KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS

Department of Mathematics and Statistics

Math 201 First Major Exam

Semester (172)

Feb. 28, 2018 at 5.45 PM-7.45 PM

Name: ..... KEY .....

I.D: ..... **Section:** ..... **Serial #:** .....

Instructions

1. No electronic device (such as calculator, mobile phone, smart watch) is allowed in this exam.
2. Justify your answers. No credit is given for (correct) answers not supported by work.

Question	Points
1	/11
2	/16
3	/12
4	/10
5	/15
6	/12
7	/12
8	/12
Total	/100

Question 1 (6+5 points)

a. Consider the following parametric curve

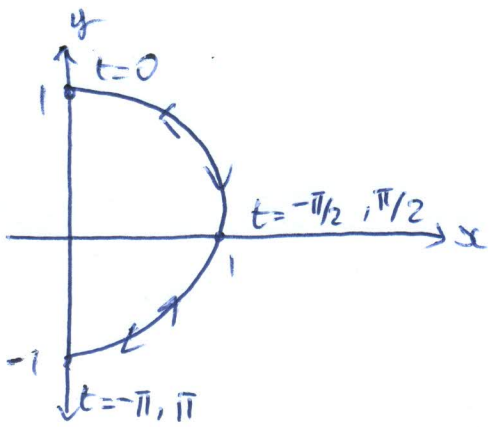
$$x = \sin^2 t, \quad y = \cos t, \quad t \in [-\pi, \pi]$$

i) Write the cartesian equation for the above curve.

ii) Sketch the graph and indicate with an arrow the direction of the curve.

(i) $x + y^2 = 1$, $0 \le x \le 1$ (2) + (1)

(ii)



t	x	y
$-\pi$	0	-1
$-\pi/2$	1	0
0	0	1
$\pi/2$	1	0
π	0	-1

sketch = (1)

direction = (1)

table = (1)

b. Find parametric equations to represent the line segment from

$$(2, -5) \text{ to } (-3, 1).$$

the cartesian eqn of the line through these points is

$$y + 5 = \frac{6}{5}(2 - x) \qquad (1)$$

Call $t = 2 - x$ (1)

Then a parametrization of this line segment is

$$x = 2 - t, \quad y = \frac{6}{5}t - 5, \quad t \in [0, 5] \qquad (1)+(1)+(1)$$

Question 2 (4+5+7 points)

Consider the following parametric curve

$$C: x = t^2 - 3t + 2, \quad y = t^2 - 4, \quad t \in (-\infty, \infty).$$

a. Find an equation for the tangent line to C at the point where $t = 1$.

b. Find the point/s on C where the tangent is vertical.

c. Find the area of the region enclosed by the curve C and the $y-axis$.

a) $\text{slope} = \left.\frac{dy}{dx}\right|_{t=1} = \left.\frac{2t}{2t-3}\right|_{t=1} = -2$ (2)

the point where $t=1$ is $(0, -3)$ (1)

the eqn of the tangent line is $y+3 = -2x$ (1)

b) V.T when $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} \neq 0$ (1)

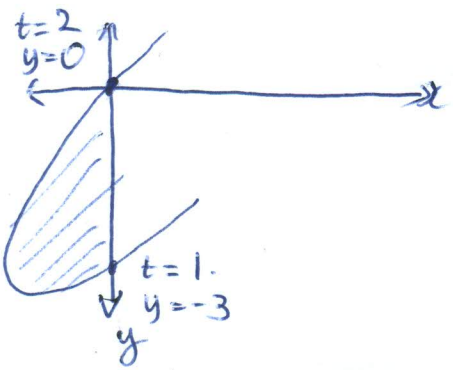
$\frac{dx}{dt} = 2t-3 = 0 \Rightarrow t = \frac{3}{2}$. (1)

$\frac{dy}{dt} = 2t$ and $\left.\frac{dy}{dt}\right|_{t=\frac{3}{2}} = 3 \neq 0$ (1)

So there is V.T at $t = \frac{3}{2}$ (1)

The point assoc. to $t=3/2$ is $(-1/4, -7/4)$ (1)

c) $x = 0 \Rightarrow t^2 - 3t + 2 = 0 \Rightarrow t = 1, t = 2 \Rightarrow y(1) = -3, \; y(2) = 0.$



$\text{Area} = \int_{-3}^{0} -x \, dy$. (4)

$= -\int_{2}^{1} (t^2 - 3t + 2) \cdot 2t \, dt$. (1)

$= -2 \left(\frac{t^4}{4} - t^3 + t^2\right)\Big|_1^2 = 1/2$ (1)+(1)