1. If f, g and h are differentiable functions, and $h(x) \neq 0$ for any x in its domain, then $\left(\frac{fg}{h}\right)'$ is equal to

$$\left(\frac{fg}{h}\right)' = \frac{(f\cdot g)'h - (fg)h'}{h^2} = \frac{(f'g + fg')h - (fg)h'}{h^2} = \frac{f'gh + fg'h - fgh'}{h^2}$$

a) $\dfrac{f'gh + fg'h - fgh'}{h^2}$

b) $\dfrac{f'g'h - fgh'}{h^2}$

c) $\dfrac{f'gh - fgh'}{h^2}$

d) $\dfrac{fgh' + fg'h - f'gh}{h^2}$

e) $\dfrac{fg'h - fgh'}{h^2}$

$$y = \frac{\sin^2(x)\ \tan^4(x)}{(x^2+1)^2}$$

$$\ln y = \ln \sin^2(x) + \ln \tan^4(x) - \ln(x^2+1)^2$$

$$= 2\ln(\sin(x)) + 4\ln(\tan(x)) - 2\ln(x^2+1)$$

2. If $y = \dfrac{\sin^2(x)\ \tan^4(x)}{(x^2+1)^2}$, then $y' =$

a) $\dfrac{\sin^2(x)\ \tan^4(x)}{(x^2+1)^2}\left(2\cot(x) + \dfrac{4\sec^2(x)}{\tan(x)} - \dfrac{4x}{x^2+1}\right)$

b) $2\cot(x) + \dfrac{4\sec^2(x)}{\tan(x)} - \dfrac{4x}{x^2+1}$

c) $\dfrac{\sin^2(x)\ \tan^4(x)}{(x^2+1)^2}\left(2\cot(x) + \dfrac{4\sec(x)}{\tan(x)} - \dfrac{4x}{x^2+1}\right)$

d) $\dfrac{\sin^2(x)\ \tan^4(x)}{(x^2+1)^2}\left(2\cot(x) + \dfrac{4\sec^2(x)}{\tan(x)} - \dfrac{2}{x^2+1}\right)$

e) $\dfrac{\sin^2(x)\ \cot^4(x)}{(x^2+1)^2}\left(2\tan(x) + \dfrac{4\sec^2(x)}{\tan(x)} - \dfrac{4x}{x^2+1}\right)$

$$\frac{y'}{y} = \frac{2}{\sin(x)}\cdot\cos(x) + \frac{4}{\tan(x)}\cdot\sec^2(x) - \frac{\cancel{2}}{x^2+1}\cdot 2x$$

$$\therefore\ y' = y\left(2\cdot\cot x + \frac{4\sec^2(x)}{\tan(x)} - \frac{4x}{x^2+1}\right)$$

$$\boxed{y' = \frac{\sin^2(x)\tan^4(x)}{(x^2+1)^2}\left(2\cot x + \frac{4\sec^2(x)}{\tan(x)} - \frac{4x}{x^2+1}\right)}$$

3. If $f(x) = e^{\frac{-4}{\pi}\cot^{-1}(ex)}$, then $f'\left(\frac{1}{e}\right) =$

(a) $\frac{2}{\pi}$

(b) $\frac{\pi}{4e}$

(c) $\frac{4}{\pi}$

(d) $\frac{\pi e}{4}$

(e) e

$$f'(x) = e^{-\frac{4}{\pi}\cot^{-1}(ex)} \cdot \frac{-1}{1+(ex)^2} \cdot \left(-\frac{4}{\pi}\right) \cdot e$$

$$f'\left(\tfrac{1}{e}\right) = e^{-\frac{4}{\pi}\cot^{-1}(1)} \cdot \frac{-1}{1+1} \cdot \left(-\frac{4}{\pi}\right)(e)$$

$$= e^{-\frac{4}{\pi}\cdot\frac{\pi}{4}} \cdot \frac{-1}{2} \cdot \frac{-4}{\pi} e$$

$$= e^{-1} \cdot \boxed{\frac{-1}{2} \cdot \frac{-4}{\pi}} \cdot e$$

$$= \frac{2}{\pi}$$

4. If $x^2 - y^2 = 1$, then $\frac{d^2y}{dx^2} =$

(a) $\frac{-1}{y^3}$

(b) $\frac{x}{y^2}$

(c) $\frac{x^2}{y}$

(d) $\frac{1}{y^3}$

(e) xy^3

$$x^2 - y^2 = 1$$

$$2x - 2y \cdot \frac{dy}{dx} = 0$$

$$\boxed{\frac{dy}{dx} = \frac{x}{y}}$$

$$\frac{d^2y}{dx^2} = \frac{1 \cdot y - \frac{dy}{dx} \cdot x}{y^2}$$

$$= \frac{y - \frac{x}{y} \cdot x}{y^2}$$

$$= \frac{y^2 - x^2}{y^3}$$

$$= \frac{-1}{y^3}$$

5. If $g(x) + x\,\sin(g(x)) = x^2$ and $g(0) = \theta$, then $g'(0) =$

(a) $-\sin(\theta)$

(b) θ

(c) 0

(d) $\csc(\theta) + \theta$

(e) $\theta - \sin(\theta)$

$g'(x) + (1 \cdot \sin(g(x)) + x \cdot \cos(g(x)) \cdot g'(x) = 2x$

$\therefore\ g'(0) + \sin(\theta) + 0 = 0$

$\boxed{g'(0) = -\sin\theta}$

6. If $y\cos(2x) + \sin^2(y) = \pi$, then $\dfrac{dy}{dx} =$

(a) $\dfrac{2y\,\sin(2x)}{\cos(2x) + \sin(2y)}$

(b) $\dfrac{2y\,\cos(y)}{\cos(2x) + 2\sin(y)\,\cos(x)}$

(c) $\dfrac{2x\,\sin(x)}{\cos(2y)}$

(d) $\dfrac{2y\,\sin(y)}{\cos(2y) + \sin(x)}$

(e) $\dfrac{\sin(2y) + \cos(2x)}{y\,\sin(x)}$

$\boxed{y' \cdot \cos(2x)} + y \cdot (-\sin(2x)(2) + \boxed{2\sin(y)\cos(y)y'} = 0$

$y' = \dfrac{2y\sin(2x)}{\cos(2x) + \sin(2y)}$

Notice that:

$\boxed{2\sin(y)\cos(y) = \sin(2y)}$

7. If $y = (ex)^{\sqrt{x}}$, then $y'(\frac{1}{e}) =$

(a) $\sqrt{e}$

(b) $\frac{-1}{\sqrt{e}}$

(c) $-\frac{1}{e}$

(d) $\frac{1}{\sqrt{e}}$

(e) e

$\ln y = \ln\left((ex)^{\sqrt{x}}\right) = \sqrt{x}\ln(ex)$

$\frac{1}{y} \cdot y' = \frac{1}{2\sqrt{x}}\ln(ex) + \sqrt{x} \cdot \frac{1}{ex} \cdot e$

At $x = \frac{1}{e}$: $\frac{1}{1} \cdot y' = \frac{1}{2\sqrt{\frac{1}{e}}} \overset{\text{Zero}}{\ln(1)} + \frac{1}{\sqrt{\frac{1}{e}}}$

$\boxed{y' = \sqrt{e}}$

8. If r and q are positive constants, then $\lim_{m \to 0}\left(1 + \frac{m}{r}\right)^{\frac{r}{mq}} = L$

(a) $e^{\frac{1}{q}}$

(b) e^{q}

(c) $e^{\frac{1}{r}}$

(d) e^{-rq}

(e) e^{rq}

Put $\frac{m}{r} = u$; $\frac{r}{mq} = \boxed{\frac{r}{m} \cdot \frac{1}{q} = \frac{1}{u} \cdot \frac{1}{q}}$

$L = \lim_{u \to 0} (1 + u)^{\frac{1}{u} \cdot \frac{1}{q}}$

$= \left(\lim_{u \to 0} (1 + u)^{\frac{1}{u}}\right)^{\frac{1}{q}}$

$= e^{\frac{1}{q}}$

9. Let the position of a particle be given by the equation $s(t) = t^3 - 9t^2 + 24t$, where s is measured in meters and t in seconds. The total distance (in meters) traveled in the first 3 seconds is

$v(t) = 3t^2 - 18t + 24$

$= 3(t^2 - 6t + 8) = 3(t-4)(t-2)$

(a) 22

(b) 38

(c) 20

(d) 18

(e) 12

$[\; +++ \quad \times \quad - - - \;] \; - \quad \times \quad ++++$

$0 \qquad 2 \qquad 3 \qquad 4$

distance $= |s(2) - s(0)| + |s(3) - s(2)|$

$= |20 - 0| + |18 - 20|$

$= 20 + 2 = 22$ m.

10. A ball is thrown up vertically with an initial velocity of $20\,m/s$. If its height after t seconds is $h(t) = 20t - 2t^2$, then the maximum height reached by the ball (in meters) is

$\boxed{\text{max. height when } v(t) = 0}$

(a) 50

(b) 20

(c) 40

(d) 10

(e) 25

$v(t) = 20 - 4t$.

$v(t) = 0 \iff t = 5$

$h(5) = 20(5) - 2(5)^2$

$= 100 - 50$

$= 50$ m.

11. A particle moves with **velocity** function

$$v(t) = t(t-3)^2,\ t > 0$$

Then the particle is slowing down when ______.

Notice that $v(t) > 0 \ \forall t > 0$.

(a) $1 < t < 3$

(b) $3 < t < 5$

(c) $0 < t < 1$

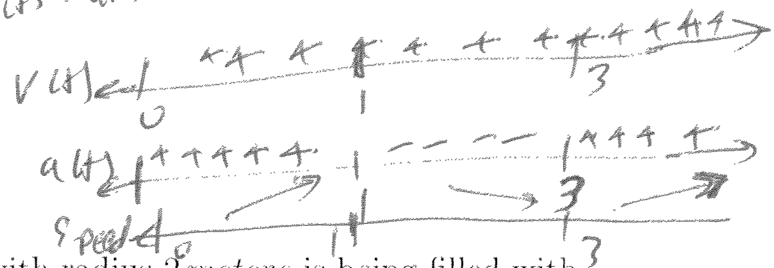
(d) $0 < t < 2$

(e) $2 < t < 5$

$$a(t) = v'(t) = 1 \cdot (t-3)^2 + 2(t-3) \cdot t$$
$$= t^2 - 6t + 9 + 2t^2 - 6t$$
$$= 3t^2 - 12t + 9$$
$$= 3(t^2 - 4t + 3)$$
$$= 3(t-1)(t-3)$$

slows down when $v(t) \cdot a(t) < 0$,

i.e. on $(1, 3)$



12. A cylindrical tank with radius $2\,meters$ is being filled with water at a rate of $8\,m^3/min$. The rate of the increase of the height (in m/min) is
(**Hint**: Recall that the volume of a cylinder with radius r and height h is $V = \pi r^2 h$)

(a) $\frac{2}{\pi}$

(b) 4

(c) $\frac{4}{\pi}$

(d) 2

(e) $\frac{\pi}{4}$

$$V = \pi r^2 h = 4\pi h \quad \boxed{(r = 2\ m)}$$

$$\boxed{\frac{dV}{dt} = 4\pi \cdot \frac{dh}{dt}}$$

$$\boxed{\frac{dh}{dt} = \frac{1}{4\pi} \cdot \frac{dV}{dt}}$$

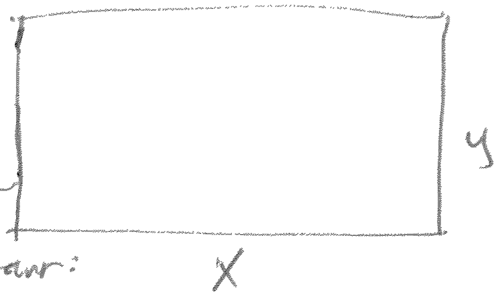
$$\left.\frac{dh}{dt}\right|_{\text{any time}} = \frac{1}{4\pi} \cdot 8 = \frac{2}{\pi}\ m/min.$$

13. The length of a rectangle is increasing at a rate of $4\,cm/s$ and its width is increasing at a rate of $2\,cm/s$. When the length is $15\,cm$, and the width is $10\,cm$, the rate (in cm^2/s) at which the area of the rectangle increases is

x = length
y = width

$A = xy$

$$\boxed{\frac{dA}{dt} = \frac{dx}{dt} y + x \cdot \frac{dy}{dt}}$$



(a) 70

(b) 60

(c) 40

(d) 30

(e) 8

At the given instant:

$$\frac{dA}{dt} = 4 \cdot (10) + 2 \cdot (15)$$
$$= 40 + 30$$
$$= 70\ cm^2/s$$

14. The x-coordinate of the point on $y = 1 + e^x - 3x$ where the tangent line is parallel to $\boxed{2x + y = 5}$ is

$y = -2x + 5 \; ; \; m = -2.$

(a) 0

(b) e

(c) $\ln 5$

(d) $\ln 3$

(e) $\ln 2$

$$\boxed{y' = e^x - 3}$$

$y' = m$

$e^x - 3 = -2$

$e^x = 1$

$$\boxed{x = \ln(1) = 0}$$

15. If the function

$$f(t) = \begin{cases} a\,t^{\frac{1}{4}} - e^t & 0 \le t < 1 \\ -\dfrac{1}{t^5} - be^t + 21 & t \ge 1 \end{cases}$$

is differentiable on $(0, \infty)$, where a and b are real constants, then $a + b =$

f(x) diff. & cts. at $(0,\infty)$, in Particular at $x=1$.

(a) 21

(b) 20

(c) 19

(d) 5

(e) 1

$\lim_{t\to 1^-} f(t) = \lim_{t\to 1^+} f(t)$

$\boxed{a - e = 20 - be}$ — ①

$f'(t) = \begin{cases} \frac{a}{4} t^{-\frac{3}{4}} - e^t & : 0 < t < 1 \\ \frac{5}{t^6} - be^t & : t > 1 \end{cases}$

$f'_-(1) = f'_+(1)$

$\boxed{\frac{a}{4} - e = 5 - be}$ — ②

Solve:

$a - e = 20 - be$

$-\left(\frac{a}{4} - e = 5 - be\right)$

$\frac{3a}{4} = 15$

$\boxed{a = 20}$

$\boxed{b = 1}$ from ①

So, $\boxed{a + b = 21}$

16. If $f(x) = \dfrac{x}{e^x}$, then $f^{(n)}(0) =$

$\boxed{f(x) = xe^{-x}}$

(a) $(-1)^{n+1}\, n$

(b) n

(c) $n!$

(d) $-(n!)$

(e) $-n$

$f'(x) = e^{-x} + x \cdot e^{-x}(-1)$

$= \boxed{e^{-x}(1 - x)}$

$f''(x) = -e^{-x}(1 - x) + e^{-x}(-1)$

$= \boxed{e^{-x}(x - 2)}$

$f'''(x) = -e^{-x}(x - 2) + e^{-x}(1)$

$= \boxed{e^{-x}(3 - x)}$

$\boxed{f^{(n)}(x) = e^{-x}(n - x)(-1)^{n+1}}$

$f'(0)$	1
$f''(0)$	-2
$f'''(0)$	3
$\vdots$	
n	$(-1)^{n+1} n$

17. The equation of normal line to the curve $f(x) = \dfrac{e^x}{1+x}$ at $(0,1)$ is

$$f'(x) = \frac{e^x(1+x) - 1\cdot(e^x)}{(1+x)^2}$$

(a) $x = 0$

(b) $y = 1$

$$= \frac{(e^x)(x)}{(1+x)^2}$$

(c) $y = x + 1$

(d) $y = 2x + 1$

$f'(0) = 0 = m_{\text{tangent}}$

(e) $y = -x + 1$

$\therefore$ The tangent line is horizontal.

The normal line is vertical and passes through $(0,1)$, whence its equation is $\boxed{x = 0}$

18. Let $y = f^{-1}(x)$. If $f(2) = 5$ and $f'(2) = 3$, then $y'(5) =$

(a) $\frac{1}{3}$

$y = f^{-1}(x)$

(b) $\frac{1}{5}$

$$\boxed{y' = \frac{1}{f'(f^{-1}(x))}}$$

(c) 15

(d) $\frac{1}{15}$

$$y'(5) = \frac{1}{f'(f^{-1}(5))}$$

(e) $\frac{3}{5}$

$$= \frac{1}{f'(2)}$$

$$= \frac{1}{3}$$

19. $\lim_{x\to 1} \dfrac{\sin(2x-2)}{x^2-1} =$

$\lim_{x\to 1} \dfrac{\sin(2(x-1))}{(x-1)(x+1)}$

(a) 1

(b) 2

(c) -1

(d) 0

(e) ∞

$\boxed{\text{set } \; u = x-1, \quad x\to 1;\ u\to 0}$

$= \left(\lim_{u\to 0} \dfrac{\sin 2u}{u}\right)\left(\lim_{x\to 1}\dfrac{1}{x+1}\right)$

$= 2\cdot\left(\lim_{u\to 0}\dfrac{\sin 2u}{2u}\right)\cdot\dfrac{1}{2}$

$= (2)(1)\left(\frac{1}{2}\right)$

$= 1$

20. The equation of the tangent line to $y = \cos(x) - \sin(x)$ at $(0, 1)$ is

(a) $y = -x + 1$

(b) $y = x + 1$

(c) $y = 1$

(d) $x = 0$

(e) $y = 2x + 1$

$y' = -\sin(x) - \cos(x)$

$y'(0) = 0 - 1$

$= -1.$

$\boxed{m_{\text{tangent}} = -1}$

$y - y_0 = m(x - x_0)$

$y - 1 = -1(x - 0)$

$\boxed{y = -x + 1}$