King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

MATH 102

EXAM II

173

Monday 23/7/2018

Net Time Allowed: 120 minutes

MASTER VERSION

1. $\displaystyle\int \frac{5x+1}{2x^2-x-1}\,dx =$

(a) $\frac{1}{2}\ln|2x+1| + 2\ln|x-1| + C$

(b) $\frac{1}{3}\ln|2x-1| + 2\ln|x+1| + C$

(c) $\frac{1}{4}\ln|2x+1| + \frac{1}{3}\ln|x-1| + C$

(d) $\frac{1}{5}\ln|2x-1| + \frac{1}{6}\ln|x-2| + C$

(e) $\frac{1}{6}\ln|2x| - \frac{1}{5}\ln|3x| + C$

2. $\displaystyle\int_{\pi/4}^{\pi/2} \csc^4(x)\,dx =$

(a) $\frac{4}{3}$

(b) $\frac{-2}{3}$

(c) 0

(d) $\frac{16}{31}$

(e) $\frac{16}{3}$

3. The average value of the function

$$f(x) = \sec^2 x \quad \text{on} \quad \left[0, \frac{\pi}{4}\right]$$

is

(a) $\frac{4}{\pi}$

(b) $\frac{4\pi}{3}$

(c) $\frac{3}{\pi}$

(d) $\frac{2}{3\pi}$

(e) $\frac{1}{2}$

4. The value of c that satisfies the Mean Value Theorem of integrals for $f(x) = 1 + 3\sqrt{x}$ over the interval $[0, 1]$ is

(a) $\frac{4}{9}$

(b) $\frac{25}{4}$

(c) $\frac{1}{16}$

(d) $\frac{9}{25}$

(e) $\frac{36}{4}$

5. The general form of the partial fractions for

$$\frac{3x^2+1}{(x^2-5x+6)(x-2)(x^3+4x)}$$

is given by

(a) $\dfrac{A}{x-2}+\dfrac{B}{(x-2)^2}+\dfrac{C}{x}+\dfrac{D}{x-3}+\dfrac{Ex+F}{x^2+4}$

(b) $\dfrac{A}{(x-2)^2}+\dfrac{B}{x}+\dfrac{C}{x-3}+\dfrac{Dx+E}{x^2+4}$

(c) $\dfrac{A}{x-2}+\dfrac{B}{(x-2)^2}+\dfrac{C}{x}+\dfrac{D}{x-3}+\dfrac{E}{x^2+4}$

(d) $\dfrac{A}{x-2}+\dfrac{B}{x^2-2}+\dfrac{C}{x}+\dfrac{D}{x-3}+\dfrac{Ex+F}{x^2+4}$

(e) $\dfrac{A}{x-2}+\dfrac{B}{x-2}+\dfrac{C}{x}+\dfrac{D}{x-3}+\dfrac{Ex+F}{x-4}$

6. The length of the curve

$$y=\sqrt{x-x^2}+\sin^{-1}\left(\sqrt{x}\right)$$

from $x=\frac{1}{9}$ to $x=\frac{1}{4}$ is equal to

(a) $\dfrac{1}{3}$

(b) 1

(c) $\dfrac{1}{5}$

(d) $\dfrac{\pi}{2}$

(e) 2π

7. $\int_0^2 \frac{x^3+4}{x^2+4}\,dx =$

(a) $\frac{4+\pi}{2} - 2\ln(2)$

(b) $\frac{\pi}{2} + \ln\left(\frac{1}{2}\right)$

(c) $\pi + 2\ln\left(\frac{4}{5}\right)$

(d) $\frac{2\pi}{3} + \ln\left(\frac{1}{2}\right)$

(e) $\frac{1+\pi}{2} + \ln(3)$

8. The region bounded by the graphs of $y = -x^2 + 4x - 3$, and $y = 0$ is rotated about the line $x = -1$. Then the volume of resulting solid is given by

(a) $-2\pi \int_1^3 (x+1)(x-1)(x-3)\,dx$

(b) $-2\pi \int_0^1 (x+1)(x-1)(x-3)\,dx$

(c) $-2\pi \int_{-1}^3 (x+1)(x+2)(x-3)\,dx$

(d) $2\pi \int_0^3 (x+1)(x-1)(x+3)\,dx$

(e) $2\pi \int_1^3 (x-1)(x-2)(x-3)\,dx$

9. $\int_0^{\pi/2} \sin^4 x \, dx =$

(a) $\frac{3\pi}{16}$

(b) $\frac{\pi}{4}$

(c) $\frac{3}{8}$

(d) $\frac{35}{2}$

(e) $\frac{16}{3}$

10. $\int_1^e 4x \ln x \, dx =$

(a) $e^2 + 1$

(b) $e + 4$

(c) $\frac{1}{4} e^{1/2}$

(d) $1 - \ln 4$

(e) $2 + \frac{1}{3} e^{1/3}$

11. $\int_{\pi/4}^{\pi/2} x \csc^2 x \, dx =$

(a) $\frac{\pi}{4} + \ln \sqrt{2}$

(b) $\frac{\pi}{2} - \ln 2\sqrt{2}$

(c) $\frac{\pi}{\sqrt{2}} - 4 \ln \sqrt{3}$

(d) $\frac{3\pi}{4\sqrt{2}} - \ln \sqrt{3}$

(e) $\frac{\pi}{4} + \ln \sqrt{5}$

12. $\int_{0}^{\frac{\pi}{3}} \frac{\tan^3 x}{\sec x} \, dx =$

(a) $\frac{1}{2}$

(b) $\frac{1}{3}$

(c) $\frac{1}{5}$

(d) $\frac{1}{6}$

(e) $\frac{1}{4}$

13. $\int_{\pi/2}^{\pi} e^{2x} \cos 2x \, dx =$

(a) $\dfrac{e^{\pi} + e^{2\pi}}{4}$

(b) $\dfrac{e^{2\pi} - e^{3\pi}}{3}$

(c) $\dfrac{e^{\pi}}{2}$

(d) $\dfrac{e^{2\pi}}{4}$

(e) $\dfrac{e^{\pi} - e^{2\pi}}{2}$

14. The region bounded by $x = y^2 - 1, \; x = y + 1$ and $y \geq 0$ is rotated about the x-axis. Then the volume of resulting solid is given by

(a) $2\pi \int_0^2 (2y + y^2 - y^3) \, dy$

(b) $2\pi \int_1^3 (2y + y^2 - y^3) \, dy$

(c) $2\pi \int_1^3 (\sqrt{x+1} - (x-1)) \, dx$

(d) $2\pi \int_0^2 (1 + y - y^2) \, dy$

(e) $2\pi \int_0^2 (1+y)^2 - (y^2)^2 \, dy$

15. $\int_0^{2\pi} \sin\left(\frac{x}{2}\right) \cos\left(\frac{x}{4}\right)\ dx =$

(a) $\frac{8}{3}$

(b) $\frac{9}{5}$

(c) $\frac{5}{3}$

(d) $\frac{2}{5}$

(e) $\frac{2}{7}$

16. $\int \frac{\sqrt{1+x}}{x}\, dx =$

(a) $2\sqrt{1+x} + \ln\left(\frac{|\sqrt{1+x}-1|}{\sqrt{1+x}+1}\right) + C$

(b) $\frac{\sqrt{1+x}}{2} + \tan^{-1}\left(\frac{\sqrt{1+x}-1}{\sqrt{1+x}+1}\right) + C$

(c) $x + \ln\left(\sqrt{1+x}+1\right) + \tan^{-1}\left(\sqrt{1+x}-1\right) + C$

(d) $\frac{\sqrt{1+x}}{2} + \frac{1}{1-x} + C$

(e) $\ln\sqrt{\frac{x+1}{x-1}} + C$

17. Using the substitution $t = \tan\left(\frac{x}{2}\right)$ where $-\pi < x < \pi$,
$\displaystyle\int \frac{dx}{\cos\, x - \sin\, x - 1} =$

(a) $\ln\left|\dfrac{t+1}{t}\right| + C$

(b) $\ln\left|\dfrac{t^2-1}{t}\right| + C$

(c) $\dfrac{t^2-1}{t} + C$

(d) $\ln\left|\dfrac{-1}{t-1}\right| + C$

(e) $\dfrac{1}{t-1} + C$

18. If $\displaystyle\int_0^{\alpha} \frac{dx}{(\alpha^2 + x^2)^{\frac{3}{2}}} = 2\sqrt{2}$ and $\alpha > 0$, then $\alpha =$

(a) $\dfrac{1}{2}$

(b) $\dfrac{1}{3}$

(c) $\dfrac{1}{5}$

(d) $\dfrac{1}{6}$

(e) $\dfrac{2}{3}$

19. $\int_0^1 \sqrt{4x - x^2}\, dx =$

(a) $\dfrac{2\pi}{3} - \dfrac{\sqrt{3}}{2}$

(b) $\dfrac{2\pi}{5}\sqrt{3}$

(c) $\dfrac{\pi}{5} + \dfrac{\sqrt{3}}{4}$

(d) $\dfrac{2\sqrt{3}}{7}$

(e) $-\dfrac{7\sqrt{3}}{4}$

20. The arc length of the curve $y = \dfrac{x^3}{12} + \dfrac{1}{x}$ from $x = 1$ to $x = 4$ is

(a) 6

(b) 9

(c) 3

(d) 7

(e) 8