KFUPM **QUIZ (4a)** **MATH 201-171**

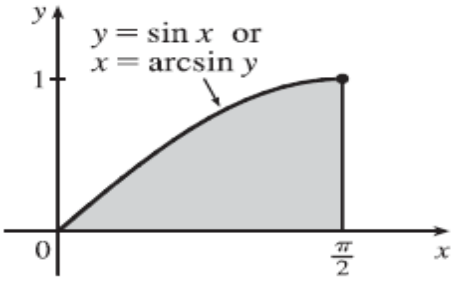
Name:_____ S.ID:_____ Sec.( ) Ser.# ( )

Q1) Evaluate the integral by reversing the order of integration.

$$\int_0^1 \int_{\arcsin y}^{\pi/2} \cos x \sqrt{1+\cos^2 x}\, dxdy.$$

Q2) Find the volume of the solid in the first octant bounded by the cylinder $z = 16 - x^2$ and the plane $y = 5$. What is the average of z over that area?

Q1)



$$\int_0^1 \int_{\arcsin y}^{\pi/2} \cos x \sqrt{1+\cos^2 x}\, dx\, dy$$

$$= \int_0^{\pi/2} \int_0^{\sin x} \cos x \sqrt{1+\cos^2 x}\, dy\, dx$$

$$= \int_0^{\pi/2} \cos x \sqrt{1+\cos^2 x}\, \big[\, y \,\big]_{y=0}^{y=\sin x}\, dx$$

$$= \int_0^{\pi/2} \cos x \sqrt{1+\cos^2 x}\, \sin x\, dx \qquad \left[\text{Let } u = \cos x,\ du = -\sin x\, dx,\ dx = du/(-\sin x)\right]$$

$$= \int_1^0 -u\sqrt{1+u^2}\, du = -\tfrac{1}{3}\left(1+u^2\right)^{3/2}\Big]_1^0$$

$$= \tfrac{1}{3}\left(\sqrt{8}-1\right) = \tfrac{1}{3}\left(2\sqrt{2}-1\right)$$

Q2)

The cylinder intersects the xy-plane along the line $x = 4$, so in the first octant, the solid lies below the surface $z = 16 - x^2$ and above the rectangle $R = [0,4] \times [0,5]$ in the xy-plane.

$$V = \int_0^5 \int_0^4 (16 - x^2)\, dx\, dy = \int_0^4 (16 - x^2)\, dx \int_0^5 dy = \left[16x - \tfrac{1}{3}x^3\right]_0^4 \big[\, y \,\big]_0^5 = \left(64 - \tfrac{64}{3} - 0\right)(5-0) = \tfrac{640}{3}$$

$$f_{ave} = \frac{640}{3} \times \frac{1}{20} = \frac{32}{3}$$