Math201.10, Quiz #1, Term 181

Name:

ID#: Solutions

Serial #:

---

1. **[3 points]** Find a Cartesian equation for the parametric curve and sketch it indicating with arrows the direction on the curve as t increases:

$$x = 2 + tant, \qquad y = 1 - 2sect, \qquad -\frac{\pi}{4} \le t \le \frac{\pi}{4}.$$

2. **[3 points]** Find the slope of the tangent line to the polar curve $r = 2\cos(\frac{\theta}{2})$ at the point corresponding to $\theta = \frac{\pi}{2}$.
3. **[4 points]** Let R be the region inside the circle $r = 2$ and outside the circle $r = 4sin\theta$. Sketch the region R and find its area.

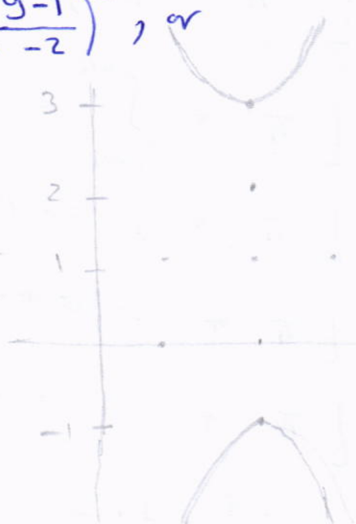
Good luck,

Ibrahim Al-Rasasi

$\boxed{1}$ Since $1 + \tan^2 t = \sec^2 t$, then $1 + (x-2)^2 = \left(\frac{y-1}{-2}\right)^2$, or

1.5

$$\frac{(y-1)^2}{4} - (x-2)^2 = 1, \text{ a hyperbola}$$

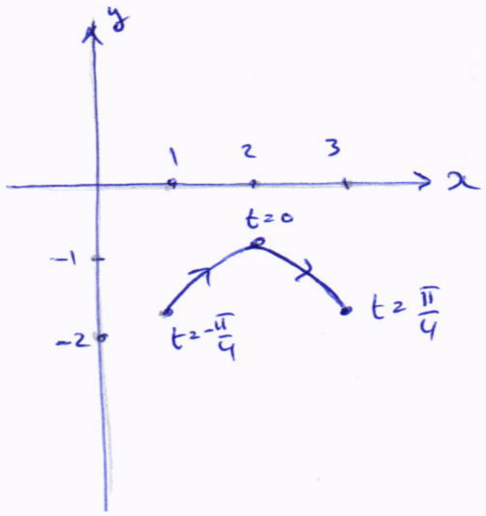


For direction

t	(x, y)	
$-\frac{\pi}{4}$	$(1, 1-2\sqrt{2})$	Initial
0	$(2, -1)$	
$\frac{\pi}{4}$	$(3, 1-2\sqrt{2})$	terminal

• For $t \in [-\frac{\pi}{4}, \frac{\pi}{4}]$, $y < 0$.

~~We~~



2 $r = 2\cos\left(\frac{\theta}{2}\right)$, $\theta = \frac{\pi}{2}$, $f(\theta) = 2\cos\left(\frac{\theta}{2}\right)$

- $$\frac{dy}{dx} = \frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta} = \frac{-2\sin\left(\frac{\theta}{2}\right)\cdot\frac{1}{2}\cdot\sin\theta + 2\cos\left(\frac{\theta}{2}\right)\cos\theta}{-2\sin\left(\frac{\theta}{2}\right)\cdot\frac{1}{2}\cdot\cos\theta - 2\cos\left(\frac{\theta}{2}\right)\sin\theta}$$

2

- $$\text{slope} = \left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{2}} = \frac{-\frac{\sqrt{2}}{2}\cdot 1 + 2\cdot\frac{\sqrt{2}}{2}\cdot 0}{-\frac{\sqrt{2}}{2}\cdot 0 - 2\cdot\frac{\sqrt{2}}{2}\cdot 1}$$

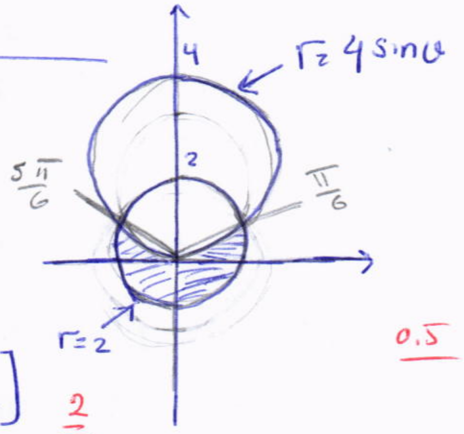
$$= \frac{-\frac{\sqrt{2}}{2}}{-\sqrt{2}} = \frac{1}{2}$$

---

3 points of intersection:

$4\sin\theta = 2 \implies \sin\theta = \frac{1}{2}$

$\implies \theta = \frac{\pi}{6}, \frac{5\pi}{6}$ 0.5



0.5

By symmetry about the y-axis.

$$A = 2\cdot\left[\int_{-\frac{\pi}{2}}^{0} \frac{1}{2}\cdot 2^2 \, d\theta + \int_{0}^{\pi/6} \frac{1}{2}\left[2^2 - (4\sin\theta)^2\right] d\theta\right]$$ 2

$$= 2\left[\left.2\theta\right]_{-\frac{\pi}{2}}^{0} + \int_0^{\pi/6} \frac{1}{2}\left(4 - 16\sin^2\theta\right) d\theta\right]$$ 0.5

$$= 2\left[\pi + \int_0^{\pi/6} 2 - 8\cdot\frac{1-\cos(2\theta)}{2} \, d\theta\right.$$

$$= 2\left[\pi + \int_0^{\pi/6} 2 - (4 - 4\cos(2\theta)) \, d\theta\right]$$

$$= 2\left[\pi + \int_0^{\pi/6} -2 + 4\cos(2\theta) \, d\theta\right.$$

$$= 2\left[\pi + \left(-2\theta + 2\sin(2\theta)\right)\Big]_0^{\pi/6}\right]$$

$$= 2\left[\pi + \left(-\frac{\pi}{3} + 2\sin\left(\frac{\pi}{3}\right)\right)\right]$$

$$= 2\left[\frac{2\pi}{3} + 2\cdot\frac{\sqrt{3}}{2}\right]$$

$$= \frac{4\pi}{3} + 2\sqrt{3}$$ 0.5

Math201.15, Quiz #1, Term 181

Name:

Solutions

ID#: **Serial #:**

---

1. **[3 points]** Find a Cartesian equation for the parametric curve and sketch it indicating with arrows the direction on the curve as t increases:

$$x = 2 + sint, \qquad y = 1 + 2cost, \qquad \frac{\pi}{2} \leq t \leq 2\pi.$$

2. **[3 points]** Find an equation for the tangent line to the following parametric curve at the point corresponding to $t = 1$:

$$x = 1 - lnt, \qquad y = t + t^2.$$

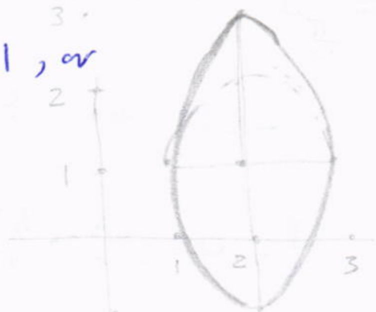
3. **[4 points]** Let R be the region inside the lemniscate $r^2 = 2\sin(2\theta)$ and outside the circle $r = 1$. Sketch the region R and find its area.

Good luck,

Ibrahim Al-Rasasi

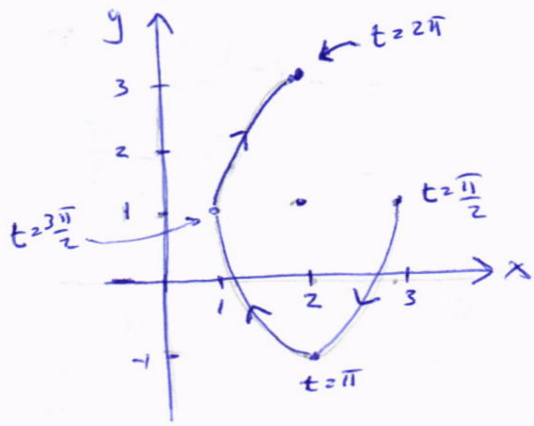
$\boxed{1}$ Since $\sin^2 t + \cos^2 t = 1$, then $(x-2)^2 + \left(\frac{y-1}{2}\right)^2 = 1$, or

1.5 $\quad (x-2)^2 + \frac{(y-1)^2}{4} = 1$, an ellipse



For direction,

t	(x,y)	
$\frac{\pi}{2}$	$(3,1)$	initial point
π	$(2,-1)$	
$\frac{3\pi}{2}$	$(1,1)$	
2π	$(2,3)$	terminal point



2 $x = 1 - \ln t \,,\; y = t + t^2 \,,\quad t = 1$

- point : $t = 1 \Rightarrow x = 1 - \ln(1) = 1 - 0 = 1 \,;\; y = 1 + 1^2 = 2 \Rightarrow (x, y) = (1, 2)$
- $\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{1 + 2t}{-\frac{1}{t}} = -t(1+2t)$
- slope $= \left.\dfrac{dy}{dx}\right|_{t=1} = -(1)(1 + 2(1)) = -3$

Eq. of tangent line : $y - 2 = -3(x - 1) \Rightarrow y = -3x + 5$

---

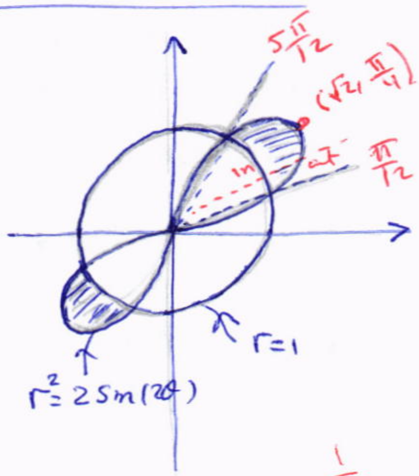
3 . points of intersection

$r^2 = r^2 \Rightarrow 2\sin(2\theta) = 1$

$\Rightarrow \sin(2\theta) = \frac{1}{2}$

$\Rightarrow$ ~~$\sin 2\theta$~~ $2\theta = \frac{\pi}{6} \,,\; \frac{5\pi}{6}$

$\Rightarrow \theta = \frac{\pi}{12} \,,\; \frac{5\pi}{12} \in QI$ 0.5



1

By Symmetry ,

$$A = 2 \cdot \int_{\pi/12}^{5\pi/12} \frac{1}{2}\left(2\sin(2\theta) - 1\right) d\theta$$ 1.5

$$= \int_{\pi/12}^{5\pi/12} 2\sin(2\theta) - 1 \; d\theta$$

$$= \left. -\cos(2\theta) - \theta \right]_{\pi/12}^{5\pi/12}$$ 0.5

$$= \left[-\cos\left(\tfrac{5\pi}{6}\right) - \tfrac{5\pi}{12}\right] - \left[-\cos\left(\tfrac{\pi}{6}\right) - \tfrac{\pi}{12}\right]$$

$$= -\cos\left(\tfrac{5\pi}{6}\right) + \cos\left(\tfrac{\pi}{6}\right) - \tfrac{5\pi}{12} + \tfrac{\pi}{12}$$

$$= -\left(-\tfrac{\sqrt{3}}{2}\right) + \tfrac{\sqrt{3}}{2} - \tfrac{4\pi}{12}$$

$$= \sqrt{3} - \frac{\pi}{3}$$ 0.5

OR $\longrightarrow$ $$A = 4 \cdot \int_{\pi/12}^{\pi/4} \frac{1}{2}\left(2\sin(2\theta) - 1\right) d\theta .$$