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ON THE SPECTRAL REPRESENTATION OF THE RAYLEIGH WAVE OPERATOR -I MATHEMATICAL FORMULATION

M.H. KAZI and D.G. WILLMER

1. Introduction

In many problems of scattering of seismic surface waves (Love waves and Rayleigh waves) at a laterally discontinuous change in elevation or in material properties of a stratified elastic medium, we need to express the displacement fields on either side of the discontinuity in terms of a complete set of eigenfunctions, proper or improper, associated with the corresponding elastodynamic operators. Kazi (1976) gave a method to obtain the spectral representation of the two-dimensional Love wave operator, associated with the propagation of monochromatic SH waves in a laterally uniform layered strip or half-space. Explicit spectral representations were obtained for two-layer models of an infinite strip, overlying another infinite strip or a half-space, with constant rigidity and density within each layer. Abu-Safiya (1981) used the method to obtain spectral representations of the Love wave operator for three-layer models. Kazi (1978a,b;1979) and Niazy and Kazi (1980,1982) have successfully used the representations found in Kazi (1976) in the SH wave diffraction problems for discontinuous wave guides.

Mathematically, the eigenvalue problem involved in the spectral representation of the Love wave operator for a stratified elastic half-space is a singular Sturm-Liouville problem with discontinuous coefficients and interface conditions. The essential step in obtaining the spectral representation in Kazi (1976) is to construct a Green function G as a function of a certain parameter λ, and to integrate it around a large circle $|\lambda| = R$ in the complex λ-plane.

In this report we set up the eigenvalue problem associated with coupled seismic (P-SV) wave propagation in a uniform half-space. We show that the two components of the displacement field satisfy a fourth order ordinary differential equation along with appropriate boundary conditions. We call the operator associated with the fourth order differential equation the Rayleigh wave operator. The singular eigenvalue problem encountered here is different from that normally encountered since the parameter in the operator occurs quadratically in the coefficients of the second order and zeroth order terms as well as in the boundary conditions.

We proceed to construct the Green function for the Rayleigh wave operator. We find that the Green function for both components of the displacement field has, in addition to poles, two branch-point singularities. The Green function is, however, not symmetric.

The possibility of using the theory of quadratic bundles developed by Roach and Sleeman (1977,1979) to examine the possibility of obtaining an expansion theorem and of using the Green functions obtained in this report for obtaining spectral representation of the Rayleigh wave operator will be investigated later.

2. Basic Equations

We confine our attention to the two dimensional problem of Rayleigh wave propagation in an isotropic elastic half-space of density ρ with Lamé parameters λ and μ in the absence of body forces. The Cartesian system of co-ordinates $x_i = (x,y,z)$ is chosen so that the positive z-axis is directed vertically downward into the medium, the xy-plane coinciding with the upper free surface. The x-axis can be thought of as lying in the plane of the paper with the y-axis pointing directly out of the paper.

The stress - strain relationship for isotropic elastic materials is

$$\tau_{\ell j} = \lambda e_{kk}\,\delta_{\ell j} + 2\mu\, e_{\ell j} \tag{2.1}$$

with $e_{\ell j}$ being the Cauchy strain tensor :

$$e_{\ell j} = \frac{1}{2}\,(\partial_j\, u_\ell + \partial_\ell\, u_j) \;;\quad \partial_j \equiv \frac{\partial}{\partial x_j} \tag{2.2}$$

u_ℓ being the displacement vector and $\tau_{\ell j}$ being the stress tensor. Applying (2.1) to the equation of linear momentum in the absence of body forces i.e. $\partial_j\, \tau_{\ell j} = \rho\ddot{u}_\ell$ (dots denoting time differentiation) gives

$$(\lambda + \mu)\partial_\ell(\partial_j u_j) + \mu\, \partial_j\partial_j\, u_\ell = \rho\ddot{u}_\ell \tag{2.3}$$

We now take all stresses and displacements to be independent of the co-ordinate y by examining harmonic waves with positive real frequency ω

and wave number k , of the form

$$u_\ell = u_\ell(z) \exp\{i(\omega t - kx)\} \tag{2.4}$$

Applying (2.4) to (2.3), and suppressing the explicit dependence on z of $u_\ell(z)$, gives

$$(\lambda + \mu)\{\delta_{\ell 3} \frac{d^2u_3}{dz^2} - ik\,(\delta_{\ell 1} \frac{du_3}{dz} + \delta_{\ell 3} \frac{du_1}{dz}) - k^2\delta_{\ell 1}u_1\}$$

$$+ \mu \frac{d^2u_\ell}{dz^2} + (\rho\omega^2 - \mu k^2)u_\ell = 0 \tag{2.5}$$

The $\ell = 2$ equation is associated with SH wave propagation, and will not interest us here. We are concerned with the equations for $\ell = 1$ and $\ell = 3$, which represent P and SV wave propagation. We have from (2.5)

$$\left.\begin{aligned} \{\rho\omega^2 - k^2(\lambda + 2\mu)\}u_1 + \mu \frac{d^2u_1}{dz^2} &= ik\,(\lambda + \mu)\,\frac{du_3}{dz} \\ \{\rho\omega^2 - \mu k^2\}u_3 + (\lambda + 2\mu)\frac{d^2u_3}{dz^2} &= ik\,(\lambda + \mu)\,\frac{du_1}{dz} \end{aligned}\right\} \tag{2.6}$$

Introduce now the longitudinal wave velocity $\alpha = \sqrt{\dfrac{\lambda + 2\mu}{\rho}}$, and the shear wave velocity $\beta = \sqrt{\dfrac{\mu}{\rho}}$, and define quantities $\nu_\alpha^{\,2}$, $\nu_\beta^{\,2}$, σ by

$$\nu_\alpha^{\,2} = \frac{\omega^2}{\alpha^2} - k^2 \;,\; \nu_\beta^{\,2} = \frac{\omega^2}{\beta^2} - k^2 \;,\; \sigma = \frac{ik(\lambda + \mu)}{\mu(\lambda + 2\mu)} \tag{2.7}$$

Equations (2.6) now read

$$\frac{d^2u_1}{dz^2} + \frac{\alpha^2}{\beta^2}\,\nu_\alpha^{\,2}u_1 = \sigma\,\rho\alpha^2\,\frac{du_3}{dz} \tag{2.8a}$$

$$\frac{d^2u_3}{dz^2} + \frac{\beta^2}{\alpha^2}\nu_\beta^{\,2}\,u_3 = \sigma\rho\beta^2\frac{du_1}{dz} \tag{2.8b}$$

Decoupling of u_1 and u_3 is easily achieved by differentiating (2.8a) twice with respect to z , and then using (2.8b) followed by (2.8a). We immediately obtain the following equation for u_1 :

$$\frac{d^4u_1}{dz^4} + \frac{d^2u_1}{dz^2}\left(\frac{\alpha^2}{\beta^2}\nu_\alpha^{\,2} + \frac{\beta^2}{\alpha^2}\nu_\beta^{\,2} - \sigma^2\rho^2\alpha^2\beta^2\right) + \nu_\alpha^{\,2}\nu_\beta^{\,2}u_1 = 0 \tag{2.9}$$

The same equation can be obtained for u_3. The coefficient of $\frac{d^2u_p}{dz^2}$ $(p = 1,3)$ is seen, after a little working, to be $\nu_\alpha^{\,2} + \nu_\beta^{\,2}$, whence the equation for u_p is finally

$$L\,u_p = 0\ ;\quad L \equiv \frac{d^4}{dz^4} + (\nu_\alpha^{\,2} + \nu_\beta^{\,2})\frac{d^2}{dz^2} + \nu_\alpha^{\,2}\nu_\beta^{\,2} \tag{2.10}$$

We shall call the operator L the Rayleigh wave operator. The p subscript will be understood from now on to take the values 1,3 only.

3. Boundary Conditions

Together with the fourth order Rayleigh wave operator L we must impose suitable boundary conditions. Since the plane $z = 0$ is stress free we must have

$$\tau_{13} = \tau_{33} = 0 \quad \text{at} \quad z = 0 \tag{3.1}$$

A suitable condition at infinity must also be imposed, namely that u_p be square integrable :

$$\int_0^\infty |u_p(z)|^2 \, dz < \infty \tag{3.2}$$

Let us now translate conditions (3.1) into conditions on u_p and its derivatives

$\tau_{13} = \tau_{33} = 0$ at $z = 0$ yield , from (2.1),(2.2) and (2.4),

$$\frac{du_1}{dz} = ik\, u_3 \qquad \text{at} \quad z = 0 \tag{3.3a}$$

$$\frac{du_3}{dz} = \left(\frac{ik\lambda}{\lambda + 2\mu} \right) u_1 \qquad \text{at} \quad z = 0 \tag{3.3b}$$

Substituting for du_1/dz , du_3/dz from (2.8), and simplifying the resulting coefficients of u_1, u_3, we obtain the first pair of boundary conditions

$$B_1(u_1) \;:\; \frac{d^2u_1}{dz^2} = \{2k^2 \frac{(\nu_\beta{}^2 - \nu_\alpha{}^2)}{(\nu_\beta{}^2 + k^2)} - \nu_\beta{}^2\}u_1 \qquad \text{at} \quad z = 0 \tag{3.4a}$$

$$B_1(u_3) \;:\; \frac{d^2u_3}{dz^2} = \{2k^2 \frac{(\nu_\alpha{}^2 - \nu_\beta{}^2)}{(\nu_\beta{}^2 + k^2)} - \nu_\alpha{}^2\}u_3 \qquad \text{at} \quad z = 0 \tag{3.4b}$$

By combining (2.8b) with (3.3a), and (2.8a) with (3.3b), we have

$$\frac{d^2u_3}{dz^2} = \{\sigma\rho\beta^2 + \frac{i}{k} \; \frac{\beta^2}{\alpha^2} \, \nu_\beta{}^2\} \, \frac{du_1}{dz} \qquad \text{at} \quad z = 0 \tag{3.5a}$$

$$\frac{d^2u_1}{dz^2} = \{\sigma\rho\alpha^2 + \frac{i(\lambda + 2\mu)}{k\lambda} \; \frac{\alpha^2}{\beta^2} \, \nu_\alpha{}^2\} \, \frac{du_3}{dz} \qquad \text{at} \quad z = 0 \tag{3.5b}$$

Now differentiating (2.8a) and (2.8b), and using the results of (3.5a), (3.5b) gives the second pair of boundary conditions

$$B_2(u_1) \ : \ \frac{d^3u_1}{dz^3} = \left\{2k^2 \frac{(\nu_\beta{}^2-\nu_\alpha{}^2)}{(\nu_\beta{}^2+k^2)} + \nu_\alpha{}^2 - 2\nu_\beta^2\right\} \frac{du_1}{dz} \quad \text{at } z = 0 \qquad (3.6a)$$

$$B_2(u_3) \ : \ \frac{d^3u_3}{dz^3} = \left\{ \frac{k^2\nu_\beta{}^2 + 2\nu_\alpha{}^4-\nu_\beta{}^4}{\nu_\beta{}^2-2\nu_\alpha{}^2-k^2} \right\} \frac{du_3}{dz} \quad \text{at } z = 0 \qquad (3.6b)$$

4. Green's Function Belonging to the Rayleigh Wave Operator

We wish to find Green's function for the operator L given in (2.10) subject to either the pair of boundary conditions $B_1(u_1)$, $B_2(u_1)$ or the pair $B_1(u_3)$, $B_2(u_3)$. The Green's functions corresponding to u_1 and u_3 will be labelled G_1, G_3 respectively. The Green's functions $G_p(z,\zeta)$ must satisy the following conditions

(G1) In each of the intervals $[0,\zeta)$ and (ζ,∞), $G_p(z,\zeta)$, considered as a function of z, satisfies the equation

$$L(G_p) = 0 \qquad (4.1)$$

(G2) G_p as a function of z satisfies the boundary conditions $B_1(u_p)$, $B_2(u_p)$ and the condition (3.2) i.e.

$$B_1(G_1) \equiv \frac{d^2G_1}{dz^2} - \varepsilon_1 G_1 = 0 \quad \text{at } z = 0; \ \varepsilon_1 \equiv \frac{2k^2(\nu_\beta{}^2-\nu_\alpha{}^2)}{\nu_\beta{}^2 + k^2} - \nu_\beta^2 \qquad (4.2a)$$

$$B_2(G_1) \equiv \frac{d^3G_1}{dz^3} - (\nu_\alpha^{\ 2} - \nu_\beta^{\ 2} + \varepsilon_1)\frac{dG_1}{dz} = 0 \quad \text{at} \quad z = 0 \tag{4.2b}$$

$$\int_0^\infty |u_1(z)|^2 dz < \infty \tag{4.2c}$$

or

$$B_1(G_3) \equiv \frac{d^2G_3}{dz^2} - \varepsilon_2 G_3 = 0 \quad \text{at} \quad z = 0; \ \varepsilon_2 = \frac{2k^2(\nu_\alpha^{\ 2} - \nu_\beta^{\ 2})}{\nu_\beta^{\ 2} + k^2} - \nu_\alpha^{\ 2} \tag{4.3.a}$$

$$B_2(G_3) \equiv \frac{d^3G_3}{dz^3} - \left(\frac{k^2\nu_\beta^{\ 2} + 2\nu_\alpha^{\ 4} - \nu_\beta^{\ 4}}{\nu_\beta^{\ 2} - 2\nu_\alpha^{\ 2} - k^2}\right)\frac{dG_3}{dz} = 0 \quad \text{at} \quad z = 0 \tag{4.3b}$$

$$\int_0^\infty |u_3(z)|^2 dz < \infty \tag{4.3c}$$

(G3) $G_p(z,\zeta)$, $\dfrac{\partial G_p}{\partial z}$, $\dfrac{\partial^2 G_p}{\partial z^2}$ are continuous at $z = \zeta$

(G4) $\dfrac{\partial^3}{\partial z^3} G_p\ (\zeta+0,\zeta) - \dfrac{\partial^3}{\partial z^3} G_p\ (\zeta-0,\zeta) = 1.$

The equation (4.1)

$$\frac{d^4G_p}{dz^4} + (\nu_\alpha^{\ 2} + \nu_\beta^{\ 2})\frac{d^2G_p}{dz^2} + \nu_\alpha^{\ 2}\nu_\beta^{\ 2}G_p = 0 \tag{4.4}$$

has the auxiliary equation

$$D^4 + (\nu_\alpha^{\ 2} + \nu_\beta^{\ 2})\, D^2 + \nu_\alpha^{\ 2}\nu_\beta^{\ 2} = 0 \ ; \quad D \equiv \frac{d}{dz} \tag{4.5}$$

with roots $\pm\, i\nu_\alpha$, $\pm\, i\nu_\beta$. Hence (4.4) has four linearly independent solutions $e^{i\nu_\alpha z}$, $e^{-i\nu_\alpha z}$, $e^{i\nu_\beta z}$, $e^{-i\nu_\beta z}$. Therefore we take

$$G_p(z,\zeta) = A_p(\zeta)e^{i\nu_\alpha z} + B_p(\zeta)e^{-i\nu_\alpha z} + C_p(\zeta)e^{i\nu_\beta z} + D_p(\zeta)e^{-i\nu_\beta z}, \qquad z < \zeta$$

$$= E_p(\zeta)e^{i\nu_\alpha z} + F_p(\zeta)e^{i\nu_\beta z}\,, \qquad z > \zeta \tag{4.6}$$

with $I(\nu_\alpha) > 0$, $I(\nu_\beta) > 0$ so that the conditions at infinity are satisfied.

The conditions (G3) and (G4) applied to (4.6) yield the following set of equations

$$(A_p - E_p)e^{i\nu_\alpha\zeta} + B_p\ e^{-i\nu_\alpha\zeta} = (F_p - C_p)\ e^{i\nu_\beta\zeta} - D_p\ e^{-i\nu_\beta\zeta} \tag{4.7a}$$

$$\nu_\alpha[(A_p - E_p)e^{i\nu_\alpha\zeta} - B_p\ e^{-i\nu_\alpha\zeta}] = \nu_\beta[(F_p - C_p)e^{i\nu_\beta\zeta} + D_p\ e^{-i\nu_\beta\zeta}] \tag{4.7b}$$

$$\nu_\alpha^2[(A_p - E_p)e^{i\nu_\alpha\zeta} + B_p\ e^{-i\nu_\alpha\zeta}] = \nu_\beta^2[(F_p - C_p)e^{i\nu_\beta\zeta} - D_p\ e^{-i\nu_\beta\zeta}] \tag{4.7c}$$

$$\nu_\alpha^3[(A_p - E_p)e^{i\nu_\alpha\zeta} - B_p\ e^{-i\nu_\alpha\zeta}] = \nu_\beta^3[(F_p - C_p)e^{i\nu_\beta\zeta} + D_p e^{-i\nu_\beta\zeta}] - i \tag{4.7d}$$

Multiplying (4.7a) by $\nu_\beta^2, \nu_\alpha^2$ respectively, and subtracting the resulting equations from (4.7c) leads to

$$A_p - E_p = -B_p\ e^{-2i\nu_\alpha\zeta} \tag{4.8a}$$

$$F_p - C_p = D_p\ e^{-2i\nu_\beta\zeta} \tag{4.8b}$$

Substitution of (4.8) in (4.7b) immediately gives

$$\nu_\alpha \, B_p \, e^{-i\nu_\alpha\zeta} = \nu_\beta D_p \, e^{-i\nu_\beta\zeta} \tag{4.9}$$

whence putting (4.8), (4.9) into (4.7d) leads to

$$B = B_p = \frac{ie^{i\nu_\alpha\zeta}}{2\nu_\alpha(\nu_\alpha{}^2-\nu_\beta{}^2)} \, , \quad D = D_p = \frac{-ie^{i\nu_\beta\zeta}}{2\nu_\beta(\nu_\alpha{}^2-\nu_\beta{}^2)} \tag{4.10}$$

The only remaining conditions to consider are the boundary equations $B_1(G_p)$ and $B_2(G_p)$ which, from (4.6), immediately give

$$(A_1 + B)(\nu_\alpha{}^2 + \varepsilon_1) = -(C_1 + D)(\nu_\beta{}^2 + \varepsilon_1) \tag{4.11a}$$

$$(A_1 - B)\nu_\alpha(2\nu_\alpha{}^2 - \nu_\beta{}^2 + \varepsilon_1) = -(C_1 - D)\nu_\beta(\nu_\alpha{}^2 + \varepsilon_1) \tag{4.11b}$$

and

$$(A_3 + B)(\nu_\alpha{}^2 + \varepsilon_2) = -(C_3 + D)(\nu_\beta{}^2 + \varepsilon_2) \tag{4.12a}$$

$$(A_3 - B)(\nu_\beta{}^2 - k^2) = -2(C_3 - D)\nu_\alpha\nu_\beta \tag{4.12b}$$

These give pairs of simultaneous equations in A_p, C_p, solvable in terms of the known functions B, D given in (4.10). Defining Δ, Δ', Δ'' by

$$\Delta = (\nu_\beta{}^2 - k^2)^2 + 4\nu_\alpha\nu_\beta k^2 \tag{4.13a}$$

$$\Delta' = (\nu_\beta{}^2 - k^2)^2 - 4\nu_\alpha\nu_\beta k^2 \tag{4.13b}$$

$$\Delta'' = -4k^2(\nu_\beta{}^2 - k^2) \tag{4.13c}$$

gives, using (4.10) $\rightarrow$ (4.12),

$$A_1 = \frac{i}{2\Lambda(\nu_\alpha{}^2-\nu_\beta{}^2)} \left\{ \frac{e^{i\nu_\beta\zeta}\Lambda''}{\nu_\beta} - \frac{e^{i\nu_\alpha\zeta}\Lambda'}{\nu_\alpha} \right\} \tag{4.14a}$$

$$C_1 = \frac{i}{2\Lambda(\nu_\alpha{}^2-\nu_\beta{}^2)} \left\{ \frac{-\nu_\beta\Lambda''\, e^{i\nu_\alpha\zeta}}{k^2} - \frac{e^{i\nu_\beta\zeta}\Lambda'}{\nu_\beta} \right\} \tag{4.14b}$$

and

$$A_3 = \frac{i}{2\Lambda(\nu_\alpha{}^2-\nu_\beta{}^2)} \left\{ \frac{e^{i\nu_\alpha\zeta}\Lambda'}{\nu_\alpha} + \frac{\nu_\alpha\, \Lambda''\, e^{i\nu_\beta\zeta}}{k^2} \right\} \tag{4.15a}$$

$$C_3 = \frac{i}{2\Lambda(\nu_\alpha{}^2-\nu_\beta{}^2)} \left\{ \frac{e^{i\nu_\beta\zeta}\Lambda'}{\nu_\beta} - \frac{e^{i\nu_\alpha\zeta}\Lambda''}{\nu_\alpha} \right\} \tag{4.15b}$$

The quantities E_p, F_p now follow immediately from (4.8). Thus from (4.8),(4.10),(4.14) and (4.15) the Green's functions given in (4.6) are

$$\begin{aligned} G_1(z,\zeta) = {} & \frac{i}{2\Lambda(\nu_\alpha{}^2-\nu_\beta{}^2)} \Big\{\Lambda''\Big(\frac{e^{i(\nu_\beta\zeta+\nu_\alpha z)}}{\nu_\beta} - \frac{\nu_\beta e^{i(\nu_\alpha\zeta+\nu_\beta z)}}{k^2}\Big) \\ & -\Lambda'\Big(\frac{e^{i\nu_\alpha(\zeta+z)}}{\nu_\alpha} + \frac{e^{i\nu_\beta(\zeta+z)}}{\nu_\beta}\Big)\Big\} + \frac{i}{2(\nu_\alpha{}^2-\nu_\beta{}^2)}\Big\{\Big(\frac{e^{i\nu_\alpha(z-\zeta)}}{\nu_\alpha} - \frac{e^{i\nu_\beta(z-\zeta)}}{\nu_\beta}\Big) \\ & \cdot\theta(z-\zeta) + \Big(\frac{e^{i\nu_\alpha(\zeta-z)}}{\nu_\alpha} - \frac{e^{i\nu_\beta(\zeta-z)}}{\nu_\beta}\Big)\theta(\zeta-z)\Big\} \end{aligned} \tag{4.16a}$$

and

$$\begin{aligned} G_3(z,\zeta) = {} & \frac{i}{2\Lambda(\nu_\alpha{}^2-\nu_\beta{}^2)} \Big\{\Lambda''\Big(\frac{\nu_\alpha e^{i(\nu_\beta\zeta+\nu_\alpha z)}}{k^2} - \frac{e^{i(\nu_\alpha\zeta+\nu_\beta z)}}{\nu_\alpha}\Big) \\ & +\Lambda'\Big(\frac{e^{i\nu_\alpha(\zeta+z)}}{\nu_\alpha} + \frac{e^{i\nu_\beta(\zeta+z)}}{\nu_\beta}\Big)\Big\} + \frac{i}{2(\nu_\alpha{}^2-\nu_\beta{}^2)}\Big\{\Big(\frac{e^{i\nu_\alpha(z-\zeta)}}{\nu_\alpha} - \frac{e^{i\nu_\beta(z-\zeta)}}{\nu_\beta}\Big) \\ & \cdot\theta(z-\zeta) + \Big(\frac{e^{i\nu_\alpha(\zeta-z)}}{\nu_\alpha} - \frac{e^{i\nu_\beta(\zeta-z)}}{\nu_\beta}\Big)\theta(\zeta-z)\Big\} \end{aligned} \tag{4.16b}$$

θ being the Heaviside unit function.

We note that the G_p are not symmetric and that, considered as functions in the complex k^2-plane, they contain branch point singularities when $\nu_\alpha = 0$, $\nu_\beta = 0$ i.e. $k^2 = \omega^2/\alpha^2$, $k^2 = \omega^2/\beta^2$. The situation with regard to the singularities will be considered in detail in a subsequent report, when we integrate the G_p in the k^2-plane in an attempt to obtain the spectral representation of the Rayleigh wave operator L.

5. Operator Bundles

Given a Hilbert space H on which are defined linear self-adjoint operators $A_i : H \to H$, $i = 0, 1, 2, \ldots, n$, an operator bundle of order n, $L_n(\xi)$, is defined by

$$L_n(\xi)u = A_0 u - \xi A_1 u - \xi^2 A_2 u - \ldots - \xi^n A_n u \tag{5.1}$$

for $u \in H$ where ξ is a parameter (real or complex). The spectral theory of these bundles is closely linked with multiparameter spectral theory, and has been studied by Roach and Sleeman (1977, 1979) and Sleeman (1978). Of particular interest to us are the operators $L_2(\xi)$, called quadratic bundles. If we consider the operator (2.10) we may, using (2.7), rewrite this as

$$L \equiv A - \xi B - \xi^2 C \tag{5.2}$$

where $\xi = k^2$, and

$$\begin{aligned} A &= \left(\frac{d^2}{dz^2} + \frac{\omega^2}{\alpha^2} \right) \left(\frac{d^2}{dz^2} + \frac{\omega^2}{\beta^2} \right) = \frac{d^4}{dz^4} + \left(\frac{\omega^2}{\alpha^2} + \frac{\omega^2}{\beta^2} \right) \frac{d^2}{dz^2} + \frac{\omega^4}{\alpha^2\beta^2} \\ B &= \left(\frac{d^2}{dz^2} + \frac{\omega^2}{\alpha^2} \right) + \left(\frac{d^2}{dz^2} + \frac{\omega^2}{\beta^2} \right) \\ C &= -1. \end{aligned} \tag{5.3}$$

Thus the Rayleigh wave operator is a quadratic bundle. The applicability of the theory of quadratic bundles to the problem of the spectral representation of L will be considered in a subsequent report.

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