Math 201.10, Quiz # 4, Term 171

Name:

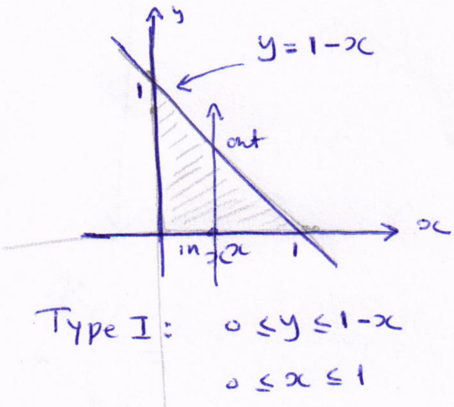
Solutions

ID#: **Serial #:**

1. **[3 points]** Set up an integral, but **DO NOT EVALUATE**, for finding the volume of the solid that lies under the surface $f(x,y) = 1 + 2x^2 + y^2$ and above the triangular region, in the $xy-plane$, with vertices $(0,0)$, $(1,0)$, $(0,1)$.
2. **[4 points]** Evaluate $\int_0^4 \int_{\sqrt{x}}^2 3y^3 e^{xy} dydx$.
3. **[3 points]** Evaluate $\iint_R \frac{\ln(x^2+y^2)}{x^2+y^2} dA$, where $R = \{(x,y): 1 \le x^2 + y^2 \le e^2, x \le 0\}$.

Good luck,

Ibrahim Al-Rasasi

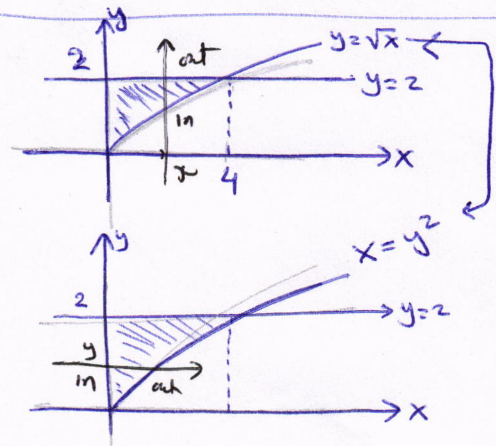


$\boxed{1}$ $V = \iint_R f(x,y)\, dA$ (1)

$= \int_0^1 \int_0^{1-x} (1+2x^2+y^2)\, dy\, dx$ (2)

Type I: $0 \le y \le 1-x$
$0 \le x \le 1$

$\boxed{2}$ Reverse the order of integration



$R = \{(x,y): 0 \le x \le 4, \sqrt{x} \le y \le 2\}$, Type I

$= \{(x,y): 0 \le x \le y^2, 0 \le y \le 2\}$, Type II

$$\int_0^4 \int_{\sqrt{x}}^2 3y^3 e^{xy}\, dy\, dx = \int_0^2 \int_0^{y^2} 3y^3 e^{xy}\, dx\, dy \quad (2)$$

$$= \int_0^2 3y^3 \cdot \frac{1}{y} e^{xy} \Big]_{x=0}^{x=y^2} dy$$

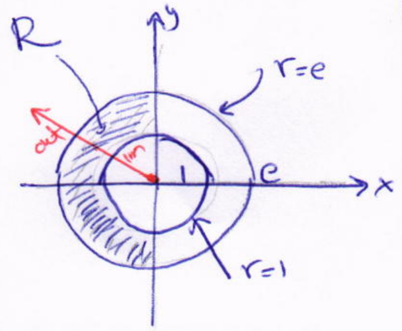
$$\boxed{\int e^{xy}\, dx = \frac{1}{y} e^{xy} + C}$$

$$= \int_0^2 3y^2 (e^{y^3} - 1)\, dy \quad (1)$$

$$= \int_0^2 3y^2 e^{y^3} - 3y^2\, dy$$

$$= e^{y^3} - y^3 \Big]_0^2 = (e^8 - 8) - (1 - 0) = e^8 - 9 \quad (1)$$

$\boxed{3}$ Change to polar Coord.



$$R = \{(r,\theta) : 1 \le r \le e,\ \tfrac{\pi}{2} \le \theta \le \tfrac{3\pi}{2}\}$$

$$\iint_R \frac{\ln(x^2+y^2)}{x^2+y^2}\, dA = \int_{\frac{\pi}{2}}^{3\pi/2}\int_1^e \frac{\ln(r^2)}{r^2}\cdot r\, dr\, d\theta \quad 2$$

$$= \int_{\frac{\pi}{2}}^{3\pi/2}\underbrace{\int_1^e \frac{2\ln r}{r}\, dr}\, d\theta$$

$u = \ln r$ $\quad \cancel{dv = \frac{1}{r}}$

$du = \frac{1}{r}\, dr$ $\quad \cancel{v =}$

$$\int \frac{\ln r}{r}\, dr = \int u\, du = \frac{1}{2}u^2 + C$$
$$= \frac{1}{2}(\ln r)^2 + C$$

$$= \int_{\pi/2}^{\frac{3\pi}{2}} 2\cdot \frac{1}{2}(\ln r)^2\Big]_{r=1}^{r=e}\, d\theta$$

$$= \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 1\, d\theta = \theta\Big]_{\frac{\pi}{2}}^{\frac{3\pi}{2}} = \pi$$

0.5 $\qquad$ 0.5