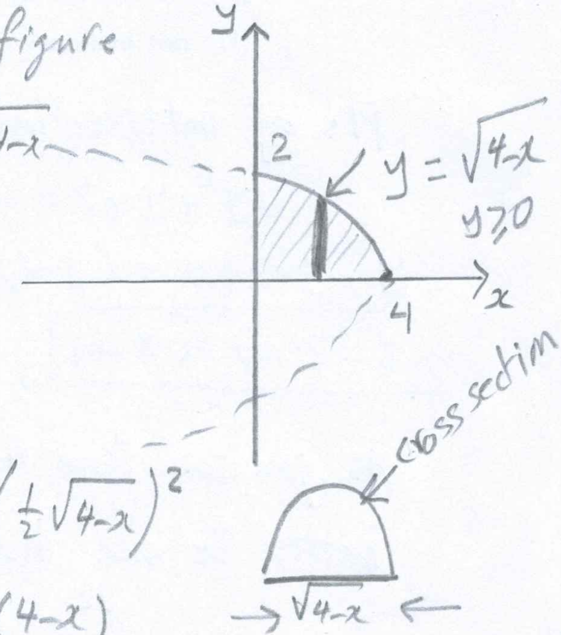
$= 2\,(3 - 4x - 6x^2)^{-1/2} + C$

2. (5 points) The base of a solid S is a **region in the first quadrant** bounded by $x = -y^2 + 4$. $x = 0$ and $y = 0$. If cross section perpendicular to the x-axis are semi circles, then find a definite integral which represents the volume of solid S. **Do not Evaluate the integral**.

The region as given in the figure

$y = \sqrt{4-x}$ or $y = \cancel{-\sqrt{4-x}}$

became $y \geq 0$



radius of a semi circle

$= \frac{1}{2}\sqrt{4-x}$

Area of semi-circle $= \frac{1}{2}\pi\left(\frac{1}{2}\sqrt{4-x}\right)^2$

$= \frac{1}{8}\pi\,(4-x)$

Volume $= \int_0^4 \frac{1}{8}(4-x)\, dx$

Math 102 - 12 Quiz II (Term 162)

3. (5 points) Use the substitution $u = 1 + e^{-x}$ to transform the definite integral

$I = \int_0^{\ln 1/2} \frac{e^{-2x}}{\sqrt{1+e^{-x}}}\, dx$ to a definite integral in terms of u.

(Do not Evaluate the new integral).

Let $u = 1 + e^{-x} \Rightarrow e^{-x} = u - 1$ and

$du = -e^{-x}\,dx \Rightarrow -du = e^{-x}\,dx$

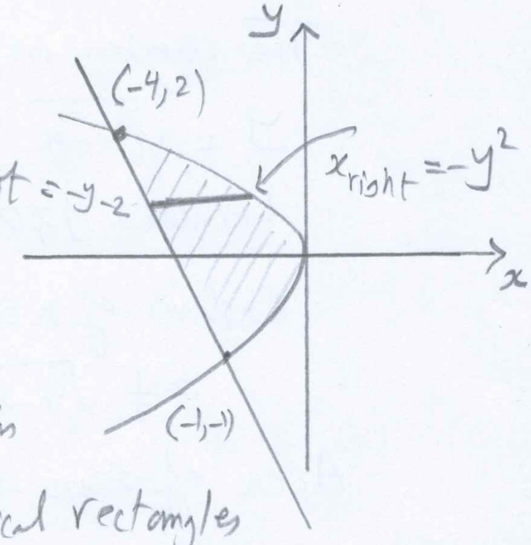
$x = 0 \Rightarrow u = 2$, $x = \ln\frac{1}{2} \Rightarrow u = 1 + e^{-\ln 1/2} = 1 + 2 = 3$

$$I = \int_0^{\ln\frac{1}{2}} \frac{e^{-x}}{\sqrt{1+e^{-x}}}\, e^{-x}\,dx = \int_2^3 \frac{u-1}{\sqrt{u}}\,(-du)$$

$$= \int_2^3 \frac{1-u}{\sqrt{u}}\,du$$

4. (5 points) Set up an integral which represents the area bounded by the parabola $y^2 = -x$ and the line $x + y + 2 = 0$

(Do not evaluate the integral)



Pts of intersection

$-y^2 + y + 2 = 0 \Rightarrow$

$y^2 - y - 2 = 0 = (y-2)(y+1)$

$\boxed{y = 2 \Rightarrow x = -4}$, $y = -1, x = -1$

As you see from the figure, it is better to use horizontal typical rectangles

$$\text{Area} = \int_{-1}^{2} [\underbrace{-y^2}_{x_{right}} - \underbrace{(y-2)}_{x_{left}}]\,dy = \int_{-1}^{2} (-y^2 + y + 2)\,dy$$