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ON THE EFFECT OF THE HIGHER MODES ON THE SCATTERING OF LOVE WAVES AT THE BOUNDARY OF WELDED LAYERED QUARTER-SPACES

By A. NIAZY AND M.H. KAZI

ABSTRACT

In this paper we investigate the effect of higher modes on the scattering of plane harmonic Love waves, normally incident (from either side) upon the plane of discontinuity in the horizontally discontinuous structure consisting of welded layered quarter-spaces with a plane surface. We find that the effect of the higher modes on the scattering of an incident fundamental mode from the hard into the soft medium is considerable for large impedance contrasts with or without body-wave conversion, and is negligible when the impedance contrast is low. Our results indicate that contributions from body-wave conversion are important in higher Love mode studies when there is large contrast in the elastic constants of the media on the two sides of the vertical plane of discontinuity.

INTRODUCTION

In a previous paper [Niazy and Kazi (1980)], we used a method based on an integral equation formulation together with the Schwinger-Levine variational principle to investigate the two-dimensional problems of the propagation of plane, harmonic, monochromatic Love waves incident normally (from either side) upon the plane of discontinuity in the structure consisting of welded layered quarter-spaces with a plane surface as shown in Figure 1. Formulas for complex reflection and transmission coefficients when the range of frequency is such that there are single (fundamental) modes in the left and right-hand domains shown in Figure 1, were obtained for the plane-wave approximation and the variational approximation which incorporates the contributions caused by body-wave conversion. Numerical computation of the results, under both approximations, indicated that when the impedance contrast across the lateral discontinuity becomes large, the amplitude of the transmitted wave from the hard medium into the soft can be amplified by more than an order of magnitude. The result is consistent with a well-known fact in earthquake engineering which is the amplification of earthquake ground motion by soft sedimentary basins. In this case the body-wave contribution was found to be of considerable significance for a frequency range of physical interest. It was also remarked that higher order Love modes, neglected in the study, could be quite important under both approximations and should be considered in any realistic computation of the ground response and response of the building to the ground motion.

In this paper we present numerical computation of results in the presence of the first higher and the second higher modes. Our results show that the effect of the higher modes on the scattering of an incident fundamental mode from hard into the soft medium is considerable, when the impedance contrast across the lateral discontinuity is high under both approximations i.e. with or without body-wave conversion, and is negligible when the impedance contrast is low. The results under the variational approximation which incorporates the contributions from body waves tend to confirm their importance in higher Love mode studies when there is large contrast in the elastic constants of the media on the two sides of the lateral discontinuity.

Basic Equations

Let us suppose that a quarter-space consisting of a material of rigidity μ_2, shear velocity β_2 and density ρ_2, overlain by a layer of depth h, density ρ_1, rigidity $\mu_1(<\mu_2)$ and shear velocity $\beta_1(<\beta_2)$, is in welded contact with a similar quarter-space of material of rigidity μ_2', shear velocity β_2' and density ρ_2', overlain by a layer of depth h, density ρ_1', rigidity $\mu_1'(<\mu_2')$ and shear velocity $\beta_1'(<\beta_2')$ (See Fig. 1). We take the vertical plane of contact between the two structures to be $x = 0$ in the coordinate system shown in the figure and regard the top plane surface $z = 0$ as stress-free. All the materials are considered to be isotropic, homogeneous and linearly elastic.

We confine our attention to the two-dimensional problems of propagation of time-harmonic Love waves incident normally (from either side) upon the plane of contact. Thus only SH wave motions need to be considered. We denote the displacement fields to the left and the right the plane of discontinuity (regions I $(x<0)$ and II $(x>0)$ in Fig. 1) by

$$e^{-i\omega t}\, v(x,z) = e^{-i\omega t}\, v_1(x,z), \quad 0 \leq z \leq h, \quad x < 0,$$

$$= e^{-i\omega t}\, v_2(x,z), \quad h \leq z, \quad x < 0,$$

and

$$e^{-i\omega t}\, v'(x,z) = e^{-i\omega t}\, v_1'(x,z), \quad 0 \leq z \leq h, \quad x > 0,$$

$$= e^{-i\omega t}\, v_2'(x,z), \quad h \leq z, \quad x > 0,$$

(ω being the angular frequency) respectively, in the coordinate system chosen in such a way that the z - axis is directed vertically downward, the free surface is given by $z = 0$ and the plane of welded contact is given by $x = 0$. The conditions at the free surface $z = 0$ and the plane of welded contact $x = 0$ imply :

$$\frac{\partial v_1}{\partial z} = 0 \quad \text{and} \quad \frac{\partial v_1'}{\partial z} = 0 \qquad \text{at} \quad z = 0, \tag{1a}$$

$$v = v' \qquad \text{at} \quad x = 0, \quad z \geq 0, \tag{1b}$$

$$\mu(z) \frac{\partial v}{\partial x} = \mu'(z) \frac{\partial v'}{\partial x} \qquad \text{at} \quad x = 0 \ , \quad z \geq 0, \tag{1c}$$

where

$$\mu(z) = \mu_1, \quad 0 \leq z < h, \quad x < 0, \tag{2}$$

$$= \mu_2, \quad h < z, \quad x < 0,$$

and

$$\mu'(z) = \mu_1', \quad 0 \leq z < h, \quad x > 0, \tag{3}$$

$$= \mu_2', \quad h < z, \quad x > 0$$

In our work on the spectral representation of the Love wave operator associated with a homogeneous layer of density ρ_1, rigidity μ_1 and shear velocity β_1 overlying a homogeneous half-space of density ρ_2, rigidity μ_2 and shear velocity β_2 (cf. Kazi 1976), we find that the spectrum of the operator is the disjoint union of the discrete spectrum, which corresponds to the ordinary Love modes, and a continuous spectrum which is the interval $(-\infty , \frac{\omega^2}{\beta_2^2})$ on the real axis of the complex λ - plane, where $\lambda = k^2$, k being the wave number. Consequently, any function of z which is square - integrable relative to the weight function $\mu(z)$ [as in (2)] may be expressed in terms of eigenfunctions corresponding to the eigenvalues belonging to the discrete spectrum and the improper eigenfunctions belonging to the continuous spectrum. We write the complete solution for the displacement $v(x,z)$ in domain I $(x<0)$, in terms of the eigenfunctions and the improper eigenfunctions for a surface-layer of depth h, rigidity μ_1 and shear velocity β_1, overlying a half-space of rigidity μ_2 and shear velocity β_2. Similarly, we write the complete solution for the displacement $v'(x,z)$ in domain II $(x<0)$ in terms of the eigenfunction and the improper eigenfunctions for the same structure as for domain I, but with a surface-layer of rigidity μ_1' and shear velocity β_1', overlying a half-space of rigidity μ_2' and shear velocity β_2'. Specifically : in domain I $(x<0,\ z\geq0)$

$$v(x,z) = -\Big[\sum_{m=1}^{r}(A_m e^{-ik_m|x|} + B_m e^{ik_m|x|})\,\chi_m(z) + \int_0^{\omega/\beta_2}\{C(k)e^{-ik|x|} + D(k)e^{ik|x|}\}\phi(z,k)\,dk + \int_0^{\infty} E(k)\, e^{-k|x|}\,\psi(z,k)\,dk\Big], \tag{4}$$

and in domain II $(x>0,\ z\geq0)$

$$v'(x,z) = \Big[\sum_{m=1}^{s}(A_m' e^{-ik'_m x} + B_m' e^{ik_m' x})\,\chi_m'(z) + \int_0^{\omega/\beta_2'}\{C'(k')\, e^{-ik'x} + D'(k')e^{ik'x}\}\phi'(z,k')\,dk' + \int_0^{\infty} E'(k')\, e^{-k'x}\,\psi'(z,k')\,dk'\Big], \tag{5}$$

where (using the formulae derived in Kazi (1976)) χ_m, χ_m' are eigenfunctions and k_m, k_m' correspond to real and positive eigenvalues for $x<0$ and $x>0$, respectively ;

$$\chi_m(z) = \phi_1^{(m)}(z) \qquad 0 \le z \le h, \tag{6}$$

$$= \phi_2^{(m)}(z) \qquad h \le z,$$

$$\chi_m'(z) = \phi_1'^{(m)}(z), \quad 0 \le z \le h, \tag{7}$$

$$= \phi_2'^{(m)}(z), \quad h \le z,$$

$$\phi_1^{(m)}(z) = F_m \frac{\cos(\sigma_1^{(m)} z)}{\cos(\sigma_1^{(m)} h)} \quad , \tag{8}$$

$$\phi_2^{(m)}(z) = F_m \, e^{\sigma_2^{(m)}(h-z)} \quad ,$$

$$F_m = \left[\frac{2\sigma_2^{(m)}}{\mu_2} \cdot \frac{(\beta_1^{-2} - U_m^{-1} C_m^{-1})}{(\beta_1^{-2} - \beta_2^{-2})} \right]^{\frac{1}{2}} \tag{9}$$

$$\phi_1'^{(m)}(z) = F_m' \frac{\cos(\sigma_1^{(m)\prime} z)}{\cos(\sigma_1^{(m)\prime} h)} \tag{10}$$

$$\phi_2'^{(m)}(z) = F_m' \, e^{\sigma_2^{(m)}(h-z)} \quad ,$$

$$F_m' = \left[\frac{2\sigma_2'^{(m)}}{\mu_2'} \cdot \frac{(\beta_1'^{-2} - U_m'^{-1} C_m'^{-1})}{(\beta_1'^{-2} - \beta_2'^{-2})} \right]^{\frac{1}{2}} \tag{11}$$

(U_m, U'_m are the group velocities and C_m C'_m are the phase velocities in the nth modes),

$$\sigma_1(\lambda) = (\frac{\omega^2}{\beta_1^2} - \lambda)^{1/2} , \quad \sigma_2(\lambda) = (\lambda - \frac{\omega^2}{\beta_2^2})^{1/2} , \tag{12}$$

$$\sigma_1^{(m)} = \sigma_1(\lambda_m) \quad , \sigma_2^{(m)} = \sigma_2(\lambda_m) \tag{13}$$

$$\lambda_m = k_m^2 , \quad k_m > 0 \tag{14}$$

and similarly for $\sigma_1'(\lambda')$, $\sigma_2'(\lambda')$, $\sigma_1'^{(m)}$, $\sigma_2'^{(m)}$ and λ_m' .

$\lambda = \lambda_m = k_m^2$, $m = 1, 2, \ldots r$ satisfy the

Love wave dispersion equation

$$\mu_1\sigma_1\tan(\sigma_1 h) - \mu_2\sigma_2 = 0 , \tag{15}$$

whereas $\lambda = \lambda_m' = k_m'^2$, $m = 1, 2, \ldots s$ satisfy the dispersion equation

$$\mu_1' \sigma_1'\tan (\sigma_1' h) - \mu_2' \sigma_2' = 0 \tag{16}$$

$\psi(z,k)$, the improper eigenfunctions in the domain $x<0$, corresponding to the continuum of improper eigenvalues $\lambda = (ik)^2$, $k>0$ and representing non-propagated modes (owing to the factor $\exp(-k|x|)$ in the integral containing ψ), are given by :

$$\psi(z,k) = \psi_1(z,k), \quad 0 \le z \le h \tag{17}$$

$$= \psi_2(z,k), \quad h \le z,$$

where

$$\psi_1(z,k) = G_k \frac{\cos(\sigma_1^{(k)} z)}{\cos(\sigma_1^{(k)} h)} , \quad 0 \le z \le h \tag{18}$$

and

$$\psi_2(z,k) = G_k \frac{\sin\{\theta^{(k)} - s_2^{(k)}(z-h)\}}{\sin\theta^{(k)}}, \quad z > h, \tag{19}$$

with

$$G_k = \frac{\sqrt{2k}\,\sin\,\theta^{(k)}}{\sqrt{\pi\,\mu_2\,s_2^{(k)}}} \tag{20}$$

$$s_2^{(k)} = \left(\frac{\omega^2}{\beta_2^2} - \lambda\right)^{\frac{1}{2}} \text{ real and positive } \quad (\lambda = -k^2,\ k > 0) \tag{21}$$

and

$$\theta^{(k)} = \tan^{-1}\left(\frac{\mu_2 s_2^{(k)} \cot\,\sigma_1^{(k)} h}{\mu_1\,\sigma_1^{(k)}}\right) \tag{22}$$

Similarly $\psi'(z,k')$, the improper eigenfunctions in the domain $x > 0$ corresponding to the improper eigenvalues $\lambda' = (ik')^2$, $k' > 0$ are given by

$$\begin{aligned} \psi'(z,k') &= \psi_1'(z,k'), & 0 \le z \le h, \\ &= \psi_2'(z,k'), & h \le z, \end{aligned} \tag{23}$$

where

$$\psi_1'(z,k') = G_{k'}' \frac{\cos(\sigma_1'^{(k')} z)}{\cos(\sigma_1'^{(k')} h)} \tag{24}$$

and

$$\psi_2'(z,k') = G_{k'}' \frac{\sin\{\theta'^{(k')} - s_2'^{(k')}(z-h)\}}{\sin\,\theta'^{(k')}} \tag{25}$$

with

$$\theta'^{(k')} = \tan^{-1}\left(\frac{\mu_2'\,s_2'^{(k')}\,\cot\,(\sigma_1'^{(k')}\,h)}{\mu_1'\,\sigma_1'^{(k')}}\right) \tag{26}$$

$G'_{k'}$, $s_2'^{(k')}$ having expressions similar to those for G_k, $s_2^{(k)}$ but in primed notation.

The improper eigenfunctions $\phi(z,k)$ and $\phi'(z,k')$ corresponding to the improper eigenvalues $\lambda = k^2$, $\lambda = k'^2$, $(0 \leq k \leq \frac{\omega}{\beta_2}$, $0 < k' < \frac{\omega}{\beta_2'})$ respectively, have expressions similar to those for $\psi(z,k)$ and $\psi'(z,k')$. Owing to the form of x - dependence in the integrals containing ϕ, ϕ', these correspond to waves travelling in the x-direction.

For Love waves incident from either side, we have to determine various coefficients in the expressions (4) and (5) subject to the matching conditions (1b) and (1c). For this purpose, we sought an integral equation formulation of the problem (see Niazy and Kazi (1980), Kazi (1978)) and introduced the concept of a scattering matrix to describe the diffraction of Love waves. As a first approximation, we neglected the modes corresponding to the continuous spectrum and found expressions for the elements of the scattering matrix. We then applied the Schwinger - Levine variational principle to improve upon the accuracy of the first approximation and to incorporate in a second approximation the effects of the non-propagated modes and the body - waves arising out of continuous spectrum. Complex reflection and transmission coefficients can be obtained through a transmission matrix related to the scattering matrix. Formulas for reflection and transmission coefficients to be computed numerically in this paper are given in the appendix.

RESULTS AND CONCLUSIONS

In all numerical computations in this paper we consider the two models shown in Figures 2. Both models have the same structure on the left-hand side of the vertical plane of discontinuity with the elastic parameters of the layer corresponding to Gabro(San Marcos, California) and those of the quarter-space to Eclogite at 1000 bars as reported by Press (1966). The material constants on the right-hand mediums in the two structures are specified in such a way as to obtain large impedance constrasts across the lateral discontinuity. The media on the left-hand and right-hand domains will sometimes be called hard and soft media respectively.Dispersion curves (k versus ω) for these models are shown in Figures 3.

Figures 4 show the absolute value of the transmission coefficients for the single (fundamental) mode of a wave incident from left to right into the fundamental mode on the right, plotted as functions of frequency for the models shown in Figures 2 (top and bottom diagrams in Figures 2 and Figures 4 correspond to each other) when :

i) higher modes are neglected in the right-hand domains (see curves marked by + which are computed from the formula in equation (A35) given in the appendix)

ii) the effect of the first higher mode is taken into consideration (see curves marked by * which are computed from equation (A41))

iii) the effect of first two higher modes is taken into account (see curves marked by x which are computed from the equation (A47)).

The formulas which we use are derived from a variational approximation which incorporates body-wave contributions. We remark that A's and B's in equations (4) and (5) are the coefficients of the normalized modes. In calculating the ratios of the surface amplitudes of incoming and outgoing waves it is, therefore, necessary to multiply the appropriate coefficients by the ratio $\chi(0)_{outgoing} / \chi(0)_{incoming}$. [$\chi_i(0)_{incoming}$ and $\chi(0)_{outgoing}$ denote the surface amplitudes of the incoming and outgoing waves respectively when they are normalized over z]. The curves clearly indicate that when all three modes are taken into account and for the 1:5 velocity contrast, there is a sudden drop in the transmission coefficient from fundamental into the fundamental mode at the onset of the first higher mode and the second higher mode. For the 1:10 contrast (upper curves) there is a rise at the onset of the first higher mode and a drop for the other modes. Moreover, the transmission coefficient from fundamental mode into the fundamental mode when the first higher mode is included but the second higher mode is neglected shows a similar behavior at the onset of the first higher mode but shows a rise at the onset of the second higher mode instead of a drop .When the two higher modes are neglected, the coefficients exhibit substantial relative maxima at the onset of the higher modes. We conclude, therefore, that neglecting the higher modes leads to a significant overestimate of the transmission coefficients when a single (fundamental) mode is incident from left to right, particularly when the impedance contrast across the lateral discontinuity is large as in the models we considered.

The curves showing the absolute values of the reflection coefficients plotted as functions of frequency, corresponding to the three cases mentioned above are shown in Figures 5. The symbols +, * and x have the same meaning as in Figures 4. The curves marked by +, * and x have been computed from the formulas given in equations (A34), (A43) and (A50) respectively.

Figures 6 show the absolute values of the transmission coefficients, plotted as functions of frequency, for the fundamental mode incident from left to right into the fundamental mode as for the two models considered in Figures 2 in the presence of the first and the second higher modes in the right-hand domain under the plane-wave approximation (when the body-wave conversion is neglected) and the variational approximation (which incorporates effects of body-wave conversion). The curves marked by + show the results under plane-wave approximation and have been computed from the formula given in equation (A24). The curves marked by * show the results under variational approximation and have been computed from the formula given in equation (A47). These results indicate that the amount of energy diffracted into body-waves is considerable in frequency ranges of physical interest but is negligible in the high frequency range and for extremely low frequencies.

Figures 7 show the absolute values of the reflection coefficients, plotted as functions of frequency, corresponding to the two cases considered in Figures 6. The symbols + and * have the same interpretation as the Figure 6. The curves marked by + have been computed from the formula given in equation (A27) whereas the curves marked by * have been computed from the formula given in equation (A50).

Figures 8 show the absolute values of the transmission coefficients for the fundamental mode incident from left to right into the first higher mode on the right for the two cases considered in Figures 6; the effect of the second higher mode is included. As in Figures 6 the curves marked by + and * correspond to the plane-wave and the variational approximations and are computed from the formulas given in equations (A22) and (A42) respectively.

The results indicate that if body-wave contributions are ignored then the transmission coefficients are overestimated, except in the higher frequency range.

Figures 9 show the absolute values of the transmission coefficients for the fundamental mode incident from left to right into the second higher mode on the right for the two cases considered in Figures 6; the effect of the first higher mode is included. Again, the curves marked by + and * correspond to the plane-wave and the variational approximations and are computed from the formulas given in equations (A26) and (A49) respectively. Here, we notice that the effect of the inclusion of body-waves is small even in the high contrast cases which we consider.

Figures 10 show the absolute values of the transmission coefficients from fundamental into the fundamental mode, plotted as functions of frequency, for the structures given in Figures 2, when a fundamental mode is incident from the right-hand domain or from the left-hand domain. These curves are marked by * and + respectively. The absolute value of the transmission coefficients of fundamental mode incident from the left into the first higher mode on the right, plotted as function of frequency, is shown by the curves marked by x; the effect of the second higher mode is neglected. Figures 11 show the corresponding reflection coefficients. The two sets of Figures show that very little energy is transmitted in the high contrast cases which we consider, when the wave is incident from the soft into the hard medium as compared to the amount of energy transmitted into the first two modes when the wave is incident from the hard to the soft medium. Figures 10 also show that the transmitted first higher mode can have an amplitude comparable to or may even exceed the amplitude of the transmitted fundamental mode in very high contrast cases and for certain frequency intervals.

In summary, we can say that when there is large contrast in the material properties on the two sides of the lateral discontinuity in our models, the transmission coefficients for the fundamental mode of a wave incident from the soft into the hard media into the fundamental mode on the other side exhibit spike-like features at frequencies corresponding to the onset of the higher modes Increasing the contrast also causes an increase in the sharpness of those spike-like features. Neglecting the modes above and including a certain higher mode, causes an exaggeration of the spike-like feature corresponding to that mode. For a given contrast, the higher the mode number, the less profound is the spike-like feature corresponding to it. Neglecting the body-wave contribution causes a serious overestimate of the transmission coefficients. The higher the mode number of a transmitted mode (generated from an incident fundamental mode) the less serious is the effect of the body waves. The larger the contrast in the material properties, the more important is the effect of the body wave. The effect of the higher modes and body waves diminishes with increasing frequency and is negligible for extremely low frequencies.

The transmission coefficients for waves incident from the soft into the hard media and all reflection coefficients exhibit somewhat similar sensitivities to the higher modes and to the body-waves effect but are of less interest in applied seismology and earthquake engineering.

APPENDIX : FORMULAS FOR REFLECTION AND TRANSMISSION COEFFICIENTS

1. Approximation based upon the neglect of the modes of the continuous spectrum.

(i) If $r = 1$ in equation (4) and $s = 1$ in equation (5) (i.e. the frequency is such that there are single (fundamental) modes in domains I and II shown in Fig.1) then we found (Niazy and Kazi (1980))

$$\underline{B} = \underline{T} \cdot \underline{A} \qquad \text{(A1)}$$

where

$$\underline{T} = \frac{1}{1 + P_{11}^2} \begin{bmatrix} -1+P_{11}^2 & -2\lambda_{11}P_{11} \\ \dfrac{-2P_{11}}{\lambda_{11}} & 1-P_{11}^2 \end{bmatrix} \qquad \text{(A2)}$$

$$\underline{A} = \begin{bmatrix} A_1 \\ A_1' \end{bmatrix} , \qquad \text{(A3)}$$

$$\underline{B} = \begin{bmatrix} B_1 \\ B_1' \end{bmatrix} \qquad \text{(A4)}$$

where A_1, B_1 are the coefficients of χ_1 in equation (4) and A_1', B_1' are the coefficients of χ_1' in equation (5)

$$\lambda_{11} P_{11} = \frac{F_1' F_1}{\mu_1'} \quad \frac{\mu_1 \mu_2' \, \overset{(1)}{\sigma}{}_2' - \mu_1' \mu_2 \, \overset{(1)}{\sigma}_2}{(k_1^2 - k_1'^2) + \frac{\omega^2}{b_1^2}} + \frac{\mu_1' \mu_2 (\overset{(1)}{\sigma}{}_2' - \overset{(1)}{\sigma}_2)}{(k_1'^2 - k_1^2) + \frac{\omega^2}{b_2^2}} \tag{A5}$$

with

$$\lambda_{11} = \left(\frac{k_1'}{k_1} \right)^{\frac{1}{2}} , \tag{A6}$$

$$\frac{1}{b_1^2} = \frac{1}{\beta_1'^2} - \frac{1}{\beta_1^2} , \tag{A7}$$

$$\frac{1}{b_2^2} = \frac{1}{\beta_2^2} - \frac{1}{\beta_2'^2} . \tag{A8}$$

The terms appearing in (A5) - (A8) have the same meaning as in the section on basic equations.

If the incident wave is travelling from <u>right</u> to <u>left</u> so that

$$A = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

then the transmission coefficient

$$\overleftarrow{B}_1 = \frac{-2\lambda_{11} P_{11}}{1 + P_{11}^2} \tag{A9}$$

and the reflection coefficient

$$\overleftarrow{B}_1' = \frac{1 - P_{11}^2}{1 + P_{11}^2} \tag{A10}$$

If the incident wave is travelling from <u>left</u> to <u>right</u> with

$$A = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

then the transmission coefficient

$$\vec{B}{}_1' = \frac{-2P_{11}}{\lambda_{11}(1 + P_{11}^2)} = \frac{k_1}{k_1'}\overset{\leftarrow}{B}_1 \qquad \text{(A11)}$$

and the reflection coefficient

$$\vec{B}_1 = \frac{P_{11}^2 - 1}{1 + P_{11}^2} = (-1)\times\overset{\leftarrow}{B}{}_1' \qquad \text{(A12)}$$

(ii) If $r = 1$, $s = 2$ in equations (4) and (5) (i.e. the frequency here is such that single (fundamental) mode exists in the left hand domain and the first two modes exist on the right) then we have

$$\underline{B} = \underline{T} \cdot \underline{A} , \qquad \text{(A13)}$$

where from the general form of the transmission matrix obtained in Niazy and Kazi (1980)

$$\underline{T} = \frac{1}{1 + P_{11}^2 + P_{21}^2} \begin{pmatrix} -1+P_{11}^2+P_{21}^2 & -2\lambda_{11}P_{11} & -2\lambda_{21}P_{21} \\ \frac{-2P_{11}}{\lambda_{11}} & -P_{11}^2 + 1 + P_{21}^2 & \frac{-2P_{11}\lambda_{21}P_{21}}{\lambda_{11}} \\ \frac{-2P_{21}}{\lambda_{21}} & \frac{-2P_{21}P_{11}\lambda_{11}}{\lambda_{21}} & -P_{21}^2 + 1 + P_{11}^2 \end{pmatrix} \tag{A14}$$

$$\underline{A} = \begin{pmatrix} A \\ A_1' \\ A_2' \end{pmatrix} , \qquad \underline{B} = \begin{pmatrix} B \\ B_1' \\ B_2' \end{pmatrix} \tag{A15}$$

(Here A_1, B_1 are the coefficients of χ_1 in equation (4) , A_1', B_1' are the coefficients of χ_1' and A_2', B_2' are the coefficients of χ_2' in equation (5))

$$\lambda_{21}P_{21} = \frac{F_2'F_1}{\mu_1'} \quad \frac{\mu_1\mu_2' \overset{(2)}{\sigma_2'} - \mu_1'\mu_2 \overset{(1)}{\sigma_2}}{(k_1^2 - k_2'^2) + \frac{\omega^2}{b_1^2}} + \frac{\mu_1'\mu_2(\overset{(2)}{\sigma_2'} - \overset{(1)}{\sigma_2})}{(k_2'^2 - k_1^2) + \frac{\omega^2}{b_2^2}} \tag{A16}$$

$$\lambda_{21} = \left(\frac{k_2'}{k_1}\right)^{\frac{1}{2}} , \left(\frac{1}{b_1^2} \text{ and } \frac{1}{b_2^2} \text{ as in (A7) \& (A8)}\right) \tag{A17}$$

(Again the terms appearing in (A16) and (A17) have the same meaning as in section on basic equations). P_{11} and λ_{11} have the same expressions as before.

Thus for an incident wave travelling from right to left with

$$\underline{A} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} ,$$

we obtain the transmission coefficient

$$\overleftarrow{B}_1 = \frac{-2\lambda_{11}P_{11}}{1 + P_{11}^2 + P_{21}^2} \tag{A18}$$

and the reflection coefficients

$$\overleftarrow{B}'_1 = \frac{1 + P_{21}^2 - P_{11}^2}{1 + P_{11}^2 + P_{21}^2} \tag{A19}$$

$$\overleftarrow{B}'_2 = \frac{-2P_{21}P_{11}\lambda_{11}}{\lambda_{21}(1 + P_{11}^2 + P_{21}^2)} \tag{A20}$$

For an incident wave travelling from left to right with

$$\underline{A} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} ,$$

we get the transmission coefficients

$$\vec{B}_1' = \frac{-2P_{11}}{\lambda_{11}(1 + P_{11}^2 + P_{21}^2)} = \frac{k_1}{k_1'} \overleftarrow{B}_1 \tag{A21}$$

$$\vec{B}_2' = \frac{-2P_{21}}{\lambda_{21}(1 + P_{11}^2 + P_{21}^2)} \tag{A22}$$

and the reflection coefficient

$$\vec{B}_1 = \frac{-1 + P_{11}^2 + P_{21}^2}{1 + P_{11}^2 + P_{21}^2} \tag{A23}$$

(iii) If $r = 1$, $s = 3$ in equations (4) and (5) (single (fundamental) mode on the left and first three modes on the right) then

$$\underline{B} = \underline{T} \cdot \underline{A} ,$$

where

$$\underline{T} = \frac{1}{N} \begin{pmatrix} N-2 & -2\lambda_{11}P_{11} & -2\lambda_{21}P_{21} & -2\lambda_{31}P_{31} \\ \frac{-2P_{11}}{\lambda_{11}} & N-2P_{11}^2 & \frac{-2P_{11}\lambda_{21}P_{21}}{\lambda_{11}} & \frac{-2P_{11}\lambda_{31}P_{31}}{\lambda_{11}} \\ \frac{-2P_{21}}{\lambda_{21}} & \frac{-2P_{21}P_{11}\lambda_{11}}{\lambda_{21}} & N-2P_{21}^2 & \frac{-2P_{31}\lambda_{31}P_{21}}{\lambda_{21}} \\ \frac{-2P_{31}}{\lambda_{31}} & \frac{-2P_{31}P_{11}\lambda_{11}}{\lambda_{31}} & \frac{-2P_{31}\lambda_{21}P_{21}}{\lambda_{31}} & N-2P_{31}^2 \end{pmatrix}$$

where $N = 1+P_{11}^2+P_{21}^2+P_{31}^2$.

For an incident wave from <u>left</u> to <u>right</u> with

$$A = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$, we get

the transmission coefficients:

$$\vec{B}'_1 = \frac{-2P_{11}}{\lambda_{11}(1 + P_{11}^2 + P_{21}^2 + P_{31}^2)} \tag{A24}$$

$$\vec{B}'_2 = \frac{-2P_{21}}{\lambda_{21}(1 + P_{11}^2 + P_{21}^2 + P_{31}^2)} \tag{A25}$$

$$\vec{B}'_3 = \frac{-2P_{31}}{\lambda_{31}(1 + P_{11}^2 + P_{21}^2 + P_{31}^2)} \tag{A26}$$

and the reflection coefficient:

$$\vec{B}_1 = \frac{-1 + P_{11}^2 + P_{21}^2 + P_{31}^2}{1 + P_{11}^2 + P_{21}^2 + P_{31}^2} \tag{A27}$$

Here λ_{11}, P_{11}, P_{21}, λ_{21} are as before,

$$\lambda_{31} = \left(\frac{k_3'}{k_1}\right)^{\frac{1}{2}}$$

and

$$\lambda_{31}P_{31} = \frac{F_3'F_1}{\mu_1'}\left[\frac{\mu_1\mu_2'\sigma_2^{(3)\prime} - \mu_1'\mu_2\sigma_2^{(1)}}{(k_1^2 - k_3'^2) + \frac{\omega^2}{b_1^2}} + \frac{\mu_1'\mu_2(\sigma_2^{(3)\prime} - \sigma_2^{(1)})}{(k_3'^2 - k_1^2) + \frac{\omega^2}{b_2^2}}\right]$$

2. Variational approximation

(i) If $r = 1$, $s = 1$ in equation (4) and (5) then (see Niazy and Kazi (1980))

$$\underline{B} = \underline{T}\cdot\underline{A}$$

$$\underline{A} = \begin{pmatrix} A_1 \\ A' \end{pmatrix}, \qquad \underline{B} = \begin{pmatrix} B_1 \\ B_1' \end{pmatrix} \tag{A28}$$

$$\underline{T} = \frac{1}{1 + P_{11}^2 - iI_{11}'}\begin{pmatrix} P_{11}^2-1-iI_{11}' & -2P_{11}\lambda_{11} \\ \frac{-2P_{11}}{\lambda_{11}} & 1-P_{11}^2-iI_{11}' \end{pmatrix} \tag{A29}$$

P_{11} and λ_{11} are given by (A5) and (A6) and

$$I'_{11} = \frac{4k_1 \overset{(1)}{\sigma}_2}{\pi\mu_2\mu'_2} \frac{(\beta_1^{-2} - U_m^{-1}C_m^{-1})}{(\beta_1^{-2} - \beta_2^{-2})} \left[\int_0^\infty \{H_2(k')\}^2 dk' + i\int_0^{\omega/\beta'_2} \{H_1(k')\}^2 dk'\right] \quad \text{(A30)}$$

where

$$H_1(k') = \frac{\left(\frac{\omega^2}{\beta_2'^2} - k'^2\right)^{\frac{1}{4}}}{\left\{\mu_1'^2\left(\frac{\omega^2}{\beta_1'^2} - k'^2\right)\tan^2\left(\frac{\omega^2}{\beta_1'^2} - k'^2\right)^{\frac{1}{2}}h + \mu_2'^2\left(\frac{\omega^2}{\beta_2'^2} - k'^2\right)\right\}^{\frac{1}{2}}} \times$$

$$\frac{\{\mu_1\mu_2'(k_1^2 - k'^2 + \frac{\omega^2}{b_2'^2}) - \mu_2\mu_1'(k_1^2 - k'^2 + \frac{\omega^2}{b_1^2})\}\{(\frac{\omega^2}{\beta_1'^2} - k'^2)^{\frac{1}{2}}\tan(h(\frac{\omega^2}{\beta_1'^2} + k'^2)^{\frac{1}{2}}) + \mu_2\mu_2' \overset{(1)}{\sigma}_2\omega^2(\frac{1}{b_1^2} - \frac{1}{b_2'^2})}{\{(k_1^2 - k'^2) + \frac{\omega^2}{b_1^2}\}\{(k_1^2 - k'^2) + \frac{\omega^2}{b_2'^2}\}}$$

(A31)

and

$$H_2(k') = \frac{\left(\frac{\omega^2}{\beta_2'^2} + k'^2\right)^{\frac{1}{4}}}{\left[\mu_1'^2\left(\frac{\omega^2}{\beta_1'^2} + k'^2\right)\tan^2\left\{\left(\frac{\omega^2}{\beta_1'^2} + k'\right)^{\frac{1}{2}}h\right\} + \mu_2'^2\left(\frac{\omega'^2}{\beta_2'^2} + k'^2\right)\right]^{\frac{1}{2}}} \times$$

$$\frac{[\{\mu_1\mu_2'(k_1^2 + k'^2 + \frac{\omega^2}{b_2'^2}) - \mu_2\mu_1'(k_1^2 + k'^2 + \frac{\omega^2}{b_1^2})\}\{(\frac{\omega^2}{\beta_1'^2} + k'^2)^{\frac{1}{2}}\tan(h(\frac{\omega^2}{\beta_1'^2} + k'^2)^{\frac{1}{2}}) + \mu_2\mu_2' \overset{(1)}{\sigma}_2\omega^2(\frac{1}{b_1^2} - \frac{1}{b_2'^2})]}{\{(k_1^2 + k'^2) + \frac{\omega^2}{b_1^2}\}\{(k_1^2 + k'^2) + \frac{\omega^2}{b_2'^2}\}}$$

(A32)

where $\frac{1}{b_1^2}$ is given as in (A7) and

$$\frac{1}{b_2'^2} = \frac{1}{\beta_2'^2} - \frac{1}{\beta_2^2} . \tag{A33}$$

Various notations appearing in formulae (A28) to (A33) have the same meaning as in the section on basic equations.

For an incident wave travelling from <u>left</u> to <u>right</u> with $\underline{A} = \begin{pmatrix}1\\0\end{pmatrix}$, we obtain the reflection coefficient

$$\vec{B}_1 = \frac{P_{11}^2 - 1 - iI_{11}'}{1 + P_{11}^2 - iI_{11}'} \tag{A34}$$

and the transmission coefficient

$$\vec{B}_1' = \frac{-2P_{11}}{\lambda_{11}(1 + P_{11}^2 - iI_{11}')} \tag{A35}$$

Similarly for an incident travelling from <u>right</u> to <u>left</u> with $\underline{A} = \begin{pmatrix}0\\1\end{pmatrix}$ we get the transmission coefficient

$$\overleftarrow{B}_1 = \frac{-2\lambda_{11}P_{11}}{1 + P_{11}^2 - iI_{11}'} = \frac{k_1'}{k_1}\overleftarrow{B}_1' \tag{A36}$$

and the reflection coefficient

$$\overleftarrow{B}_1' = \frac{1 - (P_{11}^2 + iI_{11}')}{1 + P_{11}^2 - iI_{11}'} \tag{A37}$$

(ii) If $r = 1$, $s = 2$ in equations (4) and (5) then (see Niazy and Kazi (1980))

$$\underline{B} = \underline{T} \cdot \underline{A} \tag{A38}$$

$$\underline{A} = \begin{bmatrix} A_1 \\ A_1' \\ A_2' \end{bmatrix}, \quad \underline{B} = \begin{bmatrix} B_1 \\ B_1' \\ B_2' \end{bmatrix} \tag{A39}$$

and

$$\underline{T} = \frac{1}{1+P_{11}^2+P_{21}^2-iI_{11}'} \begin{bmatrix} -1+P_{11}^2+P_{21}^2-iI_{11}' & -2\lambda_{11}P_{11} & -2\lambda_{21}P_{21} \\ \dfrac{-2P_{11}}{\lambda_{11}} & -P_{11}^2+1+P_{21}^2-iI_{11}' & \dfrac{-2P_{11}\lambda_{21}P_{21}}{\lambda_{11}} \\ \dfrac{-2P_{21}}{\lambda_{21}} & \dfrac{-2P_{21}P_{11}\lambda_{11}}{\lambda_{21}} & -P_{21}^2+1+P_{11}^2-iI_{11}' \end{bmatrix} \tag{A40}$$

P_{11}, λ_{11}, P_{21}, λ_{21} and I_{11}' are given by (A5), (A6), (A16), (A17) and (A30) respectively.

For an incident Love wave travelling from left to right with

$$\underline{A} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} ,$$

we get the transmission coefficients

$$\vec{B}_1' = \frac{-2P_{11}}{\lambda_{11}(1 + P_{11}^2 + P_{21}^2 - iI_{11}')} \tag{A41}$$

$$\vec{B}_2' = \frac{-2P_{21}}{\lambda_{21}(1 + P_{11}^2 + P_{21}^2 - iI_{11}')} \tag{A42}$$

and the reflection coefficient

$$\overleftarrow{B}_1 = \frac{-1 + P_{11}^2 + P_{21}^2 - iI_{11}'}{1 + P_{11}^2 + P_{11}^2 - iI_{11}'} \tag{A43}$$

Finally, for an incident Love wave travelling from <u>right</u> to <u>left</u> so that

$$\underline{A} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} ,$$

we obtain the transmission coefficient

$$\overleftarrow{B}_1 = \frac{-2\lambda_{11}P_{11}}{1 + P_{11}^2 + P_{21}^2 - iI_{11}'} = \frac{k_1'}{k_1}\vec{B}_1' \tag{A44}$$

and the reflection coefficients

$$\overleftarrow{B}_1^{\prime} = \frac{(1 + P_{21}^2 - P_{11}^2) - iI_{11}^{\prime}}{(1 + P_{11}^2 + P_{21}^2) - iI_{11}^{\prime}} \tag{A45}$$

$$\overleftarrow{B}_2^{\prime} = \frac{-2P_{21}P_{11}\lambda_{11}}{\lambda_{21}(1 + P_{11}^2 + P_{21}^2 - iI_{11}^{\prime})} \tag{A46}$$

(iii) For an incident wave travelling from <u>left</u> to <u>right</u> we get the transmission coefficients:

$$\overrightarrow{B}_1^{\prime} = \frac{-2P_{11}}{\lambda_{11}(1 + P_{11}^2 + P_{21}^2 + P_{31}^2 - iI_{11}^{\prime})} \tag{A47}$$

$$\overrightarrow{B}_2^{\prime} = \frac{-2P_{21}}{\lambda_{31}(1 + P_{11}^2 + P_{21}^2 + P_{31}^2 - iI_{11}^{\prime})} \tag{A48}$$

$$\overrightarrow{B}_3^{\prime} = \frac{-2P_{31}}{\lambda_{31}(1 + P_{11}^2 + P_{21}^2 + P_{31}^2 - iI_{11}^{\prime})} \tag{A49}$$

and the reflection coefficient :

$$\overrightarrow{B}_1 = \frac{-1 + P_{11}^2 + P_{21}^2 + P_{31}^2 - iI_{11}^{\prime}}{1 + P_{11}^2 + P_{21}^2 + P_{31}^2 - iI_{11}^{\prime}} \tag{A50}$$

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FIGURE CAPTIONS

1) Geometry of the problem

2) Material constants for the two models under study in this problem. The elastic properties of the layer on the left-hand side corresponds to Gabro (San Marcos, California) and those of the quarter-space correspond to Eclogite at 1000 bars as reported by Press (1966).

3) Dispersion curves (k versus ω) for the models shown in Fig.2. Bottom curves are for the left-hand side, the middle curves are for bottom model and the top curves for the top model in Fig. 2.

   + : Indicates the solution for the fundamental mode .

   * : Indicates the solution for the first higher mode .

   x : Indicates the solution for the second higher mode.

   Each symbol is plotted at every fifth sample point. When there is no real solution for the dispersion equation, then k is set arbitrarily = 0.

4) The absolute values of the transmission coefficients for the fundamental mode of a wave incident from left to right in Fig.2. The top curves correspond to the top model in Fig.2 and the bottom curves to the bottom one. In all plots, only the fundamental mode is present on the left hand-side, and the body-wave contribution is taken into account. The two curves marked by + (top and bottom) show the results when we neglect the effect of the higher modes on the right hand side.

The two curves marked by * show the results when the effect of the first higher mode is taken into account. The two curves marked by x show the results when the effect of the first two higher modes is taken into account. The unit of frequency is Hz and the numbers on the vertical axis are the ratios of the transmitted to the incident amplitudes. Each symbol is plotted at every fifth sample point. The E-format in FORTRAN IV computer language is used to give the scale on the axes.

5) The reflection coefficients corresponding to Fig.4. All conventions and symbols are the same as in Fig. 4.

6) The absolute values of the transmission coefficients for the fundamental mode corresponding to Fig.4. The effect of the first and second higher modes is taken into account. The curves marked by + show the result when the body-wave effect is neglected. The curves marked by * show the result when the body-wave effect is included in the computation. A symbol + or * is plotted at every fifth sample point.

7) The absolute values of the reflection coefficients corresponding to Fig. 6. All conventions and symbols are the same as in Fig. 6.

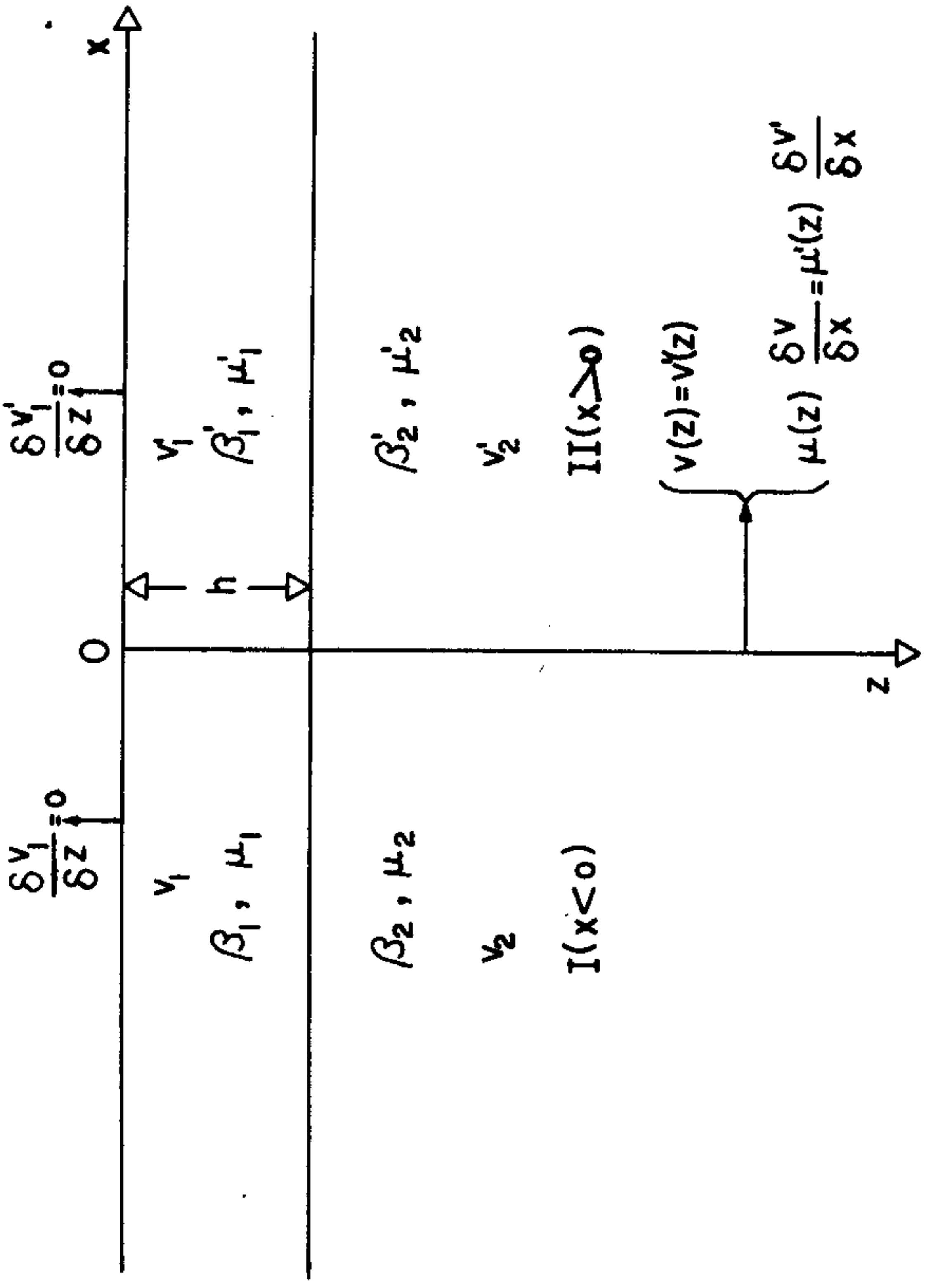
8) The absolute values of the transmission coefficients for the first higher mode corresponding to Fig. 6.

9) The absolute values of the transmission coefficients for the second higher mode corresponding to Fig. 6.

10) The absolute values of the transmission coefficients for the fundamental wave incident from left to right in Fig. 2 are marked by * and for a wave incident from right to left are marked by +. The absolute values

of the transmission coefficient for the first higher mode generated by a wave incident from left to right are marked by x. In all curves, the body-wave effect is included. The fundamental mode is incident on the left-hand side of Fig. 2. Only the effect of the first higher mode on the right-hand side is taken into account. All symbols are plotted at every fifth sample point.

11) The absolute values of the reflection coefficients corresponding to Fig. 10 with the same conventions and symbols.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z
h
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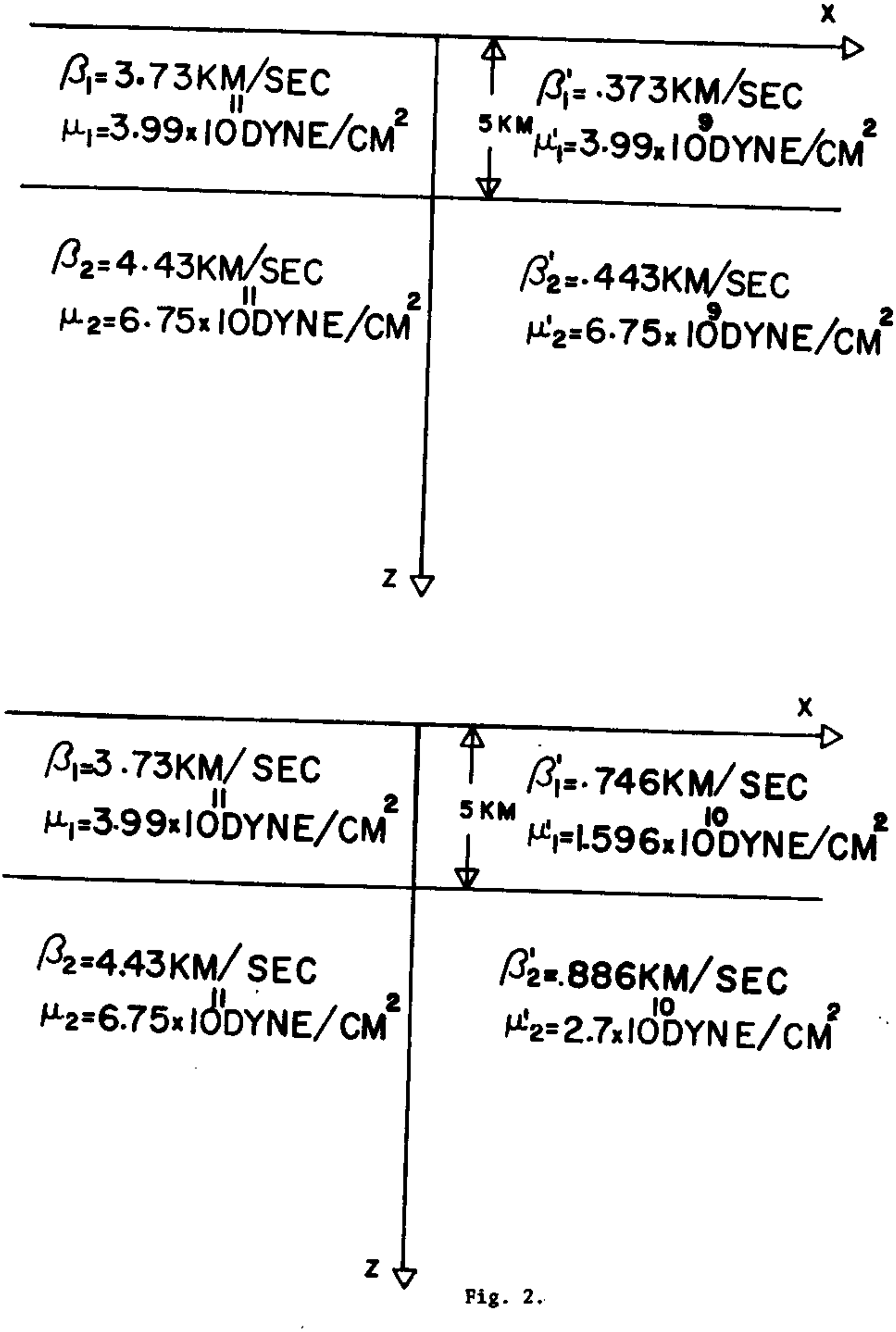
δv'₁/δz = 0
v₁
β₁ , μ₁
v'₁
β'₁ , μ'₁
β₂ , μ₂
β'₂ , μ'₂
v₂
v'₂
I(x<o)
II(x>o)
v(z)=v'(z)
μ(z) δv/δx = μ'(z) δv'/δx

FIG. 1.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5KM
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β2=4.43KM/SEC
μ2=6.75x10^11 DYNE/CM^2
β'2=.443KM/SEC
μ'2=6.75x10^9 DYNE/CM^2
Z
X
β1=3.73KM/SEC
μ1=3.99x10^11 DYNE/CM^2
5KM
β'1=.746KM/SEC
μ'1=1.596x10^10 DYNE/CM^2
β2=4.43KM/SEC
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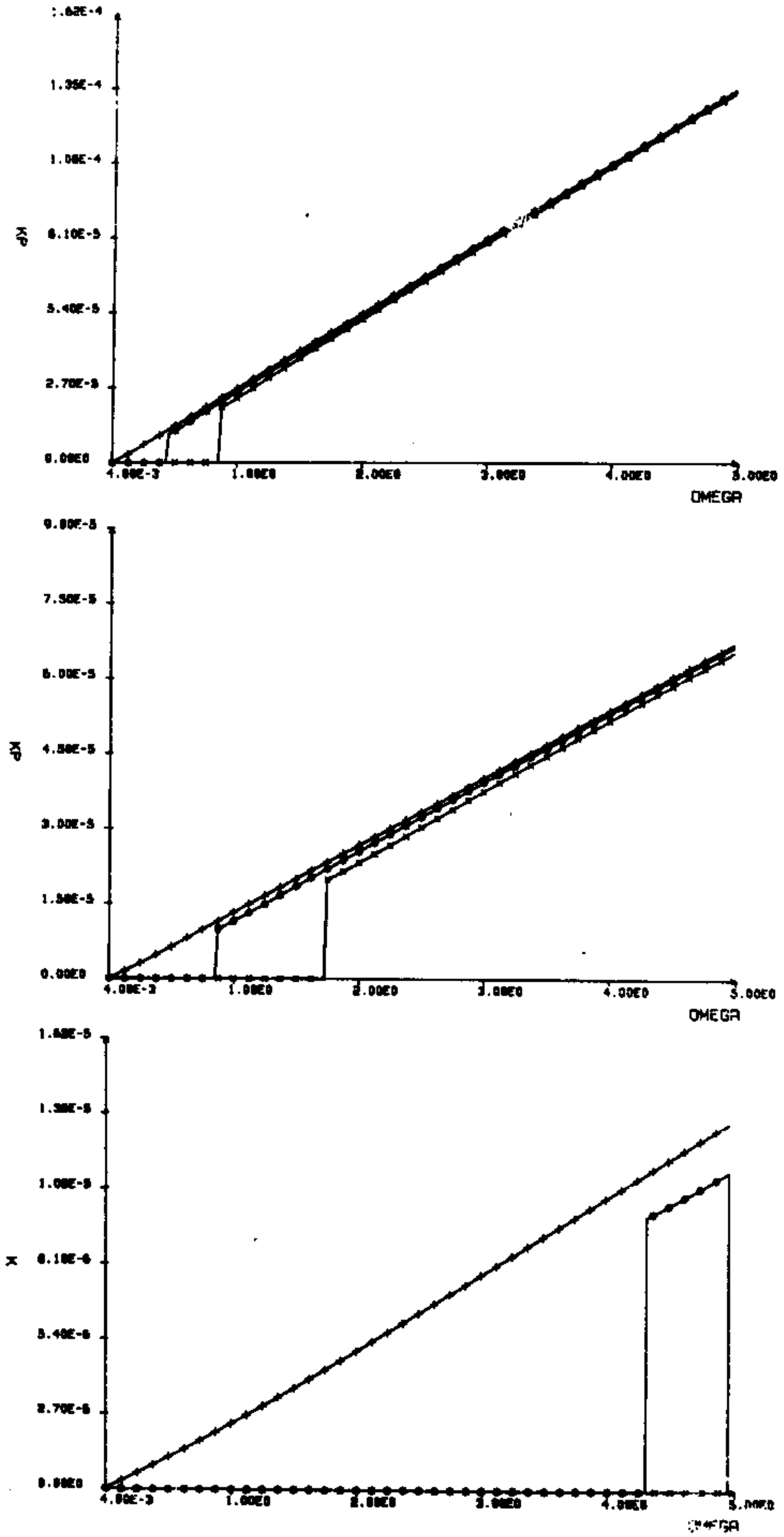
β'2=.886KM/SEC
μ'2=2.7x10^10 DYNE/CM^2
Z

Fig. 2.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4.00E0
OMEGA
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6.00E-5
4.50E-5
3.00E-5
1.50E-5
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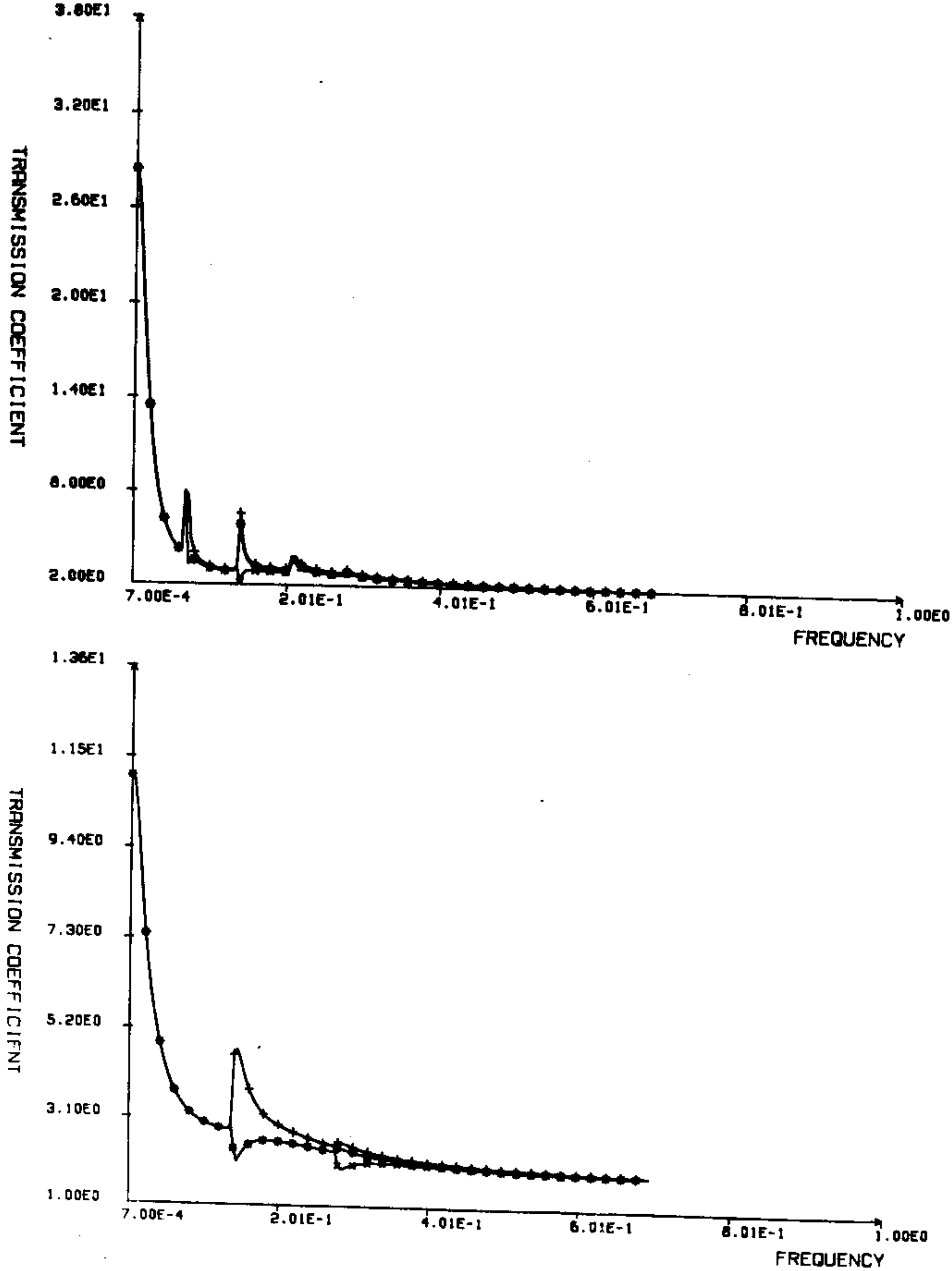
4.00E0
5.00E0
OMEGA
K

Fig. 3.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2.01E-1
4.01E-1
6.01E-1
8.01E-1
1.00E0
FREQUENCY
1.36E1
1.15E1
9.40E0
7.30E0
5.20E0
3.10E0
1.00E0
TRANSMISSION COEFFICIENT
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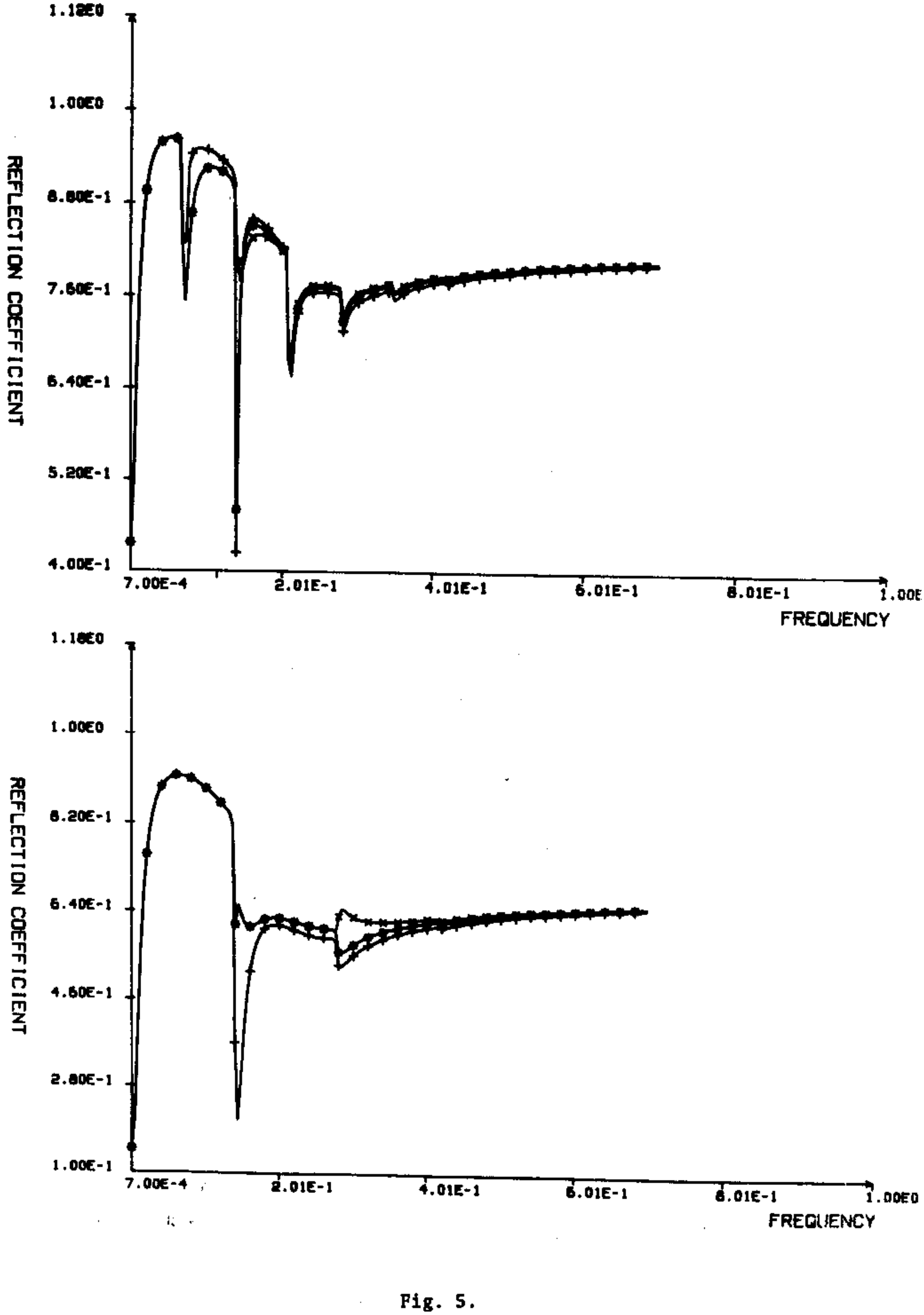
4.01E-1
6.01E-1
8.01E-1
1.00E0
FREQUENCY

Fig. 4.



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2.01E-1
4.01E-1
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8.01E-1
FREQUENCY
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6.40E-1
4.60E-1
2.80E-1
1.00E-1
REFLECTION COEFFICIENT
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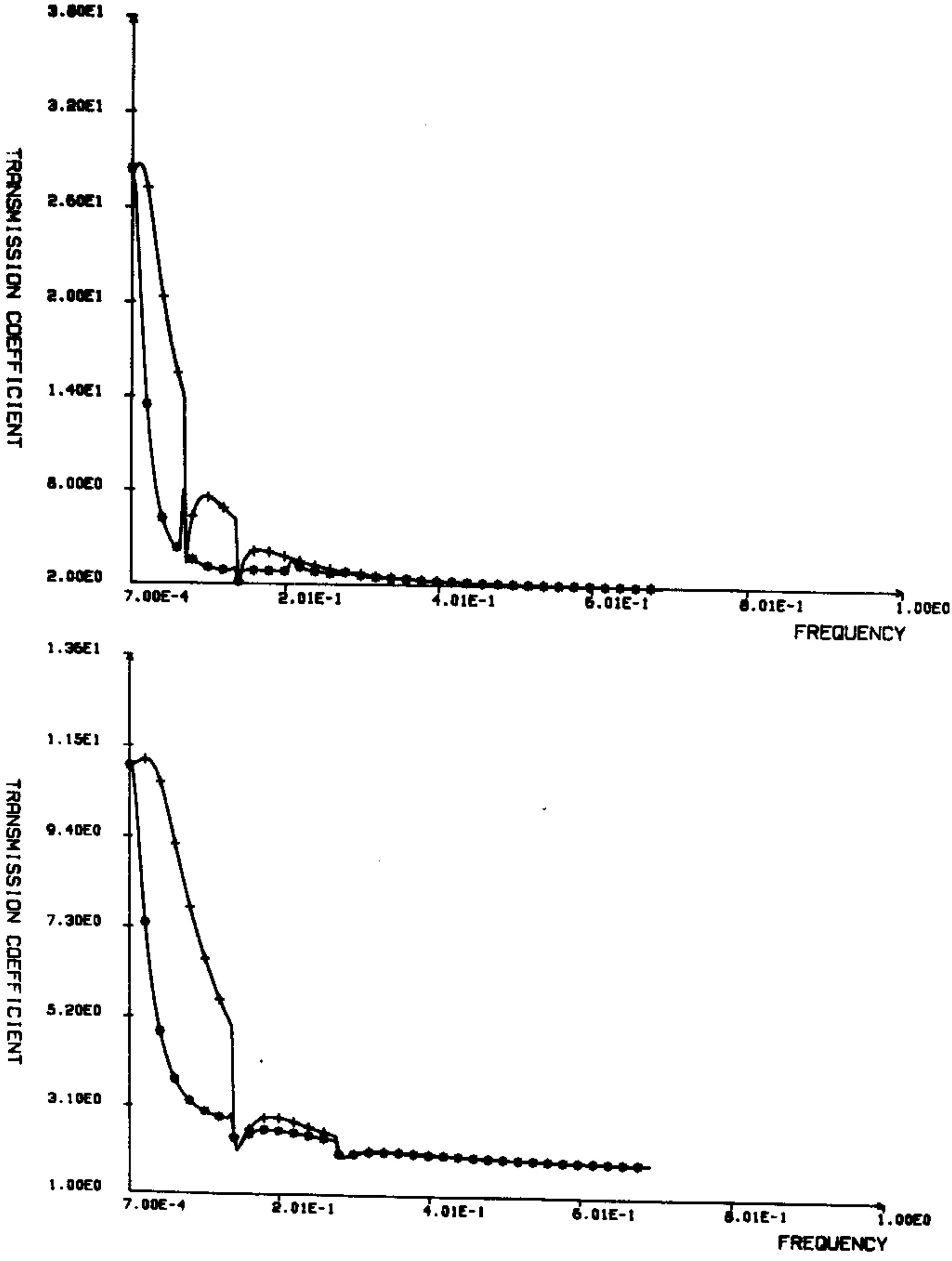
FREQUENCY

Fig. 5.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FREQUENCY
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1.00E0
TRANSMISSION COEFFICIENT
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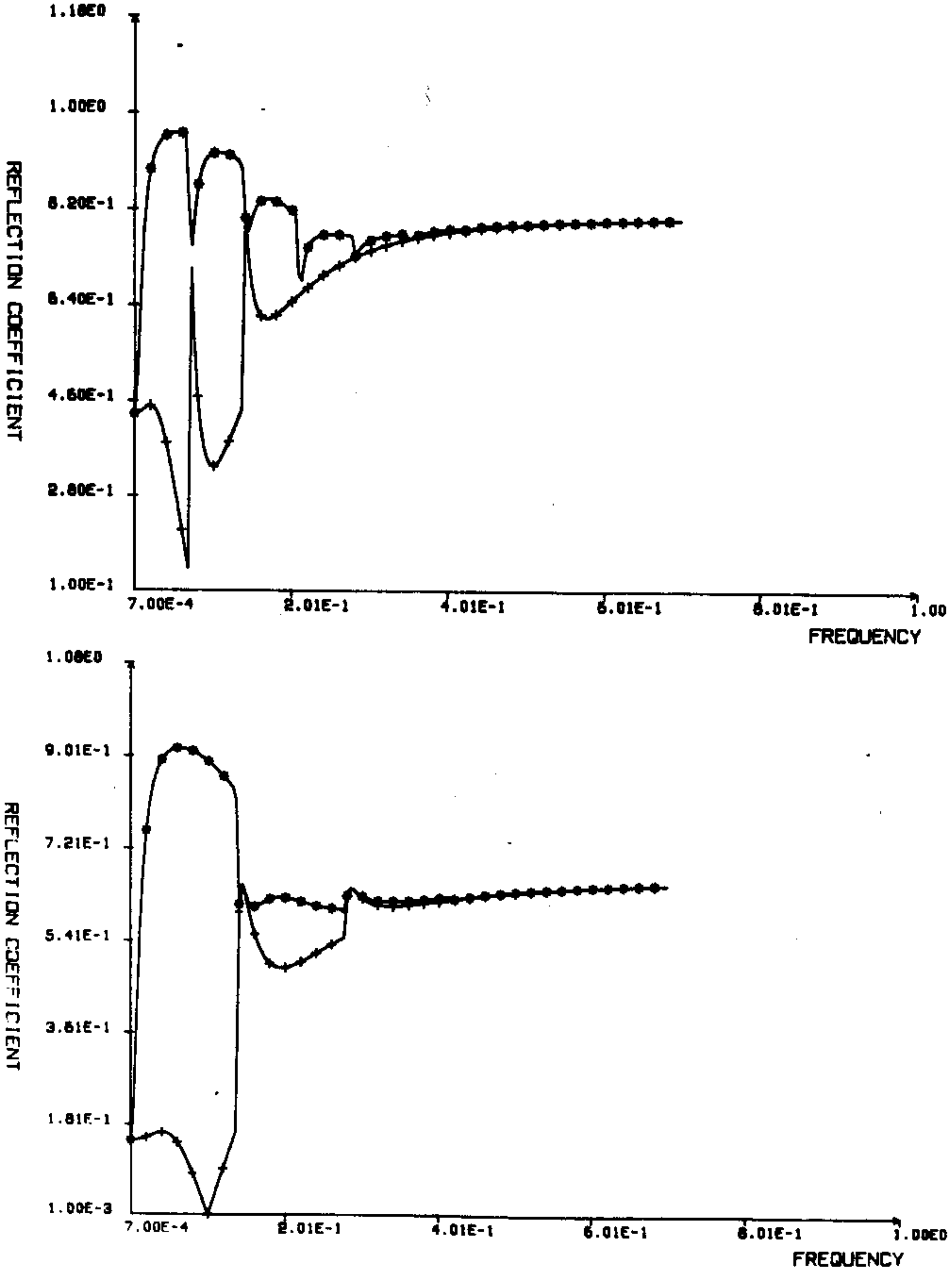
FREQUENCY

Fig. 6.



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8.01E-1
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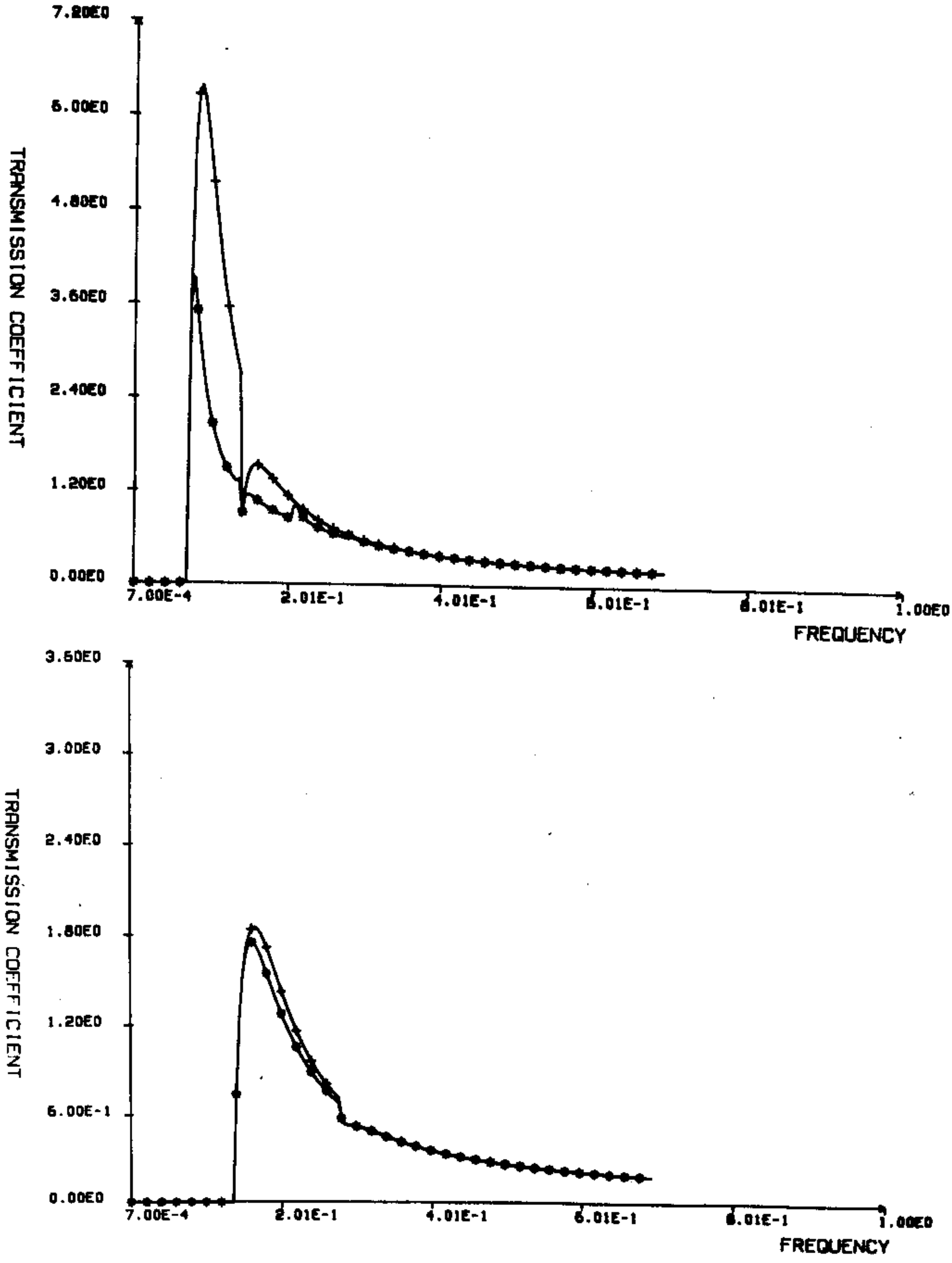
FREQUENCY

Fig. 7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4.01E-1
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8.01E-1
1.00E0
FREQUENCY
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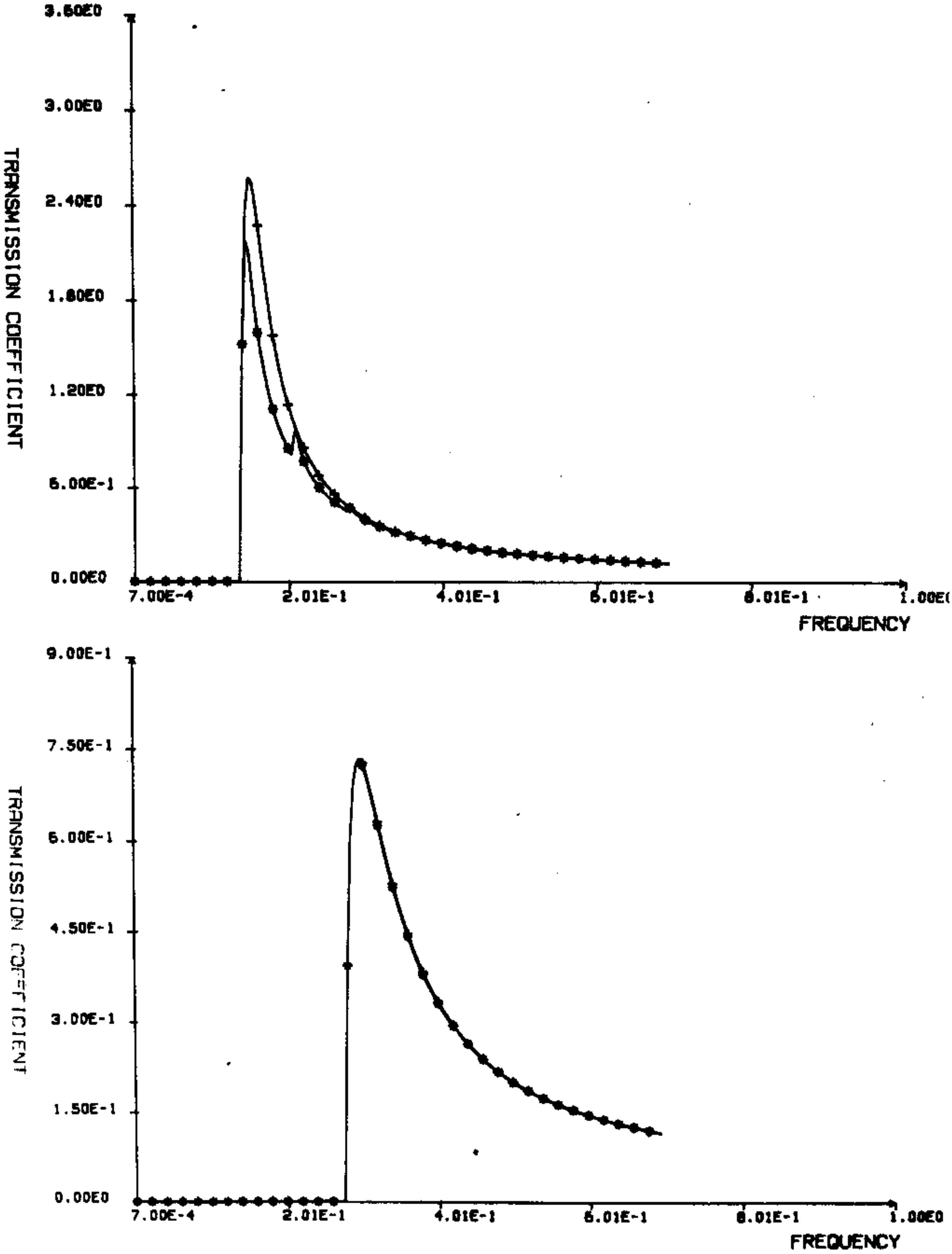
FREQUENCY

8

Fig. 8.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FREQUENCY
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TRANSMISSION COEFFICIENT
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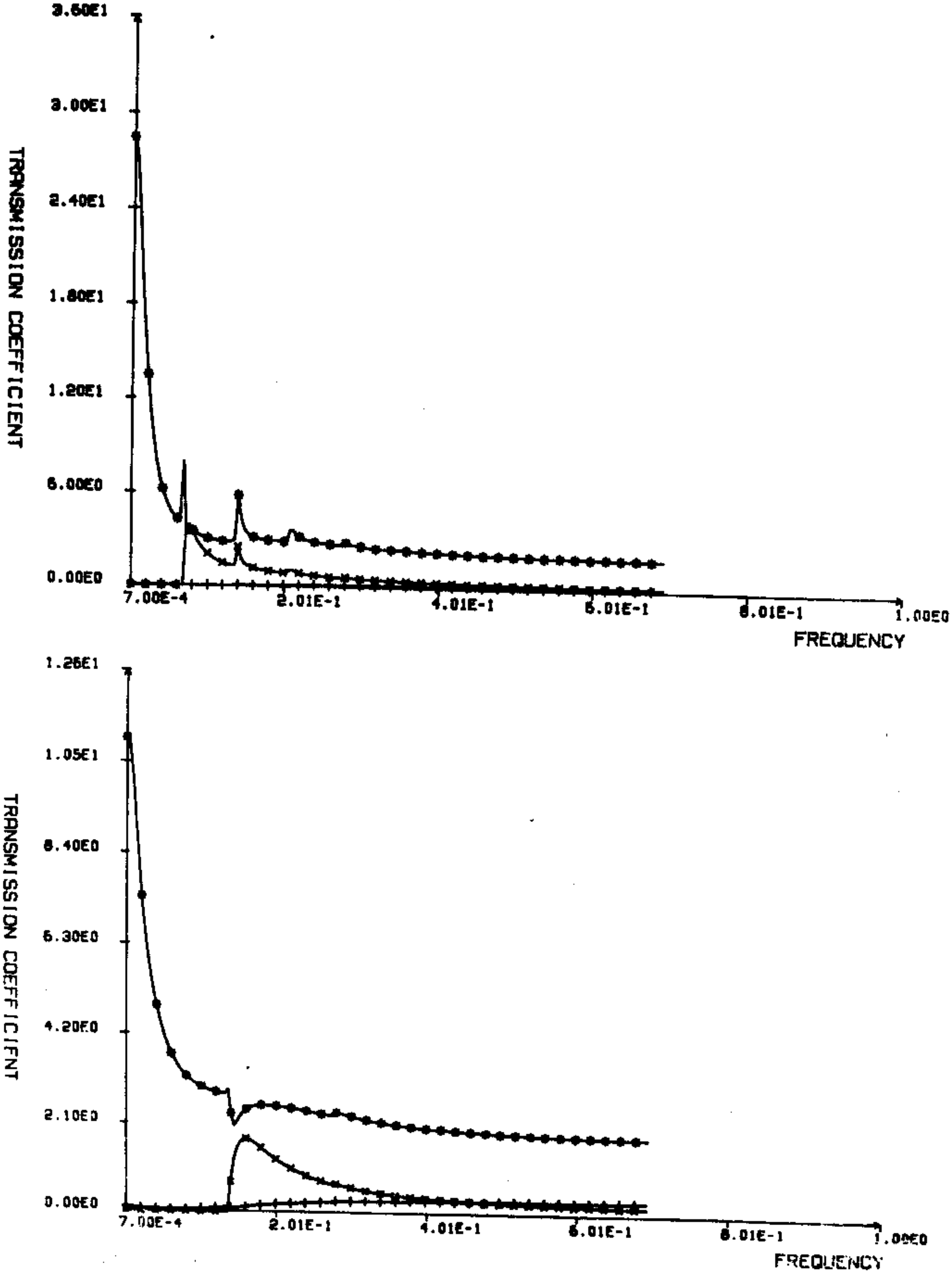
1.00E0
FREQUENCY

Fig. 9.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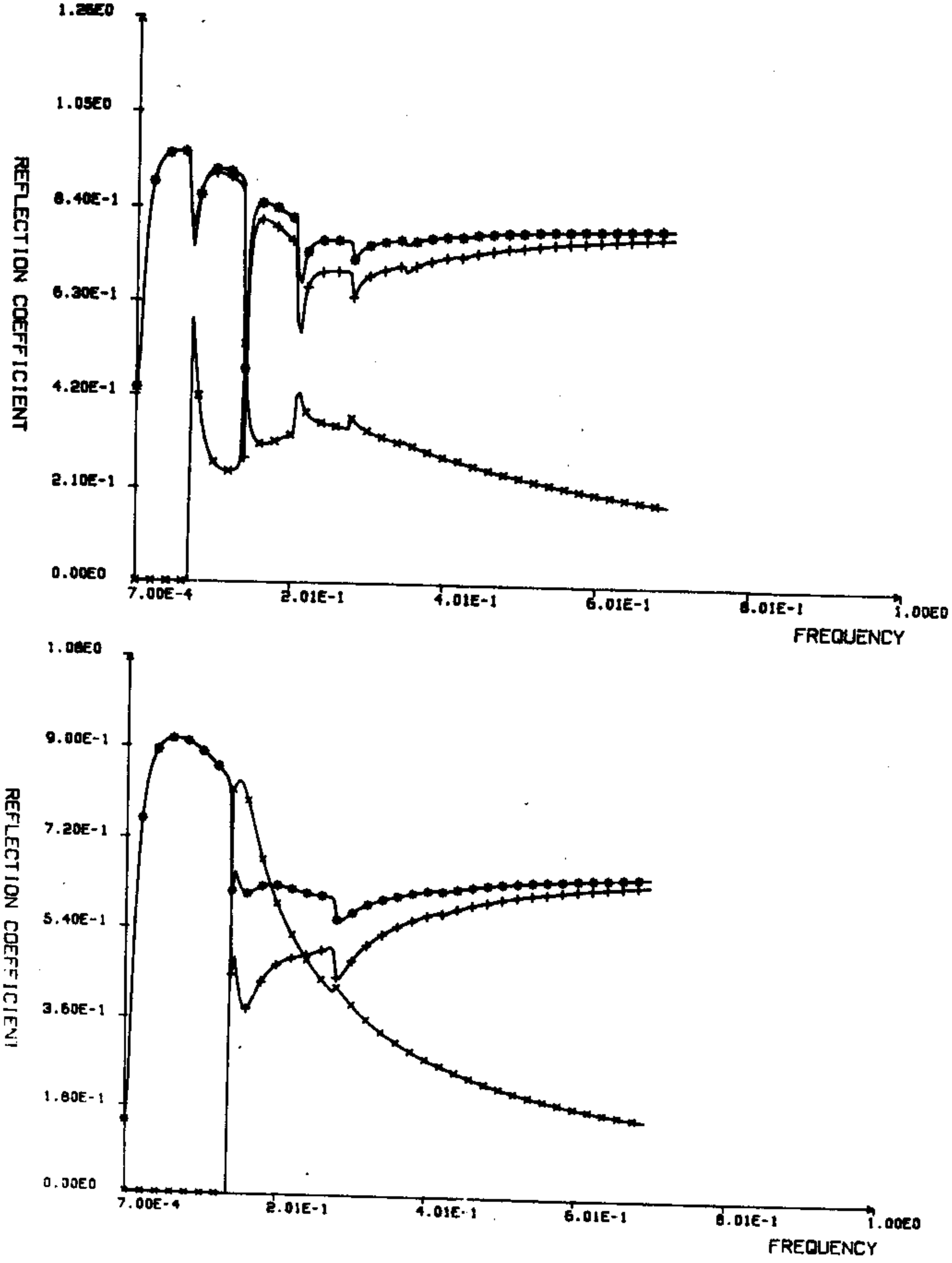
FREQUENCY

Fig. 10.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6.01E-1
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1.00E0
FREQUENCY
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REFLECTION COEFFICIENT
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1.00E0
FREQUENCY

Fig. 11.