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# Scattering of the Fundamental Mode of a Homogeneous Monochromatic Love Wave by an M-Discontinuity in a Surface Layer on Welded Quarter Spaces

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SCATTERING OF THE FUNDAMENTAL MODE
OF A HOMOGENEOUS MONOCHROMATIC LOVE WAVE
BY AN M-DISCONTINUITY IN A SURFACE LAYER
ON WELDED QUARTER-SPACES

by
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and
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ABSTRACT

We present here the numerical results of the above scattering problem in some reasonable models. The calculations are obtained by using a scattering matrix method based on the Schwinger-Levine Variational Principle first utilized in such seismological problems by M. Kazi. The method has the advantage over other methods, including numerical finite difference or finite element methods, in that it can accommodate the body wave contribution from the infinite, vertical discontinuity surface (whereas numerical methods can only accommodate the contribution from a finite surface).

The results indicate that:

i) The body waves contribution tends to zero as the frequency tends to zero.

ii) The body waves contribution is relatively more significant for the reflection coefficient than it is for the transmission coefficient.

iii) Increasing the step-size increases the body waves contribution.

iv) Increasing the material contrast to a factor of two between the two sides of the discontinuity had no profound effect on the relative significance of the body waves contribution.

v) The amplitude of the transmitted wave, incident from the harder media onto the softer media, increases (greater than the amplitude of the incident wave) in the high frequency region with increasing the size of the step.

## INTRODUCTION

In this paper we consider the two-dimensional problem of diffraction of plane, harmonic, monochromatic Love waves, incident normally (from either side) upon the vertical plane of discontinuity in a structure representing M-discontinuity step, whick idealizes the difference in the depth to the M-discontinuity under continents and oceans. We use the method based on integral representation and Schwinger-Levine principle to describe the wave-field by means of a scattering matrix. In this method, approximate expressions for the elements of the scattering matrix are obtained through the plane-wave approximation and their variational improvement is sought through the Schwinger-Levine variational principle. The details of the method are given in Kazi (1978a) and will not be given here. We shall confine ourselves to obtain formulae for complex reflection and transmission coefficients through a transmission matrix related to the scattering matrix.

The authors have already used the method for Love wave diffraction problems associated with laterally discontinuous structures involving a vertical surface step (Kazi 1978a, b) and welded layered quarter-spaces with plane top surface (Niazy and Kazi 1980, 1982).

Other numerical methods have been explored recently to study similar problems. Drake and Bolt (1980) used a finite element model to compare the dispersion curve for Love waves incident on the continental margin of California as obtained from an ocean bottom seismometer and a land based station. Such purely numerical methods have advantages and disadvantages over analytic-numerical hybrid methods like the one we are discussing here.

While purely numerical methods allow the incorporation of a complex, realistic media, they cannot incorporate infinite boundaries. In some physical problems, an infinite boundary may be a better approximation than a finite boundary. It is not possible to get an infinite boundary solution from a finite boundary solution by letting the suitable boundary recede to infinity. An infinite boundary should be incorporated in the formulation of the problem initially as the nature of spectrum is different from that of a finite boundary solution.

Another disadvantage of purely numerical methods is that they do not allow adequate physical insight into the solution. Contributions of different types of waves are often inseparable. The method we are presenting here allows the separate consideration of the body wave contribution which shall be examined in some detail here. We can have up to eight different material constants in the solution corresponding to two layers and two quarter-spaces.

We also present some numerical results for some reasonable models of the continental margins. Actual seismological applications of the solution require placing of ocean bottom seismometers and land based seismometers that are approximately on the same azimuth from an active earthquake zone that is perpendicular to the continental margin and comparing the observed versus the predicted results. In Saudi Arabia, a potential area for the application is the southwestern part of the country.

In the Jizan area, it is thought that an abrupt transition takes place from oceanic type crust to continental type with a corresponding M-type discontinuity beneath the escarpment. The interpretations of the refraction survey carried out by the USGS on behalf of the Ministry of Petroleum and Minerals (Blank, et.al. (1979)) gave some support to this view.

The placement of a horizontal, intermediate period seismometer in Jizan and another one above the escarpment may allow the recording of a suitable earthquake from the axis of the Red Sea and may allow comparison of the synthetic records with the observation and may help in verifying or refuting the hypothesis. Unfortunately, until now no such facility exists.

# EQUATIONS OF MOTION

We consider a simplified two-dimensional model of the M-discontinuity step which idealizes the difference in the depth to the M-discontinuity under continents and margins and is, therefore, more relevant to the continental margin problem. The model consists of a stepped surface-layer with plane boundary at the top overlying half-space (see fig. 1b). The surface layer is of rigidities $\mu_1$, $\mu'_1$, shear velocities $\beta_1$, $\beta'_1$, densities $\rho_1$, $\rho'_1$, and thicknesses $h_1$ and $h_2(>h_1)$ in the two regions separated by the step, whereas the substratum is a half-space with stepped boundary and of rigidities $\mu_2$ $(>\mu_1)$ $\mu'_2$ $(>\mu'_1)$, shear velocities $\beta_2(>\beta_1)$ $\beta'_2(>\beta'_1)$ and densities $\rho_2$, $\rho'_2$ on either side of the vertical plane of discontinuity (see fig. 1). We shall consider the two-dimensional problems of propagation of time-harmonic Love waves incident normally (from either side) upon the discontinuity. Thus only SH wave motions need to be considered. We denote the y-components of the seismic displacement fields to the left and right of the plane of discontinuity by $e^{-i\omega t}v\ (x,z)$ and $e^{-i\omega t}v'(x,z)$ respectively where

$$e^{-i\omega t}v(x,z) = e^{-i\omega t}v_1(x,z)\ ,\ 0 < z < h_1,\ x < 0$$

$$= e^{-i\omega t}v_2(x,z)\ ,\ h_1 < z\ ,\ x < 0$$

and

$$e^{-i\omega t}v'(x,z) = e^{-i\omega t}v'_1(x,z),\ 0 < z < h_2,\ x < 0,$$

$$= e^{-i\omega t}v'_2(x,z),\ h_2 < z\ ,\ x < 0$$

($\omega$ being the angular frequency) are the solutions of the Love wave differential equation

$$\rho(z)\ \frac{\partial^2 v}{\partial t^2} = \frac{\partial}{\partial x}\ \left(\mu(z)\ \frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial z}\ \left(\mu(z)\ \frac{\partial v}{\partial z}\right)$$

The co-ordinate system is chosen in such a way that the z-axis is directed vertically downward, the upper boundary of the surface layer is given by $z = 0$, and of the half-space is given by $z = h_1$, $x < 0$, $z = h_2$, $x > 0$ and the vertical discontinuity lies in the plane $x = 0$. The surface $z = 0$ is stress-free and so

$$\frac{\partial v_1}{\partial z} = 0 \quad \text{and} \quad \frac{\partial v_1'}{\partial z} = 0 \text{ at } z = 0 \tag{1a}$$

$$v = v' \text{ at } x = 0,\ z \geq 0 \tag{1b}$$

$$\mu(z)\,\frac{\partial v}{\partial x} = \mu'(z)\,\frac{\partial v'}{\partial x} \text{ at } x = 0,\ z \geq 0, \tag{1c}$$

where

$$\begin{aligned} \mu(z) &= \mu_1,\ 0 \leq z \leq h_1,\ x < 0 \\ &= \mu_2,\ h_1 \leq z,\ x < 0 \end{aligned} \tag{2}$$

and

$$\begin{aligned} \mu'(z) &= \mu_1',\ 0 \leq z \leq h_2,\ x > 0 \\ &= \mu_2',\ z \geq h_2,\ x > 0 \end{aligned} \tag{3}$$

Using our work on the spectral representation of the Love wave operator associated with a homogeneous layer overlying a homogeneous half-space (Kazi (1976)) we can write the complete solutions for the displacements $v$ and $v'$ as follows.

In DOMAIN I $(x < 0,\ z \geq 0)$

$$\begin{aligned} v(x,z) = -\Big[ \sum_{m=1}^{r} (A_m e^{-ik_m|x|} + B_m e^{ik_m|x|}) x_m(z) + \int_0^{\omega/\beta_2} \{C(k)e^{-ik|x|} \\ + D(k)e^{ik|x|}\}\phi(z,k)dk + \int_0^{\infty} E(k)e^{-k|x|}\psi(z,k)dk\Big] \end{aligned} \tag{4}$$

and in DOMAIN II $(x > 0,\ z \geq 0)$

$$v'(x,z) = [\sum_{m=1}^{s} (A'_m e^{-ik'_m x} + B'_m e^{ik'_m x})\chi'_m(z) + \int_0^{\omega/\beta_2} \{C'(k')e^{-ik'x} + D'(k')e^{ik'x}\}\phi'(z,k')dk' + \int_0^{\infty} E'(k')e^{-k'x}\chi'(z,k')dk'] \quad (5)$$

where $\chi_m$, $\chi'_m$ are eigenfunctions and $k_m$, $k'_m$ correspond to real and positive eigenvalues for $x < 0$ and $x > 0$, respectively:

$$\chi_m(z) = \phi_1^{(m)}(z), \quad 0 < z < h_1,$$
$$= \phi_2^{(m)}(z), \quad h_1 < z \quad (6)$$

$$\chi'_m(z) = \phi_1'^{(m)}(z), \quad 0 < z < h_2$$
$$= \phi_2'^{(m)}(z), \quad h_2 < z \quad (7)$$

$$\phi_1^{(m)}(z) = F_m \frac{\cos(\sigma_1^{(m)} z)}{\cos(\sigma_1^{(m)} h_1)}$$

$$\phi_2^{(m)}(z) = F_m e^{\sigma_2^{(m)}(h_1 - z)} \quad (8)$$

$$F_m = \left[ 2 \frac{\sigma_2^{(m)}}{\mu_2} \left( \frac{\beta_1^{-2} - U_m^{-1} C_m^{-1}}{\beta_1^{-2} - \beta_2^{-2}} \right) \right]^{1/2}, \quad (9)$$

$$\phi'^{(m)}(z) = F'_m \frac{\cos(\sigma_1'^{(m)} z)}{\cos(\sigma_1^{m\prime} h_2)},$$

$$(10)$$

$$\phi_1'^{(m)}(z) = F'_m e^{\sigma_2^{(m)\prime}(h_2 - z)}$$

$$F'_m = \left[ 2 \frac{\sigma_2'^{(m)}}{\mu'_2} \cdot \left( \frac{\beta_1'^{-2} - U_m'^{-1} C_m'^{-1}}{\beta_1'^{-2} - \beta_2'^{-2}} \right) \right]^{1/2} \quad (11)$$

($U_m$ , $U_m'$ are the group velocities and $C_m$ , $C_m'$ the phase velocities in the $m^{th}$ - mode $C_m = \frac{w}{k_m}$ , $U_m^{-1} = \frac{dk_m}{dw}$ etc.)

$$\sigma_1(\lambda) = \left( \frac{\omega^2}{\beta_1^2} - \lambda\right)^{1/2} , \quad \sigma_2(\lambda) = \left(\lambda - \frac{\omega^2}{\beta_2^2} \right)^{1/2} \tag{12}$$

$$\sigma_1(\lambda_m) = \sigma_1^{(m)} , \quad \sigma_2(\lambda_m) = \sigma_2^{(m)} , \tag{13}$$

$$\lambda_m = k_m^2 , \quad k_m > 0, \tag{14}$$

and similarly for $\sigma_1'(\lambda')$ , $\sigma_2'(\lambda')$, $\sigma_1^{(m)\prime}$, $\sigma_2^{(m)}$ and $\lambda_m'$

$\lambda = \lambda_m = k_m^2$, m = 1,2,......r, satisfy the dispersion equation

$$\mu_1\sigma_1 \tan\sigma_1 h_1 - \mu_2\sigma_2 = 0 \tag{15}$$

whereas $\lambda' = \lambda_m' = k_m'^2$, m = 1, 2, ....., s , satisfy the equation

$$\mu_1'\sigma_1' \tan\sigma_1' h_2 - \mu_2'\sigma_2' = 0 \tag{16}$$

$\psi(z,k)$, the improper eigenfunction corresponding to the improper eigenvalues $\lambda = (ik)^2$, $k > 0$ are given by

$$\begin{aligned} \psi(z,k) &= \psi_1(z,k), \quad 0 < z < h_1, \\ &= \psi_2(z,k), \quad h_1 < z \end{aligned} \tag{17}$$

where
$$\psi_1(z,k) = G_k \frac{\cos(\sigma_1^{(k)} z)}{\cos(\sigma_1^{(k)} h_1)} , \quad 0 < z < h_1 \tag{18}$$

and
$$\psi_2(z,k) = G_k \frac{\sin \left\{\theta^{(k)} - s_2^{(k)}(z-h_1)\right\}}{\sin\theta^{(k)}} , \quad z > h_1, \tag{19}$$

with
$$G_k = \sqrt{\frac{2k}{\pi\mu_2 s_2^{(k)}}} \quad \sin \theta^{(k)} \tag{20}$$

$$s_2^{(k)} = \left( \frac{\omega^2}{\beta_2^2} - \lambda \right)^{1/2} \quad \text{real and positive } (\lambda = -k^2,\ k > 0) \tag{21}$$

and
$$\theta^{(k)} = \tan^{-1} \left( \frac{\mu_2 \, s_2^{(k)} \cot \sigma_1^{(k)} h_1}{\mu_1 \sigma_1^{(k)}} \right) \tag{22}$$

Owing to the factor $e^{-k|x|}$ in the integral containing $\psi$, these represent non-propagated modes.

Similarly $\psi'(z,k')$, the improper eigenfunctions in the domain $x > 0$ corresponding to the improper eigenvalues $\lambda' = (ik')^2$, $k' > 0$ are given by

$$\begin{aligned} \psi'(z,k') &= \psi_1'(z,k'), \quad 0 < z < h_2 \\ &= \psi_2'(z,k'), \quad h_2 < z \end{aligned} \tag{23}$$

where

$$\psi_1'(z,k') = G_{k'}' \, \frac{\cos(\sigma_1'^{(k')} z)}{\cos(\sigma_1'^{(k')} h_2)} \tag{24}$$

and
$$\psi_2'(z,k') = G_{k'}' \, \frac{\sin \left\{ \theta'^{(k')} - s_2'^{(k')}(z-h_2) \right\}}{\sin \theta'^{(k')}} \tag{25}$$

with
$$\theta'^{(k')} = \tan^{-1} \left( \frac{\mu_2' \, s_2'^{(k')} \cot (\sigma_1'^{(k')} h_2)}{\mu_1' \, \sigma_1'^{(k')}} \right), \tag{26}$$

$G_{k'}'$, $s_2'^{(k')}$ having expression similar to $G_k$ and $s_2^{(k)}$ but in the primed notation.

The improper eigenfunctions $\phi(z,k)$ and $\phi'(z,k')$ corresponding to the improper eigenvalues $\lambda = k^2$, $\lambda = k'^2$, $(0 < k,\ k' < \omega/\beta^2)$ respectively have expressions similar to those for $\psi(z,k)$ and $\psi'(z,k')$. Owing to the form of x-dependence in the integrals containing $\phi, \phi'$, these correspond to waves travelling in the x-direction.

The orthonormality relations amongst various proper and improper eigenfunctions are given by (cf. Kazi 1976)

$$\int_0^\infty \mu(z)\chi_m(z)\chi_n(z)dz = \delta_{mn}\ ,\ 1 \leq m,\ n \leq r, \tag{27a}$$

$$\int_0^\infty \mu(z)\chi_m(z)\phi(z,k)dz = 0,\ 1 \leq m \leq r,\ 0 < k \leq \frac{\omega}{\beta_2} \tag{27b}$$

$$\int_0^\infty \mu(z)\chi_m(z)\psi(z,k)dz = 0,\ 1 \leq m \leq r,\ 0 < k \leq \frac{\omega}{\beta_2} \tag{27c}$$

$$\int_0^\infty \mu(z)\phi(z,k)\psi(z,k)dz = 0 \tag{27d}$$

$$\int_0^\infty \mu(z)\psi(z,k)\psi(z,l)dz = \delta(k-l),\ 0 < k,\ l < \infty \tag{27e}$$

$$\int_0^\infty \mu(z)\phi(z,k)\phi(z,l)dz = \delta(k-l),\ 0 < k,\ l \leq \frac{\omega}{\beta_2} \tag{27f}$$

The orthonormality relations amongst $\chi_m'(z)$ $(m = 1, 2 \ldots . s)$, $\phi'(z,k')$ $(0 < k' \leq \frac{\omega}{\beta_2'})$ and $\psi'(z,k')$, $0 < k' < \infty$ are the same as (27a - f) but in primed quantities.

## INTEGRAL EQUATION FORMULATION

We proceed as in Kazi (1978). Let $\tau(z)$ denote the component $\tau_{xy}$ of stress at any point of the plane x = 0. Then 1(c) implies

$$\tau(z) = \tau_{xy}\Big|_{x=0} = \mu(z)\frac{\partial v}{\partial z}\Big|_{x=0^-} = \mu'(z)\frac{\partial v'}{\partial z}\Big|_{x=0^+}, \quad z > 0, \qquad (28)$$

Thus
$$\tau(z) = \mu(z)\frac{\partial v}{\partial x}\Big|_{x\to 0^-} = -\mu(z)\Big[\sum_{m=1}^{r} ik_m(A_m - B_m)\chi_m(z) + \int_0^{\omega/\beta_2} ik\{C(k) - D(k)\}\phi(z,k)dk + \int_0^{\infty} kE(k)\psi(z,k)dk\Big] \qquad (29)$$

and
$$\tau(z) = \mu'(z)\frac{\partial v'}{\partial x}\Big|_{x\to 0^+} = -\mu'(z)\Big[\sum_{m=1}^{s} ik'_m(A'_m - B'_m)\chi'_m(z) + \int_0^{\omega/\beta'_2} ik\{C'(k') - D'(k')\}\phi'(z,k')dk' + \int_0^{\infty} k'E'(k')\psi'(z,k')dk'\Big] \qquad (30)$$

On multiplying equation (29) separately by $\chi_m(z)$ $(m = 1, 2, \ldots r)$, $\phi(z,k)$ $(0 < k < \omega/\beta_2)$ and $\psi(z,k)$ $(0 < k < \infty)$, and integrating with respect to z from 0 to $\infty$, we obtain (using orthonormality relations 27a – f):

$$-ik_m(A_m - B_m) = \int_0^\infty \tau(\eta)\chi_m(\eta)d\eta, \; m = 1, 2, \ldots r, \quad (31a)$$

$$-ik\{C(k) - D(k)\} = \int_0^\infty \tau(\eta)\phi(\eta,k)d\eta, \quad (31b)$$

and

$$-k\, E(k) = \int_0^\infty \tau(\eta)\psi(\eta,k)d\eta \quad (31c)$$

Likewise equation (30) leads to the equations:

$$-ik'_m(A'_m - B'_m) = \int_0^\infty \tau(\eta)\chi'_m(\eta)d\eta, \; m = 1, 2, \ldots s, \quad (31d)$$

$$-ik'\{C'(k') - D'(k')\} = \int_0^\infty \tau(\eta)\phi'(\eta,k')d\eta \quad (32a)$$

and

$$-k'\, E'(k') = \int_0^\infty \tau(\eta)\psi'(\eta,k')d\eta \quad (32b)$$

Eliminating $D(k)$, $D'(k')$, $E(k)$, $E'(k')$ (assuming $C(k) = C'(k') = 0$ and invoking the matching condition (1b)), we obtain:

$$\sum_{m=1}^{r}(A_m + B_m)\chi_m(z) + \sum_{m=1}^{s}(A'_m + B'_m)\chi'_m(z) = \int_0^\infty \tau(\eta)\bar{G}(z,\eta)d\eta \quad z > 0 \quad (33)$$

where

$$\bar{G}(z,\eta) = G(z,\eta) + ig(z,\eta), \quad (34)$$

$$G(z,\eta) = \int_0^\infty \frac{\psi(z,k)\psi(\eta,k)}{k}\,dk + \int_0^\infty \frac{\psi'(z,k')\psi'(\eta,k')}{k'}\,dk' \quad (35)$$

and

$$g(z,\eta) = \int_0^{\omega/\beta_2} \frac{\phi(z,k)\phi(\eta,k)}{k}\,dk + \int_0^{\omega/\beta'_2} \frac{\phi'(z),(k')\phi'(\eta,k')}{k'}\,dk' \quad (36)$$

We note that $\bar{G}(z,\eta)$ is a Green's function type symmetric kernel, whose real and imaginary parts correspond to non-propagated and propagated modes (respectively) arising out of the continuous part of the spectrum. The integral equation formulation of the problem is given by (31a,d) and (33). The scattering matrix formulation is the same as in Kazi(1978a). Moreover, the approximate formulae for the elements of the scattering matrix and the resulting reflection and transmission coefficients arising from the approximation based upon the neglect of modes corresponding to continuous spectrum and the variational approximation based upon the Schwinger – Levine variational principle, are identical in form to the formulae derived for the step problem in Kazi (1978a). We shall, therefore omit details already covered in Kazi (1978a) and confine ourselves to the derivation of explicit expressions for the reflection and transmission coefficients for various cases under both approximations.

# FORMULAE FOR REFLECTION AND TRANSMISSION COEFFICIENTS

## (i) Approximation Based Upon the Neglect of Modes Corresponding to the Continuous Spectrum

The transmission matrix is (cf. Kazi 1978a) given by the formula

$$\underline{T} = \begin{vmatrix} -\underline{Q} \\ \underline{R} \end{vmatrix}^{-1} \begin{vmatrix} \underline{Q} \\ \underline{R} \end{vmatrix}$$

where $\underline{Q}$ and $\underline{R}$ are given by

$$\underline{Q} = \left(\begin{array}{ccc|cccc} 1 & & & \lambda_{11}P_{11} & \lambda_{21}P_{21} & \cdots\cdots & \lambda_{s1}P_{s1} \\ & 1 & 0 & \lambda_{12}P_{12} & \lambda_{22}P_{22} & \cdots\cdots & \lambda_{s2}P_{s2} \\ & & & & & & \\ & & & & & & \\ 0 & & 1 & \lambda_{1r}P_{1r} & \lambda_{2r}P_{2r} & \cdots\cdots & \lambda_{sr}P_{rs} \\ & r \times r & & & r \times s & & \end{array}\right)$$

and

$$\underline{R} = \left(\begin{array}{ccc|ccc} \frac{P_{11}}{\lambda_{11}} & \frac{P_{12}}{\lambda_{12}} & \frac{P_{1r}}{\lambda_{1}} & -1 & & \\ \frac{P_{21}}{\lambda_{21}} & \frac{P_{22}}{\lambda_{22}} & \frac{P_{2r}}{\lambda_{2r}} & & -1 & \\ & & & & & 0 \\ & & & 0 & & \\ \frac{P_{s1}}{\lambda_{s1}} & \frac{P_{s2}}{\lambda_{s2}} & \frac{P_{sr}}{\lambda_{sr}} & & & -1 \\ & s \times r & & & s \times s & \end{array}\right)$$

The particular forms of $\underline{T}$ in the special cases $r = 1$, $s > 1$ and $r > 1$, $s = 1$ are given by

$$\underline{T} = \frac{1}{N}\begin{pmatrix} -2+N & -2\lambda_{11}P_{11} & -2\lambda_{21}P_{21} & -2\lambda_{s1}P_{s1} \\ \frac{-2P_{11}}{\lambda_{11}} & -2P_{11}^2+N & \frac{-2P_{11}\lambda_{21}P_{21}}{\lambda_{11}} & \frac{-2P_{11}\lambda_{s1}P_{s1}}{\lambda_{11}} \\ \frac{-2P_{21}}{\lambda_{21}} & \frac{-2P_{21}P_{11}\lambda_{11}}{\lambda_{21}} & -2P_{21}^2+N & \frac{-2P_{21}\lambda_{s1}P_{s1}}{\lambda_{21}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{-2P_{s1}}{\lambda_{s1}} & \frac{-2P_{s1}P_{11}\lambda_{11}}{\lambda_{s1}} & \cdots\cdots & -2P_{s1}^2+N \end{pmatrix}$$

where

$$N = 1 + P_{11}^2 + P_{21}^2 + \cdots\cdots + P_{s1}^2$$

and

$$\underline{T} = \frac{1}{N}\begin{pmatrix} -N+2_{11}^2 & \frac{2P_{11}P_{12}\lambda_{11}}{\lambda_{12}} & \cdots & \frac{+2P_{11}P_{1r}\lambda_{1r}}{\lambda_{1r}} & \cdots & -2P_{11}\lambda_{11} \\ \frac{2P_{11}\lambda_{12}P_{12}}{\lambda_{11}} & -N+2P_{12} & \cdots & \frac{+2P_{12}P_{1r}\lambda_{12}}{\lambda_{1r}} & \cdots & -2P_{12}\lambda_{12} \\ \vdots & \vdots & & \vdots & & \vdots \\ \frac{2P_{1r}\lambda_{1r}P_{11}}{\lambda_{11}} & \frac{2P_{12}P_{1r}\lambda_{1r}}{\lambda_{12}} & & -N+2P_{1r}^2 & & -2P_{1r}\lambda_{1r} \\ \frac{-2P_{11}}{\lambda_{11}} & \frac{-2P_{12}}{\lambda_{12}} & & \frac{-2P_{1r}}{\lambda_{1r}} & & -N+2 \end{pmatrix}$$

where

$$N = 1 + P_{11}^2 + P_{12}^2 + \cdots\cdots + P_{1r}^2$$

All the formulae for the reflection and transmission coefficients, which were derived in terms of the coupling coefficients for various special cases in Kazi (1978a) remain unchanged for the present problem. We must, however, re-evaluate the coupling coefficients $\lambda_{im}\, P_{im}$.

Here

$$I = \lambda_{im}\, P_{im} = \int_0^{\infty} \mu(z)\chi_i'(z)\chi_m(z)dz, \qquad i = 1,2,\ldots\ldots s \qquad \lambda_{im} = \left(\frac{k_i'}{k_m}\right)^{1/2}$$

$$m = 1,2,\ldots\ldots r \qquad (37)$$

where $\chi_i'(z)$ and $\chi_m(z)$ are given by (6) – (11).

We find (see appendix) that

$$\lambda_{im}P_{im} = F_i'F_m \left[ \frac{\mu_1\sigma_1'^{(i)}\sin(\sigma_1'^{(i)}h_1) - \mu_2\sigma_2^{(m)}\cos(\sigma_1'^{(i)}h_1)}{\cos(\sigma_1^{(i)\prime}h_2)\{(k_m^2 - k_i'^2) + \frac{\omega^2}{b_1^2}\}} + \frac{\mu_2(\sigma_2'^{(i)} - \sigma_2^{(m)})\, e^{-\sigma_2^{(m)}\delta}}{(k_i'^2 - k_m^2) + \frac{\omega^2}{b_2^2}} \right]$$

$$+ \frac{\mu_2 F_i' F_m}{(k_m^2 - k_i'^2) + \frac{\omega^2}{b_3^2}} \left[ \left( \frac{\mu_2'\sigma_2^{(i)\prime}}{\mu_1'} - \sigma_2^{(m)}\right) e^{-\sigma_2^{(m)}\delta} \right.$$

$$\left. + \frac{1}{\cos(\sigma_1'^{(i)}h_2)}\{\sigma_2^{(m)}\cos(\sigma_1^{(i)\prime}h_1) - \sigma_1^{(i)\prime}\sin(\sigma_1^{(i)\prime}h_1)\} \right] \qquad (38)$$

where

$$\frac{1}{b_1^2} = \frac{1}{\beta_1'^2} - \frac{1}{\beta_1^2} \qquad (39)$$

$$\frac{1}{b_2^2} = \frac{1}{\beta_2^2} - \frac{1}{\beta_2'^2} \qquad (40)$$

$$\frac{1}{b_3^2} = \frac{1}{\beta_1'^2} - \frac{1}{\beta_2^2} \tag{41}$$

and $F_m$, $F_i'$ are given by (9) and (11) respectively.

In the particular case when r = 1, s = 1, we find (see Kazi (1978a)) that

$$\underline{B} = \underline{T}.\underline{A} \quad \text{where} \quad \underline{T} = \frac{1}{1 + P_{11}^2} \begin{bmatrix} -1 + P_{11}^2 & -2\lambda_{11}P_{11} \\ \dfrac{-2P_{11}}{\lambda_{11}} & 1 - P_{11}^2 \end{bmatrix} \tag{42}$$

$$\underline{A} = \begin{bmatrix} A_1 \\ A_1' \end{bmatrix} , \tag{43}$$

$$\underline{B} = \begin{bmatrix} B_1 \\ B_1' \end{bmatrix} \tag{44}$$

Here $A_1$, $B_1$ are the coefficients of $\chi_1$ in equation (4) and $A_1'$, $B_1'$ are the coefficients of $\chi_1'$ in equation (5) and $P_{11}$ can be obtained from (38). Thus, if the incident wave is travelling from right to left so that

$$\underline{A} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{45}$$

then the transmission coefficient

$$\mathfrak{B}_1 = \frac{-2\lambda_{11} P_{11}}{1 + P_{11}^2} \tag{46}$$

and the reflection coefficient

$$\bar{B}_1' = \frac{1 - P_{11}^2}{1 + P_{11}^2} \qquad (47)$$

If the incident wave is travelling from left to right with

$$\underline{A} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad (48)$$

then the transmission coefficient

$$\bar{B}_1' = \frac{-2P_{11}}{\lambda_{11}(1 + P_{11}^2)} = \frac{k_1}{k_1'} \bar{B}_1 \qquad (49)$$

and the reflection coefficient

$$\bar{B}_1 = \frac{P_{11}^2 - 1}{1 + P_{11}^2} = (-1)\ \bar{B}_1' \qquad (50)$$

(ii) Variational Approximation

The formulae based upon the variational approximation can be derived in exactly the same manner as in Kazi (1978a).

In the simple case when r = 1, s = 1 in equations (4) and (5), we have (cf. Kazi(1978a))

$$\underline{A} = \begin{bmatrix} A \\ A_1' \end{bmatrix}, \qquad \underline{B} = \begin{bmatrix} B \\ B_1' \end{bmatrix} \qquad (51)$$

$$\underline{T} = \frac{1}{1 + P_{11}^2 - iI_{11}'} \begin{bmatrix} P_{11}^2 - 1 - iI_{11}' & -2P_{11}\lambda_{11} \\ \frac{-2P_{11}}{\lambda_{11}} & 1 - P_{11}^2 - iI_{11}' \end{bmatrix} \qquad (52)$$

where $P_{11}$ and $\lambda_{11}$ are given by (37) and (38) and $I_i'$ has the following expression (see appendix):

$$I_{11}' = k_1 \left\{ \frac{4(k_1^2 - \omega^2/\beta_2^2)}{\pi \mu_2} \quad \mu_2' \quad \frac{(\beta_1'^2 - U_m^{-1} C_m^{-1}}{(\beta_1^{-2} - \beta_2^{-2})} \right\}$$

$$\left[ \int_0^\infty \{H_3(k')\}^2 \, dk' + i \int_0^{\omega/\beta_2'} H_4(k')\}^2 dk' \right] \qquad (53)$$

where $H_3(k')$ and $H_4(k')$ are given by equations (see appendix (A25) and (A26)) respectively.

For an incident wave travelling from left to right with $A = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$,

we obtain the reflection coefficient

$$\vec{B}_1 = \frac{P_{11}^2 - 1 - iI_{11}'}{1 + P_{11}^2 - iI_{11}'} \qquad (54)$$

and the transmission coefficient

$$\vec{B}_1' = \frac{-2P_{11}}{\lambda_{11}(1 + P_{11}^2 - iI_{11}')} \qquad (55)$$

Likewise, for an incident wave travelling from right to left with

$$\underline{A} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad (56)$$

we get the transmission coefficient

$$\hat{B}_1 = \frac{-2\lambda_{11}P_{11}}{1 + P_{11}^2 - iI_{11}'} = \frac{k_1'}{k} {}^{+}B_1' \tag{57}$$

and the reflection coefficient

$$\hat{B}_1' = \frac{(1 - P_{11}^2 + iI_{11}')}{(1 + P_{11}^2 - iI_{11}')} \tag{58}$$

We remark that the integrals in (53) are convergent but must be evaluated numerically because of the complicated nature of the integrals. We note that A's and B's in equations (4) and (5) are the coefficients of normalized modes. In calculating the ratio of the surface amplitudes of incoming and outgoing waves, it is therefore necessary to multiply the appropriate coefficients by the ratio

$$\chi(0)_{\text{outgoing}} \; / \; \chi(0)_{\text{incoming}} ,$$

where $\chi(0)_{\text{incoming}}$ and $\chi(0)_{\text{outgoing}}$ denote the surface amplitudes of the incoming and outgoing waves, respectively, when they are normalized over z. Thus, if the incident fundamental Love wave travelling from right to left is of unit surface amplitude, then the absolute value of the transmission coefficient is given by $| \hat{B}_1(\chi_1(0) \,/\, \chi_1'(0)) |$ , where $\hat{B}_1$ is given by (46) under the plane wave approximation and by (57) under the variational approximation.

# NUMERICAL RESULTS AND CONCLUSIONS

We present here the results of including the body waves contribution to the problem of scattering of two dimensional Love waves at an M-step discontinuity in the material constants. We refer to Fig. 1 for the geometry of the problem. Fig. 2 shows the chosen material constants for the computation.

Figure 3 shows the plots of k and k′ as functions of $\omega$ for the material constants shown in Fig. 2 and the geometry of Fig. 1a with $h_1 = h_2 = 5$ km. The curve marked by the symbols "0" is the plot of k and that marked by the symbols "+" is for k′. The symbols are plotted at every tenth sample point. The units of $\omega$ is $sec^{-1}$ and of k is $cm^{-1}$.

Figure 4 shows the transmission coefficient without body waves for a wave incident from left to right in the media that we have just described (marked by "0"). Including the body waves contribution gives us the curve marked by "x". As we can see the two curves are coincident and the body waves contribution is negligible in this case. The transmission coefficient without body waves for a wave incident from right to left is marked by "+". Including the body waves gives us the curve marked by "#". Again, the two curves are coincident and the body waves contribution is negligible.

Figure 5 shows the reflection coefficients corresponding to Fig. 4. The curve marked by "0" shows the coincidence of the reflection coefficients for a wave incident from right to left and a wave incident from left to right when the body waves are neglected. This is a well known mathematical fact that has been demonstrated in our previously published work on the subject (Niazy and Kazi (1982))

Including the body waves contribution for a wave incident from left to right gives us the curve marked by "x". Including the body waves for a wave incident from right to left gives us the curve marked by "#".

Figure 6 shows k and k′ as functions of $\omega$ for the geometry shown in Fig. 1b with $h_1 = 5$ km, $h_2 = 6$ km and the same material constants shown in Fig. 2. The conventions and symbols are the same as those used in Fig. 1.

Figure 7 shows the transmission coefficient for the geometry of the media we have considered in talking about Fig 6. The conventions and symbols are the same as those used in Fig. 4. It is clear that the introduction of the step has emphasized the contribution of the body waves. However, it is still negligible for practical purposes.

Figure 8 shows the reflection coefficient corresponding to the transmission coefficient shown in Fig. 7. Clearly, the relative contribution of the body waves here is much more significant than it is for the transmission coefficient that we have just discussed. However, the energy involved is a small fraction of the total energy in the incident wave. This is indicated by the small amplitude of the reflection coefficients compared with the transmission coefficients.

Figure 9 shows k and k′ as functions of $\omega$ for the geometry shown in Fig. 1c with $h_2 = 7$ km. The conventions and symbols are the same as those used in Fig. 3.

Figure 10 shows the transmission coefficient for the geometry of the media we have considered in talking about the previous figure. The conventions and symbols are the same as those used in Fig. 4. Clearly, increasing the size of the step (from 1 km to 2 km) has increased the significance of the body waves contribution. However the net effect is still small.

Figure 11 shows the reflection coefficient corresponding to the transmission coefficient shown in Fig. 10.

Again the increase in the step-size has caused an increase in the body waves contribution. In this case the body wave contribution is a substantial fraction of the total reflection coefficient.

Figure 12 shows k and k′ as functions of $\omega$ for the geometry shown in Fig. 1b except that the step is on the left-hand side. The conventions and symbols are the same as those used in Fig. 3

Figure 13 shows the transmission coefficient for the geometry of Fig. 10. The conventions and symbols are the same as those used in Fig. 4. Clearly, the body wave contribution is small in the range of frequencies under consideration.

Fig. 14 shows the reflection coefficient corresponding to the transmission coefficient shown in Fig. 13. The magnitude of the relative contribution of the body waves is comparable to the case in which the step is reversed. (Fig. 8)

Figure 15 shows k and $k'$ as functions of $\omega$ for the geometry shown in Fig. 1c except that the step is reversed. The conventions and the symbols are the same as those used in Fig. 3.

Figure 16 shows the transmission coefficient for geometry of the media we have just considered. The conventions and symbols are the same as those used in Fig. 4. Again, increasing the step has increased the significance of the body wave contribution as has been observed in the previous cases. The body wave contribution tends to vanish as $\omega \to 0$ (i.e. very long wave lengths).

Figure 17 shows the reflection coefficients corresponding to the transmission coefficient shown in the previous figure. The trait that increasing the step-size causes an increase in the significance of the body wave contribution is also exemplified here. The vanishing of the body waves contribution with increasing wavelength (decreasing $\omega$) is also apparent as it has been shown for all of the other previous cases.

We then changed the material constants on the right-hand side. The velocities were made half the corresponding velocities on the left-hand side. The densities were kept the same as on the left-hand side. Thus the rigidities on the right-hand sides are one-fourth of the corresponding rigidities on the left-hand side. The geometry was changed the manner indicated in Fig. 1.

No profound differences were found by the mentioned change in the material constants.

In summary, we can say that we presented the results of incorporating the body waves contribution to the scattering of two dimensional Love waves in a layer over a half-space at an M-type step vertical discontinuity as shown in Fig. 2. We limited the range of $\omega$ so that only the fundamental mode on both sides of the discontinuity is significant. The range of $\omega$ could be extended further in the future.

The results of the analysis of the body waves contribution indicate the following conclusions:-

i) The body waves contribution tends to zero as $\omega$ tends to zero.

ii) The body waves contribution is relatively more significant for the reflection coefficient than it is for the transmission coefficient

iii) Increasing the step-size increases the body waves contribution.

iv) Increasing the material contrast to a factor of two between the two sides of the discontinuity had no profound effect on the relative significance of the body waves contribution.

v) The amplitude of the transmitted wave, incident from the harder media onto the softer media, increases (greater than the amplitude of the incident wave) in the high frequency region with increasing the size of the step.

The solution may contribute to the understanding of the developing continental margins on the west coast of this country when adequate seismic instruments are installed in appropriate locations.

## APPENDIX

(i) Evaluation of $P_{im}$: Consider $I = \lambda_{im} P_{im} = \int_0^\infty \mu(z)\chi_i'(z)\chi_m(z)\,dz,$

$i = 1, 2, \ldots, s$

$$m = 1, 2, \ldots, r, \quad \lambda_{im} = \left(\frac{k_i'}{k_m}\right)^{1/2} \qquad \text{(A1)}$$

we write

$$I = \mu_1 \int_0^{h_1} \phi_1'^{(i)}(z)\phi_1^{(m)}(z)dz + \mu_2 \int_{h_1}^{h_2} \phi_1'^{(i)}(z)\phi_2^{(m)}(z)\,dz$$

$$+ \mu_2 \int_{h_2}^{\infty} \phi_2'^{(i)}(z)\phi_2^{(m)}(z)dz$$

$$= I_1 + I_2 + I_3 \qquad \text{(A2)}$$

where

$$I_1 = \frac{\mu_1 F_1' F_m}{\cos(\sigma_1'^{(i)}h_2)\cos(\sigma_1^{(m)}h_1)} \int_0^{h_1} \cos(\sigma_1'^{(i)}z)\cos(\sigma_1^{(m)}z)dz$$

(using (8),(10))

$$= \frac{\mu_1 F_i' F_m}{2\cos(\sigma_1'^{(i)}h_2)\cos(\sigma_1^{(m} h_1)} \int_0^{h_1} \left[ \cos(\sigma_1'^{(i)}+\sigma_1^{(m)})z + \cos(\sigma_1'^{(i)}-\sigma_1^{(m)}) \right] dz$$

$$= \frac{\mu_1 F_i' F_m}{2\cos(\sigma_1'^{(i)}h_2)\cos(\sigma_1^{(m)}h_1)} \left[ \frac{\sin(\sigma_1'^{(i)}+\sigma_1^{(m)})z}{\sigma_1'^{(i)}+\sigma_1^{(m)}} + \frac{\sin(\sigma_1'^{(i)}-\sigma_1^{(m)})z}{\sigma_1'^{(i)}-\sigma_1^{(m)}} \right]_0^{h_1}$$

$$= \frac{\mu_1 F_i' F_m}{2\cos(\sigma_1'^{(i)}h_2)\cos(\sigma_1^{(m)}h_1)} \left[ \frac{1}{\sigma_1'^{(i)}+\sigma_1^{(m)}} \{\sin(\sigma_1'^{(i)}+\sigma_1^{(m)})h_1\} \right]$$

$$+ \frac{1}{\sigma_1'^{(i)}-\sigma_1^{(m)}} \{\sin(\sigma_1'^{(i)}-\sigma_1^{(m)})h_1\}$$

$$= \frac{\mu_1 F_i' F_m}{2} \frac{\cos(\sigma_1'^{(i)} h_1)}{\cos(\sigma_1'^{(i)} h_2)} \left[ \frac{\tan(\sigma_1'^{(i)} h_1) + \tan(\sigma_1^{(m)} h_1)}{\sigma_1'^{(i)} + \sigma_1^{(m)}} \right.$$

$$\left. + \frac{\tan(\sigma_1'^{(i)} h_1) - \tan(\sigma_1^{(m)} h_1)}{\sigma_1'^{(i)} - \sigma_1^{(m)}} \right]$$

$$= \frac{\cos(\sigma_1'^{(i)} h_1}{\cos(\sigma_1'^{(i)} h_2)} \frac{F_i' F_m}{(\sigma_1'^{(i)})^2 - (\sigma_1^{(m)})^2} \mu_1 \sigma_1'^{(i)} \tan(\sigma_1'^{(i)} h_1) - \mu_1 \sigma_1^{(m)} \tan \sigma_1^{(m)} h_1$$

$$= \frac{F_i' F_m}{(\sigma_1'^{(i)})^2 - (\sigma_1^{(m)})^2} \frac{\cos(\sigma_1'^{(i)} h_1)}{\cos(\sigma_1'^{(i)} h_2)} \left[ \mu_1 \sigma_1'^{(i)} \tan(\sigma_1'^{(i)} h_1) - \mu_2 \sigma_2^{(m)} \right]$$

$$= \frac{1}{\cos(\sigma_1'^{(i)} h_2)} \frac{F_i' F_m}{\{(k_m^2 - k_1'^2) + \omega^2 (1/\beta_1'^2 - 1/\beta_1^2)\}} \left[ \mu_1 \sigma_1'^{(i)} \sin(\sigma_1'^{(i)} h_1) \right.$$

$$\left. - \mu_2 \sigma_2^{(m)} \cos(\sigma_1'^{(i)} h_1) \right] \qquad \text{(A3)}$$

(using the dispersion relation (15)

and the relations:

$$(\sigma_i^{(m)})^2 = \frac{\omega^2}{\beta_1^2} - k_m^2$$

$$(\sigma_1'^{(i)})^2 = \frac{\omega^2}{\beta_1'^2} - k_i'^2)$$

$$I_2 = \frac{\mu_2 F_i' F_m}{\cos(\sigma_1'^{(i)} h_2)} \int_{h_1}^{h_2} \cos(\sigma_1'^{(i)} z) \quad e^{\sigma_2^{(m)}(h_1 - z)} dz$$

$$= \frac{\mu_2 F_i' F_m}{\cos(\sigma_1'^{(i)} h_2)} \quad e^{\sigma_2^{(m)} h_1} \int_{h_1}^{h_2} \cos(\sigma_1'^{(i)} z) \; e^{-\sigma_2^{(m)} z} \; dz$$

$$= \frac{\mu_2 F_i' F_m}{\cos(\sigma_1'^{(i)} h_2)} \quad e^{\sigma_2^{(m)} h_1} \left| \frac{e^{-\sigma_2^{(m)} z}}{(\sigma_1'^{(i)})^2 + (\sigma_2^{(m)})^2} \{-\sigma_2^{(m)} \cos(\sigma_1'^{(i)} z) + \sigma_1'^{(i)} \sin \sigma_1'^{(i)} z\} \right|_{h_2}^{h_2}$$

$$= \frac{\mu_2 F_i' F_m}{\cos(\sigma_1'^{(i)} h_2)} \quad \frac{e^{-\sigma_2^{(m)} h_1}}{(\sigma_1'^{(i)})^2 + (\sigma_2^{(m)})^2} \Big[ \sigma_1'^{(i)} \{ e^{-\sigma_2^{(m)} h_2} \sin(\sigma_1'^{(i)} h_2) - e^{-\sigma_2^{(m)} h_1} \sin(\sigma_1'^{(i)} h_1) \}$$

$$- \sigma_2^{(m)} \{ e^{-\sigma_2^{(m)} h_2} \cos(\sigma_1'^{(i)} h_2) - e^{-\sigma_2^{(m)} h_1} \cos(\sigma_1'^{(i)} h_1) \} \Big]$$

$$= \frac{\mu_2 F_i' F_m}{(\sigma_1'^{(i)2}) + (\sigma_2^{(m)2})} \left[ \sigma_1'^{(i)} \left( e^{-\sigma_2^{(m)} \delta} \tan \sigma_1'^{(i)} h_2 - \frac{\sin(\sigma_1'^{(i)} h_1)}{\cos(\sigma_1'^{(i)} h_2)} \right) - \sigma_2^{(m)} \left( e^{-\sigma_2^{(m)} \delta} - \frac{\cos(\sigma_1'^{(i)} h_1)}{\cos \sigma_1'^{(i)} h_2} \right) \right], \; \delta = h_2 - h_1$$

(using dispersion eq. (16))

$$= \frac{\mu_2 F_i' F_m}{(\sigma_1'^{(i)})^2 + (\sigma_2^{(m)})^2} \left[ \left( \frac{\mu_2' \sigma_2'^{(i)}}{\mu_1'} - \sigma_2^{(m)} \right) e^{-\sigma_2^{(m)} \delta} + \frac{1}{\cos \sigma_1'^{(i)} h_2} \left( \sigma_2^{(m)} \cos(\sigma_1'^{(i)} h_1) - \sigma_1'^{(i)} \sin(\sigma_1'^{(i)} h_1) \right) \right] \quad \text{(A4)}$$

$$I_3 = \mu_2 F_i' F_m \int_{h_2}^{\infty} \mathrm{Exp}\{\sigma_2^{(m)}(h_1 - z) + \sigma_2'^{(i)}(h_2 - z)\} dz$$

$$= \mu_2 F_i' F_m \; e^{(\sigma_2^{(m)} h_1 + \sigma_2'^{(i)} h_2)} \int_{h_2}^{\infty} e^{-(\sigma_2^{(m)} + \sigma_2'^{(i)}) z} \; dz$$

$$= \mu_2 F_i' F_m \, e^{(\sigma_2^{(m)} h_1 + \sigma_2'^{(i)} h_2)} \; \frac{e^{-(\sigma_2^{(m)} + \sigma_2'^{(i)}) h_2}}{\sigma_2^{(m)} + \sigma_2'^{(i)}}$$

$$= \frac{\mu_2 F_i' F_m}{\sigma_2^{(m)} + \sigma_2'^{(i)}} \; e^{-\sigma_2^{(m)} \delta} = \frac{\mu_2 F_i' F_m \{\sigma_2'^{(i)} - \sigma_2^{(m)}\} e^{-\sigma_2^{(m)} \delta}}{\{(k_i'^2 - k_m^2) + \omega^2 (1/\beta_2^2 - 1/\beta_2'^2)\}} \qquad \text{(A5)}$$

(using the relations $(\sigma_2'^{(i)})^2 = k_i'^2 - \omega^2/\beta_2'^2$ and

$(\sigma^{2(m)})^2 = k_m^2 - \omega^2/\beta_2^2$

whence from (A1)-(A5), we obtain

$$\lambda_{im} P_{im} = F_i' F_m \left[ \frac{\mu_1 \sigma_1'^{(i)} \sin(\sigma_1'^{(i)} h_1) - \mu_2 \sigma_2^{(m)} \cos(\sigma_1'^{(i)} h_1)}{\cos \sigma_1'^{(i)} h_2 \{(k_m^2 - k_i'^2) + \omega^2/b_1^2} + \frac{\mu_2 (\sigma_2'^{(i)} - \sigma_2^{(m)}) \; e^{-\sigma_2^{(m)} \delta}}{(k_i'^2 - k_m^2) + \omega^2/b_2^2} \right]$$

$$+ \frac{\mu_2 F_i' F_m}{(k_m^2 - k_i'^2) + \omega^2/b_3^2} \left[ \left( \frac{\mu_2' \sigma_2'^{(i)}}{\mu_1'} - \sigma_2^{(m)} \right) e^{-\sigma_2^{(m)} \delta} + \frac{1}{\cos(\sigma'^{(i)} h_2)} \left( \sigma_2^{(m)} \cos(\sigma_1'^{(i)} h_1) - \sigma_1'^{(i)} \sin(\sigma_1'^{(i)} h_1) \right) \right]$$

(A6)

with

$$\frac{1}{b_1^2} = \frac{1}{\beta_1'^2} - \frac{1}{\beta_1^2} \qquad \text{(A7)}$$

$$\frac{1}{b_2^2} = \frac{1}{\beta_2^2} - \frac{1}{\beta_2'^2} \qquad \text{(A8)}$$

$$\frac{1}{b_3^2} = \frac{1}{\beta_1'^2} - \frac{1}{\beta_2^2} \qquad \text{(A9)}$$

(ii) <u>Evaluation of $I_{11}'$</u>

$$I_{11}' = k_1 \, I_{11} = k_1 \int_0^\infty \frac{dk'}{k'} \left[ \int_0^\infty \mu(\eta) \psi'(\eta, k') \chi_1(\eta) d\eta \int_0^\infty \mu(z) \psi'(z, k') \chi_1(z) dz \right.$$

$$+ i \int_0^{\omega/\beta_2'} \frac{dk'}{k'} \int_0^\infty \mu(\eta)\phi'(\eta,k')\chi_1(\eta)d\eta \int_0^\infty \mu(z)\phi'(z,k')\chi_1(z)\, dz \Bigg] \tag{A10}$$

we first compute

$$I_1 = \int_0^\infty \mu(z)\phi'(z,k')\chi_1(z)dz \tag{A11}$$

$$= \mu_1 \int_0^{h_1} \phi_1'(z,k')\phi_1^{\prime(1)}(z)\, dz + \mu_2 \int_{h_1}^{h_2} \phi_1'(z,k')\, \phi_2^{(1)}(z)\, dz$$

$$+ \mu_2 \int_2^\infty \phi_2'(z,k')\phi_2^{(1)}(z)\, dz$$

$$= I_2 + I_3 + I_4, \tag{A12}$$

where

$$I_2 = \frac{\mu_1 G_{k'}' F_1}{\cos(\sigma_1^{\prime(k')} h_2)\cos(\sigma_1^{(1)} h_1)} \int_0^{h_1} \cos(\sigma_1^{\prime(k')} z)\cos(\sigma_1^{(1)} z)dz$$

$$= \frac{\mu_1 F_1 G_{k'}'}{2\cos(\sigma_1^{(k')} h_2)\cos(\sigma_1^{(1)} h_1)} \left[ \frac{\sin(\sigma_1^{\prime(k')} + \sigma_1^{(1)})\, z}{\sigma_1^{\prime(k')} + \sigma_1^{(1)}} + \frac{\sin(\sigma_1^{\prime(k')} - \sigma_1^{(1)}) z}{\sigma_1^{\prime(k')} - \sigma_1^{(1)}} \right]_0^{h_1}$$

$$= \frac{\mu_1 F_1 G_{k'}'}{2\cos(\sigma_1^{\prime(k')} h_2)\cos(\sigma_1^{(1)} h_1)} \left[ \frac{1}{\sigma_1^{\prime(k')} + \sigma_1^{(1)}} \{\sin(\sigma_1^{\prime(k')} + \sigma_1^{(1)}) h_1 + \frac{1}{\sigma_1^{\prime(k')} - \sigma_1^{(1)}} \{\sin(\sigma_1^{\prime(k')} - \sigma_1^{(1)}) h_1 \right]$$

$$= \frac{\mu_1 F_1 G'_{k'}}{2} \frac{\cos(\sigma_1'^{(k')} h_1)}{\cos(\sigma_1'^{(k')} h_2)} \left[ \frac{\tan(\sigma_1'^{(k')} h_1) + \tan(\sigma_1^{(1)} h_1)}{\sigma_1'^{(k')} + \sigma_1^{(1)}} + \frac{\tan(\sigma_1'^{(k')} h_1) - \tan(\sigma_1^{(1)} h_1)}{\sigma_1'^{(k')} - \sigma_1^{(1)}} \right]$$

$$= F_1 G'_{k'} \frac{\cos(\sigma_1'^{(k')} h_1)}{\cos(\sigma_1'^{(k')} h_2)} \frac{1}{(\sigma_1'^{(k')})^2 - (\sigma_1^{(1)})^2} \{\mu_1 \sigma_1'^{(k')} \tan(\sigma_1'^{(k')} h_1) - \mu_1 \sigma_1^{(1)} \tan \sigma_1^{(1)} h_1\}$$

$$= F_1 G'_{k'} \frac{\cos(\sigma_1'^{(k')} h_1)}{\cos(\sigma_1'^{(k')} h_2)} \frac{1}{(\sigma_1'^{(k')})^2 - (\sigma_1^{(1)})^2} \left[ \mu_1 \sigma_1'^{(k')} \tan(\sigma_1'^{(k')} h_1) - \mu_2 \sigma_2^{(1)} \right] \tag{A13}$$

$$I_4 = \frac{\mu_2 G'_{k'} F_1}{\sin\theta'^{(k')}} \int_{h_2}^{\infty} e^{\sigma_2^{(1)}(h_1 - z)} \sin\{\theta'^{(k')} - s_2'^{(k')}(z - h_2)\}\, dz$$

(using (8) and expression for $\phi_2'(z,k')$)

$$= - \frac{\mu_2 F_1 G'_{k'}}{(\sigma_2^{(1)})^2 + (s_2'^{(k')})^2} \frac{1}{\sin\theta'^{(k')}} \{s_2'^{(k')} \cos\theta'^{(k')} - \sigma_2^{(1)} \sin\theta'^{(k')}\}$$

$$= \frac{\mu_2 F_1 G'_{k'}}{(\sigma_2^{(1)})^2 + (s_2'^{(k')})^2} \{\sigma_2^{(1)} - \frac{\mu_1'}{\mu_2'} \sigma_1'^{(k')} \tan(\sigma_1'^{(k')} h_2)\} , \tag{A14}$$

(using the relation $\theta'^{(k')} = \tan^{-1}\left(\frac{\mu_2' s_2'^{(k')} \cot\sigma_1'^{(k')} h_2}{\mu_1' \sigma_1'^{(k')}}\right)$)

$$I_3 = \frac{\mu_2 G'_{k'} F_1}{\cos(\sigma_1'^{(k')} h_2)} \int_{h_1}^{h_2} \cos(\sigma_1'^{(k')} z)\, e^{\sigma_2^{(1)}(h_1 - z)}\, dz$$

$$= \frac{\mu_2 G'_{k'} F_1}{\cos(\sigma_1'^{(k')} h_2)} e^{\sigma_2^{(1)} h_1} \int_{h_1}^{h_2} \cos(\sigma_1'^{(k')} z)\, e^{-\sigma_2^{(1)} z}\, dz$$

$$= \frac{\mu_2 G'_{k'} F_1}{\cos(\sigma_1'^{(k')} h_2)} \; e^{\sigma_2^{(1)} h_1} \; \Big| \; \frac{e^{-\sigma_2^{(1)} z}}{(\sigma_1'^{(k')})^2 + (\sigma_2^{(1)})^2} \{-\sigma_2'^{(1)} \cos(\sigma_1'^{(k')} z)$$

$$+ \sigma_1'^{(k')} \sin(\sigma_1'^{(k')} z)\} \Big|_{h_1}^{h_2}$$

$$= \frac{\mu_2 G'_{k'} F_1}{\cos(\sigma_1'^{(k')} h_2)} \; \frac{e^{\sigma_2^{(1)} h_1}}{(\sigma_1'^{(k')})^2 + (\sigma_2^{(1)})^2} \Big[ \sigma_1'^{(k')} \{ e^{-\sigma_2^{(1)} h_2} \sin(\sigma_1'^{(k')} h_2)$$

$$- e^{-\sigma_2^{(1)} h_1} \sin(\sigma_1'^{(k')} h_1)\} - \sigma_2^{(1)} \{ e^{-\sigma_2^{(1)} h_2} \cos(\sigma_1'^{(k')} h_2)$$

$$- e^{-\sigma_2^{(1)} h_1} \cos(\sigma_1'^{(k')} h_1)\} \Big]$$

$$= \frac{\mu_2 G'_{k'} F_1}{\sigma_1'^{(k')})^2 + (\sigma_2^{(1)})^2} \Big[ \sigma_1'^{(k')} \{ e^{-\sigma_2^{(1)} \delta} \tan(\sigma_1'^{(k')} h_2) - \frac{\sin(\sigma_1'^{(k')} h_1)}{\cos(\sigma_1'^{(k')} h_2)}$$

$$- \sigma_2^{(1)} \{ e^{-\sigma_2^{(1)} \delta} - \frac{\cos(\sigma_1'^{(k')} h_1)}{\cos(\sigma_1'^{(k')} h_2)} \} \Big] , \; \delta = h_2 - h_1$$

$$= \frac{\mu_2 G'_{k'} F_1}{(\sigma_1'^{(k')}) + (\sigma_2^{(1)})^2} \Big[ \{ \sigma_1'^{(k')} \tan(\sigma_1'^{(k')} h_2) - \sigma_2^{(1)} \} \; e^{-\sigma_2^{(1)} \delta} +$$

$$+ \frac{1}{\cos\sigma_1'^{(k')} h_2} \{ \sigma_2^{(1)} \cos(\sigma_1'^{(k')} h_1) - \sigma_1'^{(k')} \sin(\sigma_1'^{(k')} h_1)\} \Big] \quad \text{(A15)}$$

From equations (A10) to (A15) we obtain

$$I_1 = F_1 G'_{k'} \Big[ \frac{\cos(\sigma_1'^{(k')} h_1)}{\cos(\sigma_1'^{(k')} h_2)} \; \frac{\mu_1 \sigma_1'^{(k')} \tan(\sigma_1'^{(k')} h_1) - \mu_2 \sigma_2^{(1)}}{(\sigma_1'^{(k')})^2 - (\sigma_1^{(1)})^2}$$

$$+ \; \frac{\mu_2 \sigma_2^{(1)} - \mu_2 \mu_1' / \mu_2' \sigma_1'^{(k')} \; \tan(\sigma_1'^{(k')} h_2)}{(\sigma_2^{(1)})^2 + (s_2'^{(k')})^2}$$

$$+ \frac{\mu_2 e^{-\sigma_2^{(1)}\delta}\{\sigma_1^{\prime(k')}\tan(\sigma_1^{\prime(k')}h_2)-\sigma_2^{(1)}\}}{(\sigma_1^{\prime(k')})^2 + (\sigma_2^{(1)})^2}$$

$$+ \left.\frac{\mu_2 \sec(\sigma_1^{\prime(k')}h_2)\ \{\sigma_2^{(1)}\cos(\sigma_1^{\prime(k')}h_1) - \sigma_1^{\prime(k')}\sin(\sigma_1^{\prime(k')}h_1)\}}{(\sigma_1^{\prime(k')})^2+(\sigma_2^{(1)})^2}\right] \tag{A16}$$

Using the relations

$$(\sigma_1^{\prime(k')})^2 - (\sigma_1^{(1)})^2 = (k_1^2 - k'^2) + \frac{\omega^2}{b_1^2}\ , \quad \frac{1}{b_1^2} = \frac{1}{\beta_1'^2} - \frac{1}{\beta_1^2}$$

$$(\sigma_2^{(1)})^2 + (s_2^{\prime(k')})^2 = (k_1^2 - k'^2) + \frac{\omega^2}{b_2'^2}\ , \quad \frac{1}{b_2'^2} = \frac{1}{\beta_2'^2} - \frac{1}{\beta_2^2}$$

$$(\sigma_2^{(1)})^2 + (\sigma_1^{\prime(k')})^2 = k_1^2 - k'^2 + \frac{\omega^2}{b_3^2}\ , \quad \frac{1}{b_3^2} = \frac{1}{\beta_1'^2} - \frac{1}{\beta_2^2}\ ,$$

we rewrite (A15):

$$I_1 = F_1 G'_{k'} H_1(k'), \tag{A17}$$

where

$$H_1(k') = \left[ \frac{\cos\{(\omega^2/\beta_1'^2-k'^2)^{1/2}h_1\}}{\cos\{(\omega^2/\beta_1'^2-k'^2)^{1/2}h_2\}} \cdot \right.$$

$$\frac{\mu_1(\omega^2/\beta_2'^2-k'^2)^{1/2}\tan\{(\omega^2/\beta_1'^2-k'^2)^{1/2}h_1\} - \mu_2(k_1^2-\omega^2/\ _2\ ^{1/2}}{k_1^2 - k'^2 + \omega^2/b_1^2}$$

$$+ \frac{\mu_2'\mu_2(k_1^2-\omega^2/\beta_2^2)^{1/2} - \mu_2\mu_1'(\omega^2/\beta_1'^2-k'^2)^{1/2}\tan\{(\omega^2/\beta_1'^2-k'^2)^{1/2}h_2\}}{\mu_2'(k_1^2 - k'^2 + \omega^2/b_2'^2)}$$

$$+ \frac{\mu_2(\exp\{-(k^2-\omega^2/\beta_2^2)^{1/2}(h_2-h_1)\})[(\omega^2/\beta_1'^2-k'^2)^{1/2}\tan\{(\omega^2/\beta_1'^2-k'^2)^{1/2}h_2\}-(k_1^2-\omega^2/\beta_2^2)^{1/2}]}{(k_1^2 - k'^2 + \omega^2/b_3^2}$$

$$+ \left. \frac{\mu_2[(k_1^2-\omega^2/\beta_2^2)^{1/2}\cos(\omega^2/\beta_2'^2-k'^2)^{1/2}h_1-(\omega^2/\beta_1'^2-k'^2)^{1/2}\sin(\omega^2/\beta_1'^2-k'^2)^{1/2}h_1]}{\cos\{(\omega^2/\beta_1'^2-k'^2)^{1/2}h_2\}(k_1^2-k'^2+\omega^2/b_3^2}\right] \tag{A18}$$

$$F_1 = \left\{ \frac{2(k_1^2 - \omega^2/\beta_2^2)^{1/2}}{\mu_2} \quad \frac{(\beta_1^{-2} - U_m^{-1}\ C_m^{-1})}{(\beta_1^{-2} - \beta_2^{-2})} \right\}^{1/2} \tag{A19}$$

$$G'_{k'} = \left(\frac{2k'}{\pi\mu'_2 s'_2{}^{(k')}}\right)^{1/2} \sin\theta'^{(k')} = \left[\frac{2k'\mu'_2 s'_2{}^{(k')}}{\pi\{\mu'^2_1(\sigma'^{(k')}_1)^2\tan^2(\sigma'^{(k')}_1 h_2)+\mu'^2_2(s'^{(k')}_2)^2\}}\right]^{1/2}$$

$$G'_{k'} = \frac{\{2k'\mu'_2(\omega^2/\beta'^2_2-k'^2)^{1/2}\}^{1/2}}{\sqrt{\pi}\ [\mu'^2_1(\omega^2/\beta'^2_1-k'^2)\tan^2\{(\omega^2/\beta'^2_1-k'^2)^{1/2}h_2\} + \mu'^2_2(\omega^2/\beta'^2_2-k'^2)]^{1/2}} \tag{A20}$$

We now consider the integral

$$I_5 = \int_0^\infty \mu(z)\psi'(z,k')\chi_1(z)dz, \tag{A21}$$

where $\psi'(z,k')$ is given by (23) and $\chi_1(z)$ is given by (6). Following the same procedure as for the evaluation of the integral $I_1$, we obtain:

$$I_5 = \frac{F_1\{2k'\mu'_2(\omega^2/\beta'^2_2+k'^2)^{1/2}\}^{1/2}H_2(k')}{\sqrt{\pi}\ [\mu'^2_1(\omega^2/\beta'^2_1+k'^2)\tan^2\{(\omega^2/\beta'^2_1+k'^2)^{1/2}h_2\}+\mu'^2_2(\omega^2/\beta'^2_2+k'^2)]^{1/2}} \tag{A22}$$

where $F_1$ is given by (A19) and

$$H_2(k') = H_1(ik'), \tag{A23}$$

where $H_1(k')$ is given by (A18)

From equations (A10)-(A23) we get

$$I'_{11} = k_1 \left\{ \frac{4(k_1^2-\omega^2/\beta_2^2)}{\pi\mu_2}\ \frac{\mu'_2(\beta_1^{-2}-U_m^{-1}C_m^{-1})}{(\beta_1^{-2}-\beta_2^{-2})} \right\} \left[ \int_0^\infty \{H_4(k')\}^2dk' + i\int_0^{\omega/\beta'_2}\{H_1(k')\}^2dk'\right] \tag{A24}$$

where

$$H_3(k') = \frac{(\omega^2/\beta'^2_2-k'^2)^{1/4}\ H_1(k')}{[\mu'^2_1(\omega^2/\beta'^2_1-k'^2)\tan^2\{(\omega^2/\beta'^2_1-k'^2)^{1/2}h_2\}+\mu'^2_2(\omega^2/\beta'^2_2-k'^2)]^{1/2}} \tag{A25}$$

$$H_4(k') = \frac{(\omega^2/\beta'^2_2+k'^2)^{1/4}\ H_2(k')}{[\mu'^2_1(\omega^2/\beta'^2_1+k'^2)\tan^2\{(\omega^2/\beta'^2_1+k'^2)^{1/2}h_2\}+\mu'^2_2(\omega^2/\beta'^2_2+k'^2)]^{1/2}} \tag{A26}$$

and H (k') and H (k') are given by (A18) and (A23) respectively.

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## FIGURES AND FIGURE CAPTIONS

Fig. 1: Geometry of the problem:
a) For the purpose of the numerical calculations, we chose $h_1 = h_2 = 5$ km
b) $h_1 = 5$ km, $h_2 = 6$ km
c) $h_1 = 5$ km, $h_2 = 7$ km

Fig. 2: Material constant chosen to represent continental margins for the calculation.

Fig. 3: k and k′ versus ω for the case shown in Fig. 1a.

Fig. 4: Transmission coefficient for the case shown in Fig. 1a. For a wave incident from left to right, the symbol "O" denotes the solution without body waves and "x" denotes the solution with body waves. For a wave incident from right to left the symbol "+" denotes the solution without body waves and "#" denotes the solution with body waves.

Fig. 5: Reflection coefficient corresponding to Fig. 4.

Fig. 6: k and k′ versus ω for the geometry shown in Fig. 1b.

Fig. 7: Transmission coefficient corresponding to Fig. 6.

Fig. 8: Reflection coefficient corresponding to Fig. 6.

Fig. 9: k and k′ versus ω for the geometry shown in Fig. 1c.

Fig. 10: The transmission coefficient corresponding to Fig. 9.

Fig. 11: The reflection coefficient corresponding to Fig. 9.

Fig. 12: k and k′ versus ω for the geometry shown in Fig. 1b, except that the step is on the left side.

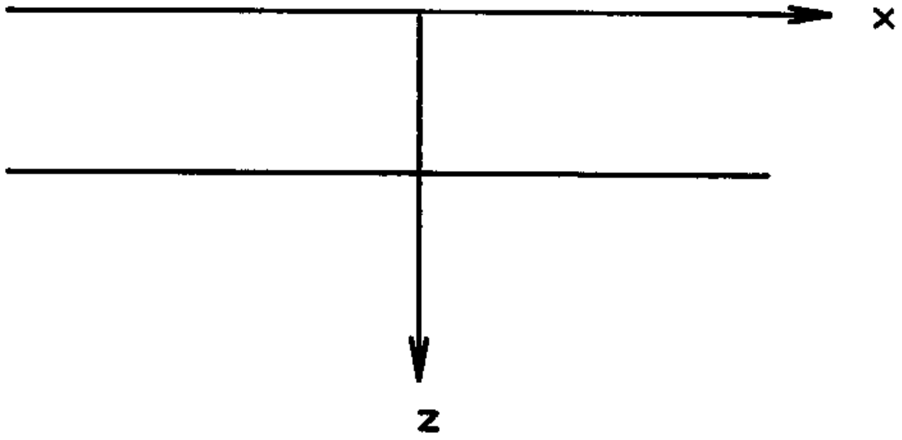
Fig. 13: Transmission coefficient corresponding to Fig. 12.

Fig. 14: Reflection coefficient corresponding to Fig. 12.

Fig. 15: k and k′ versus ω for the geometry shown in Fig. 1c, except that the step is on the left side

Fig. 16: Transmission coefficient corresponding to Fig. 15.

Fig. 17: Reflection coefficient corresponding to Fig. 15.

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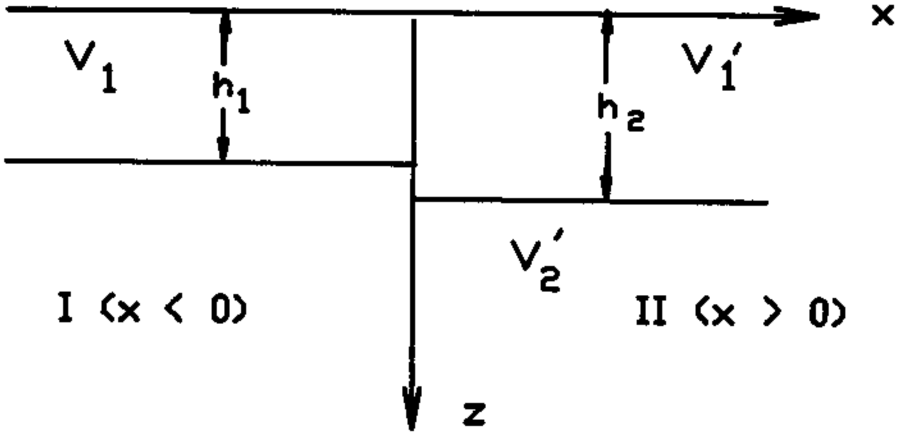


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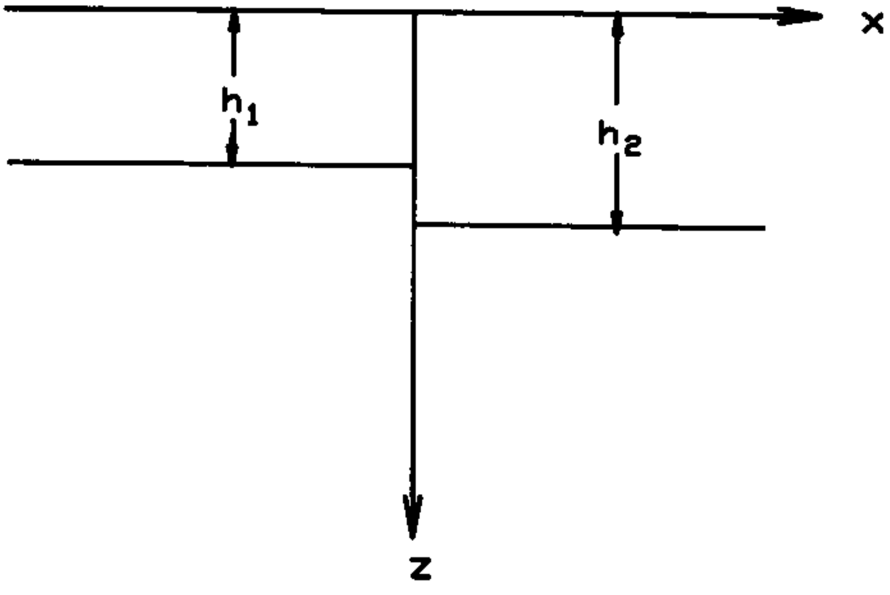


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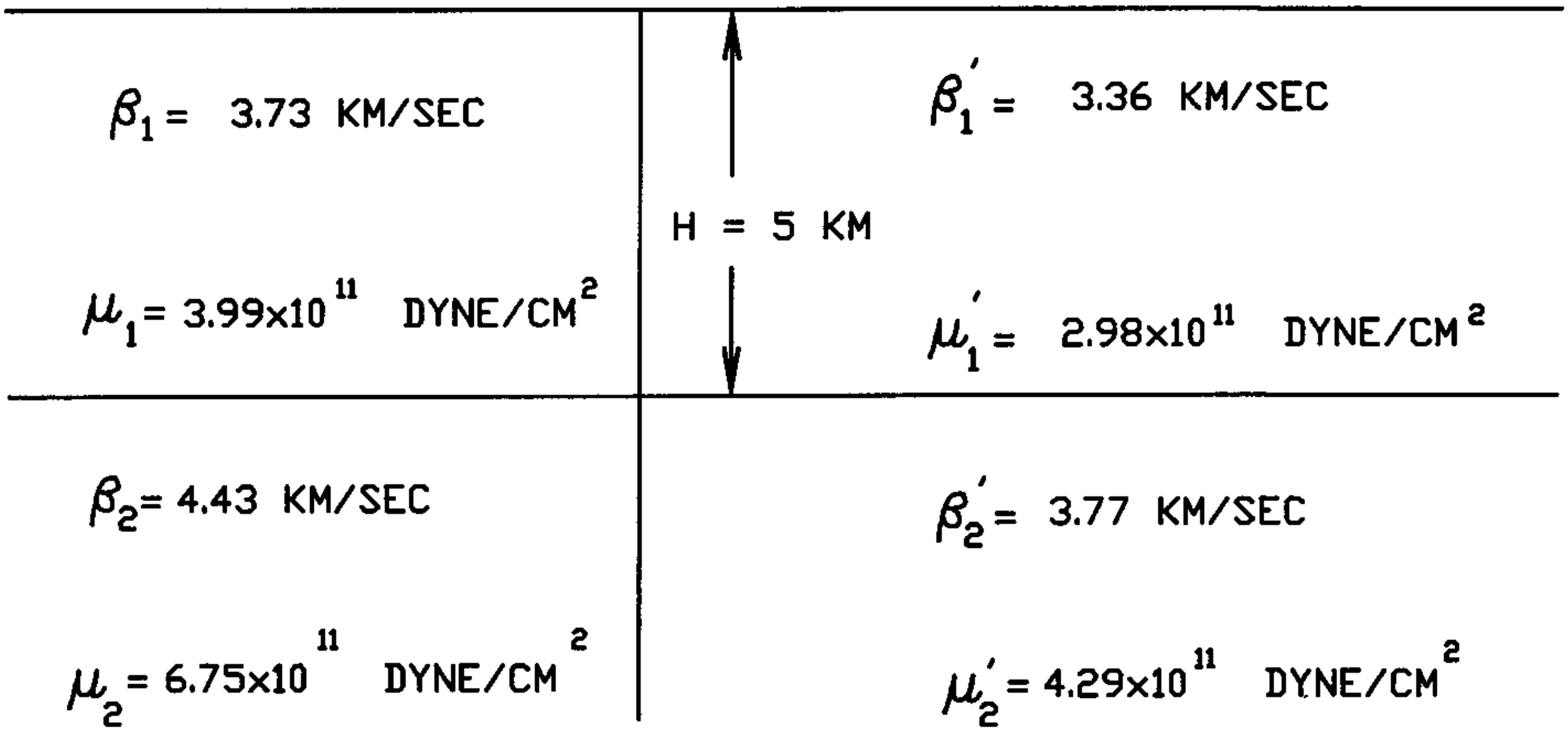
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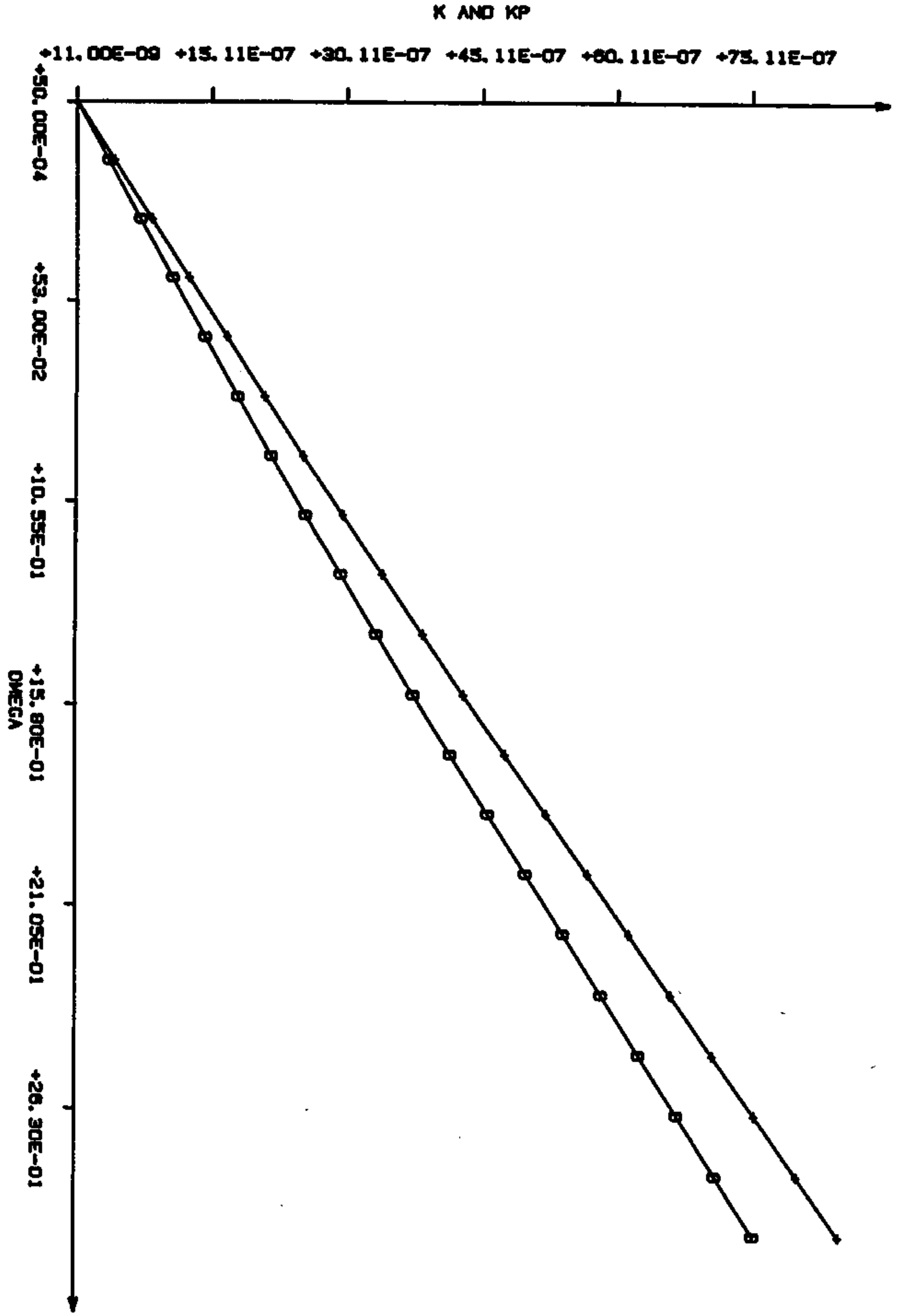
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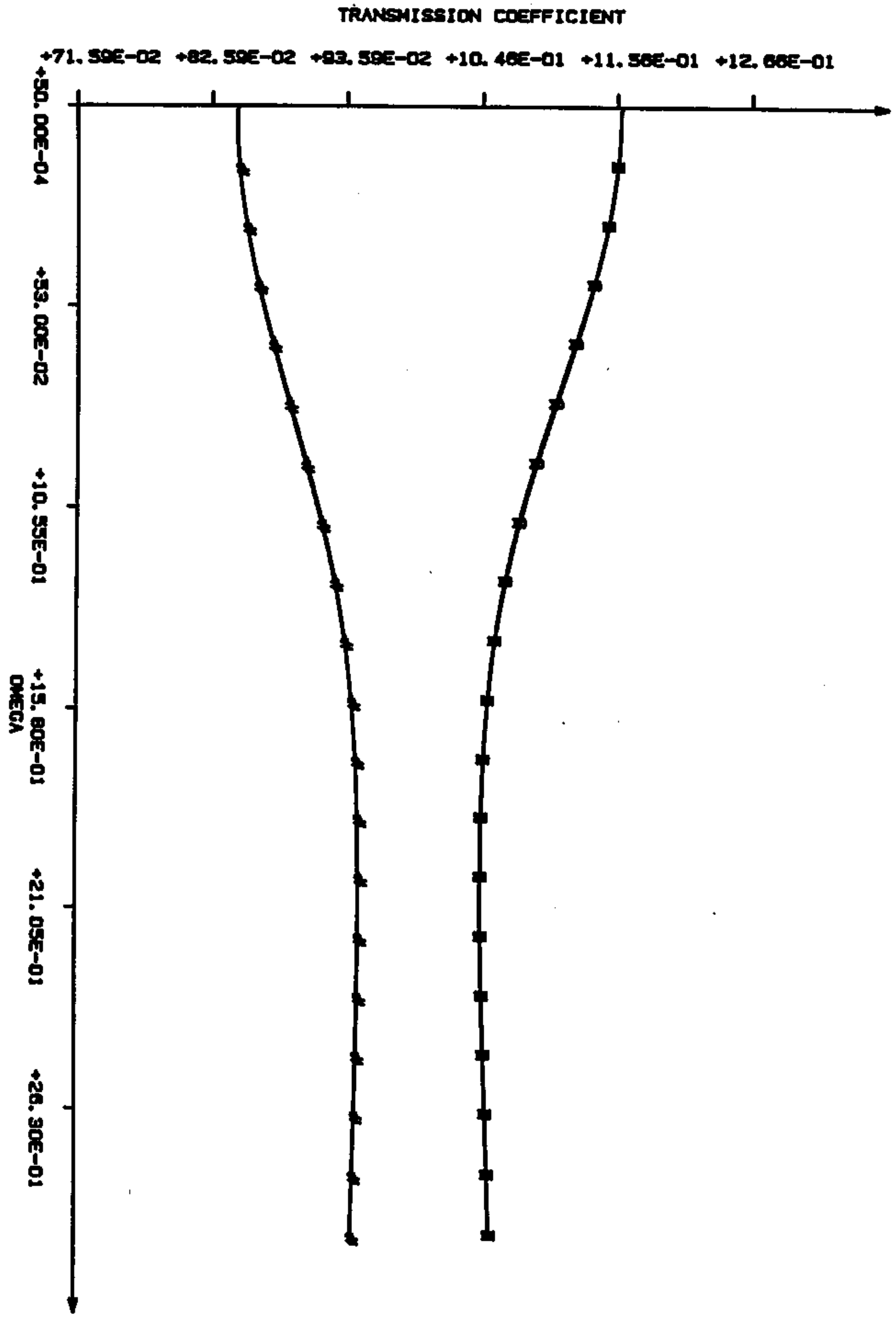
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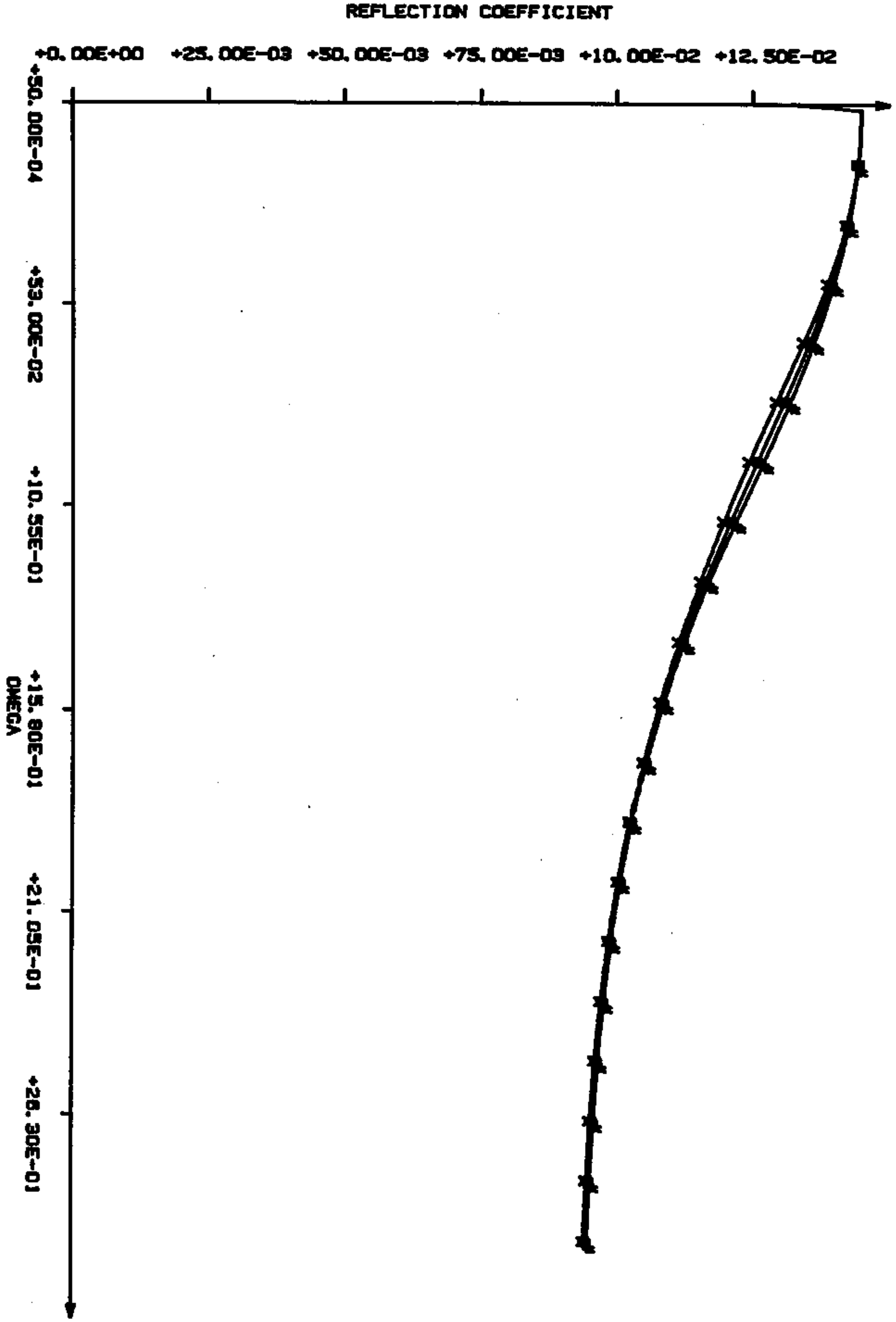
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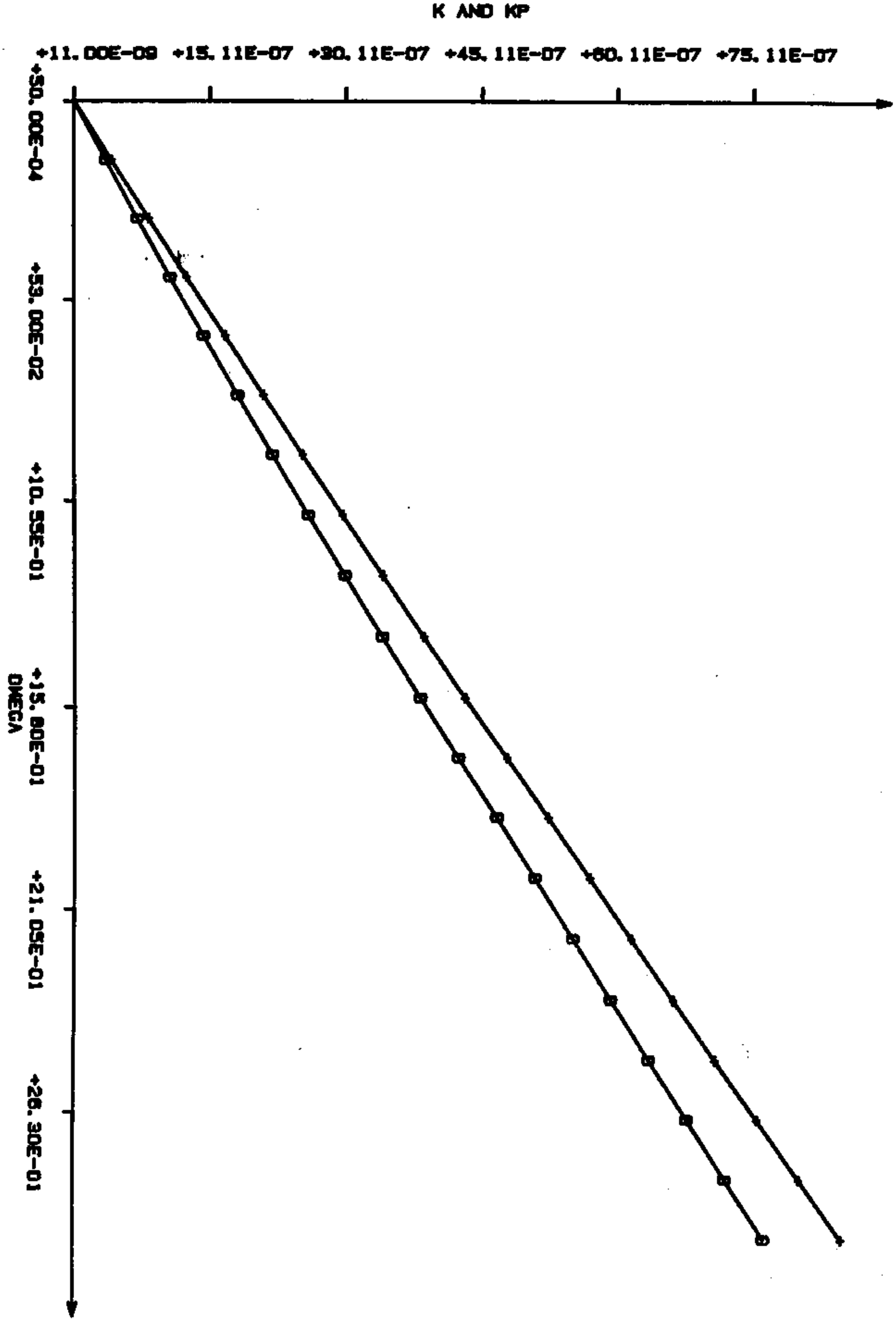
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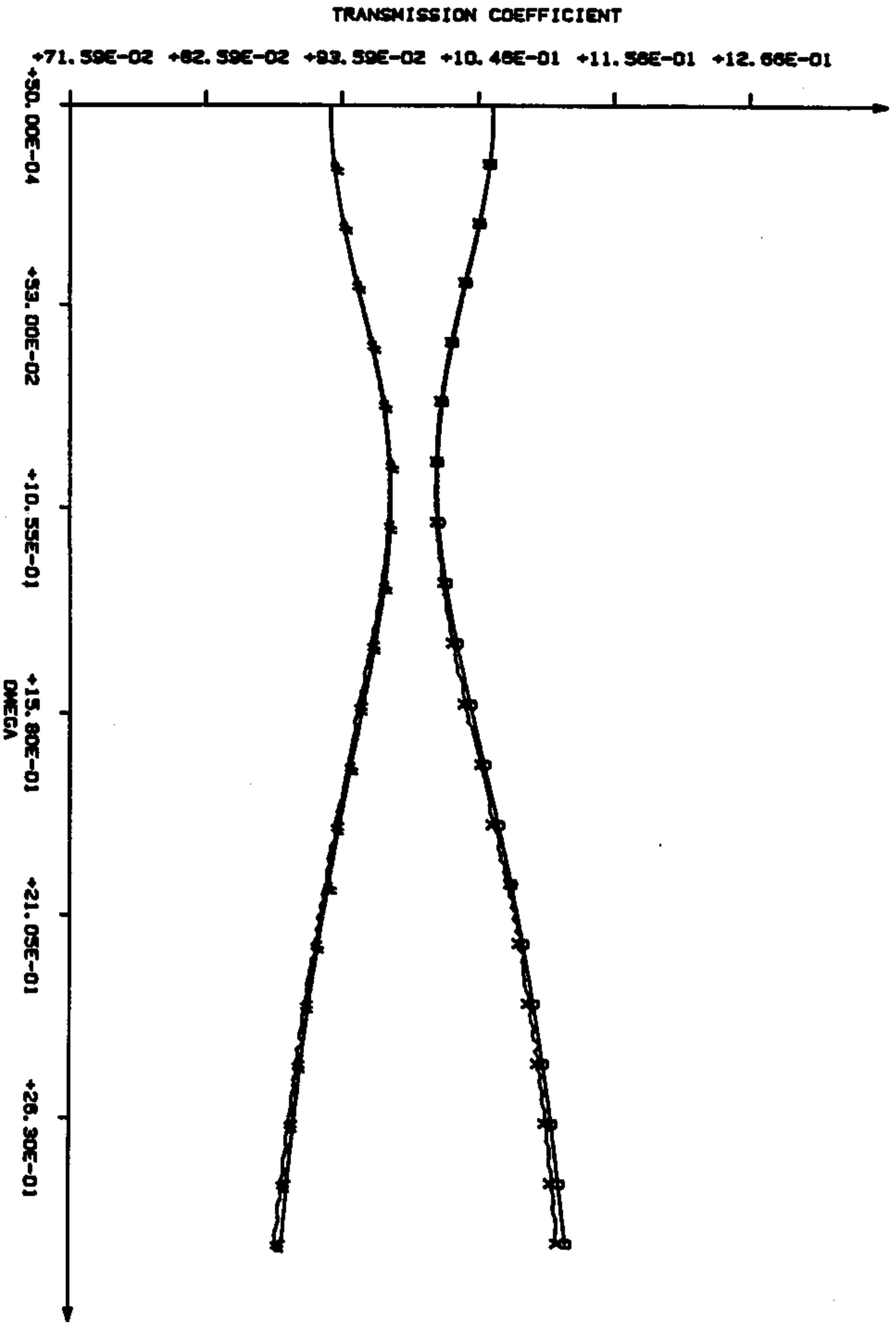
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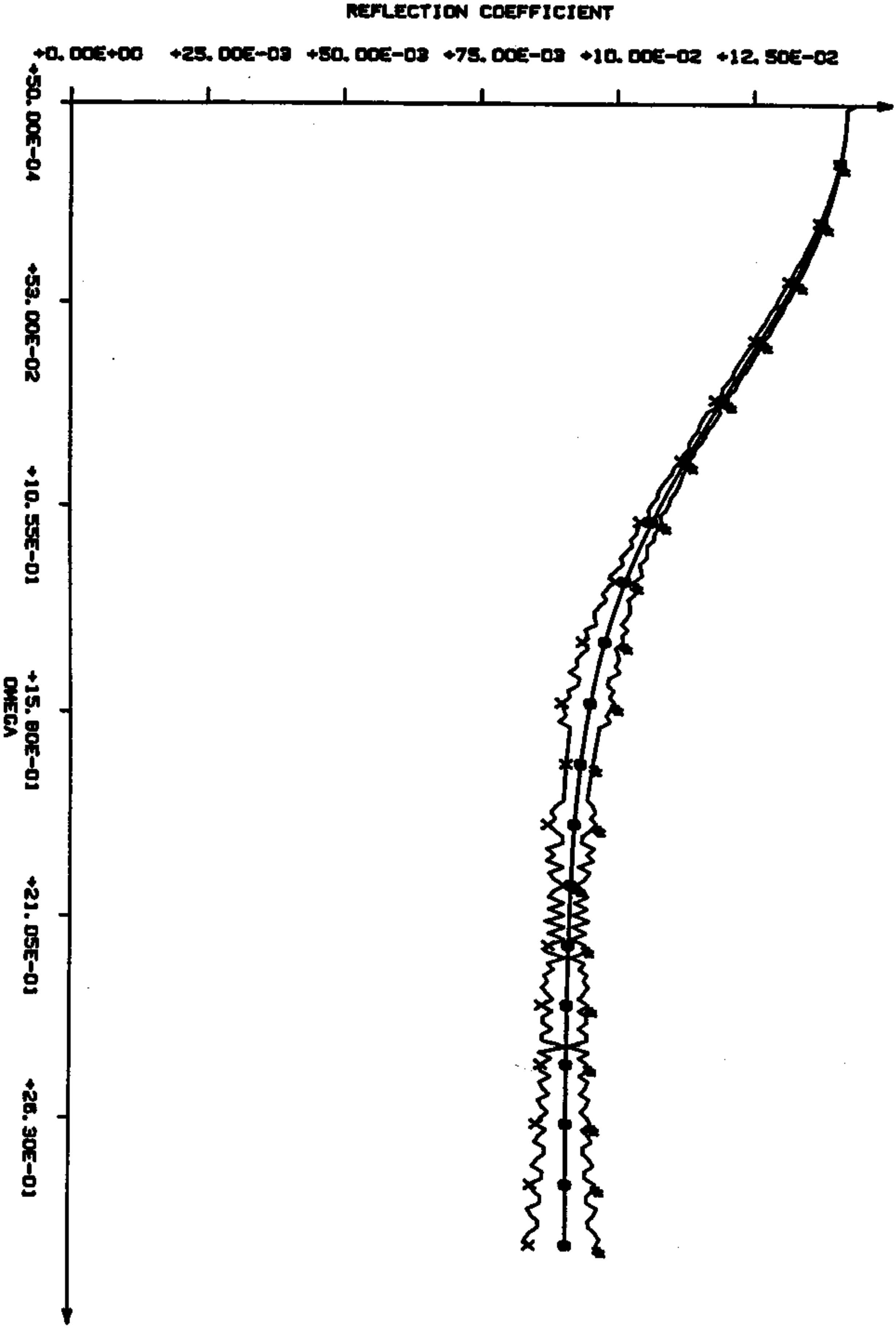
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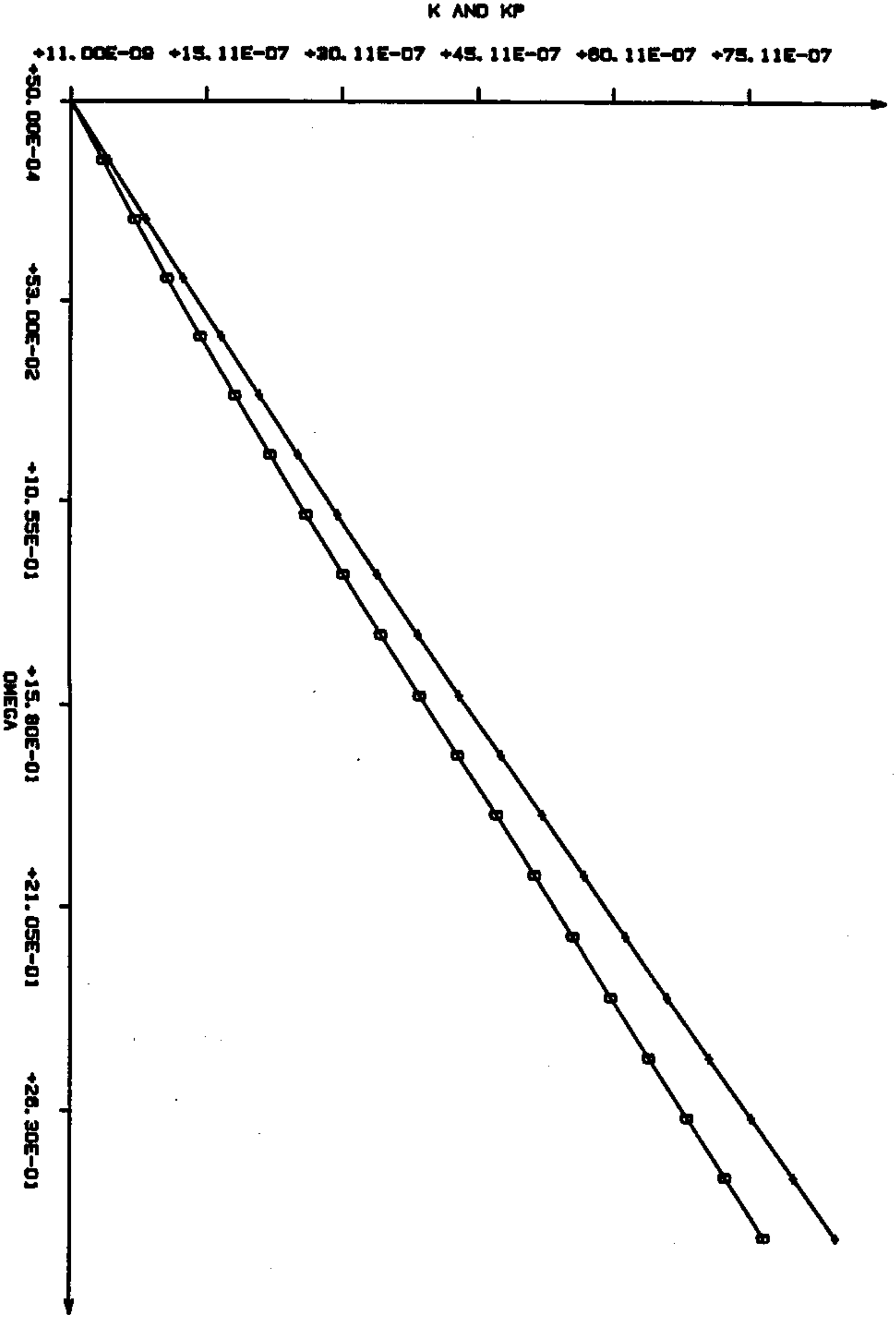
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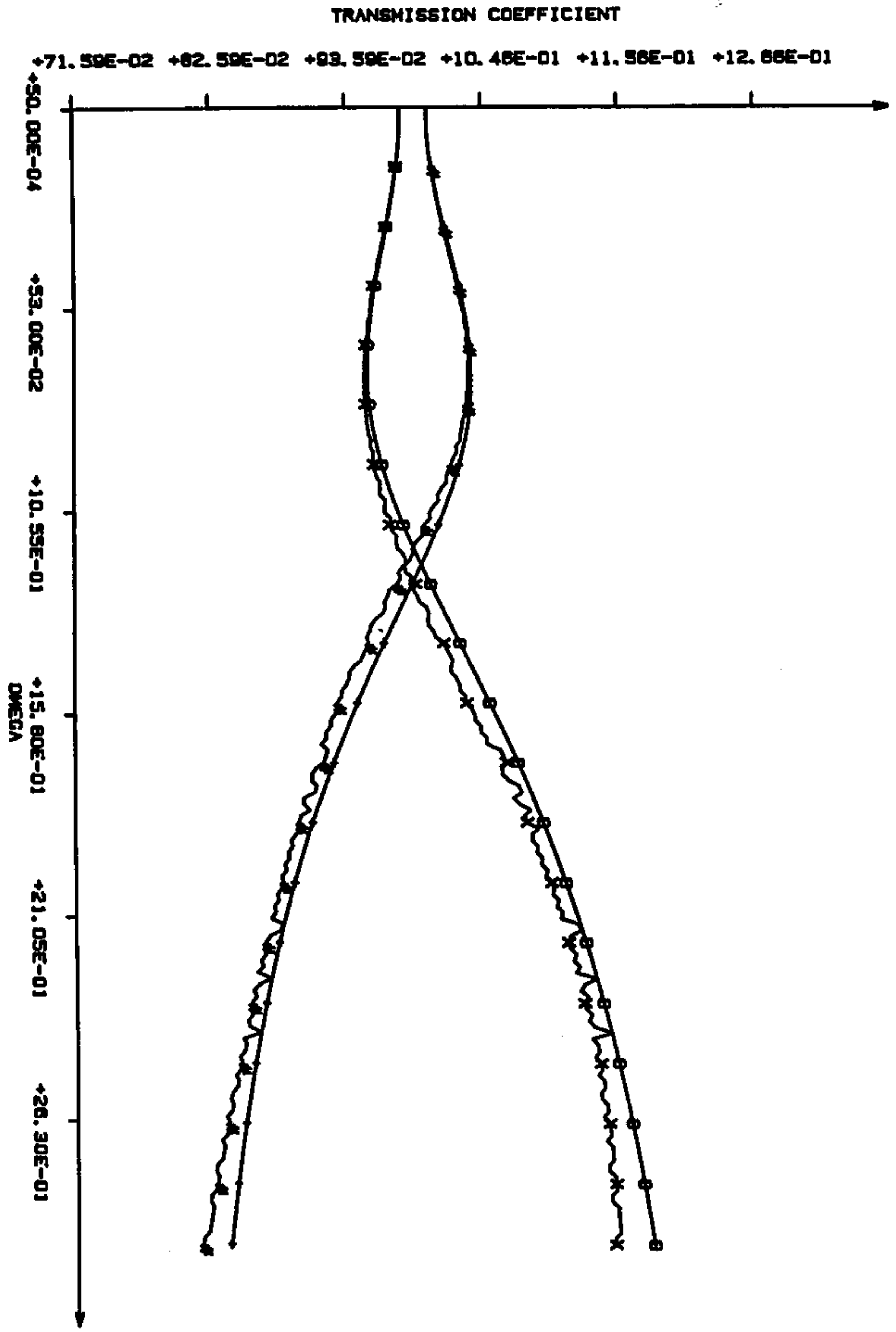
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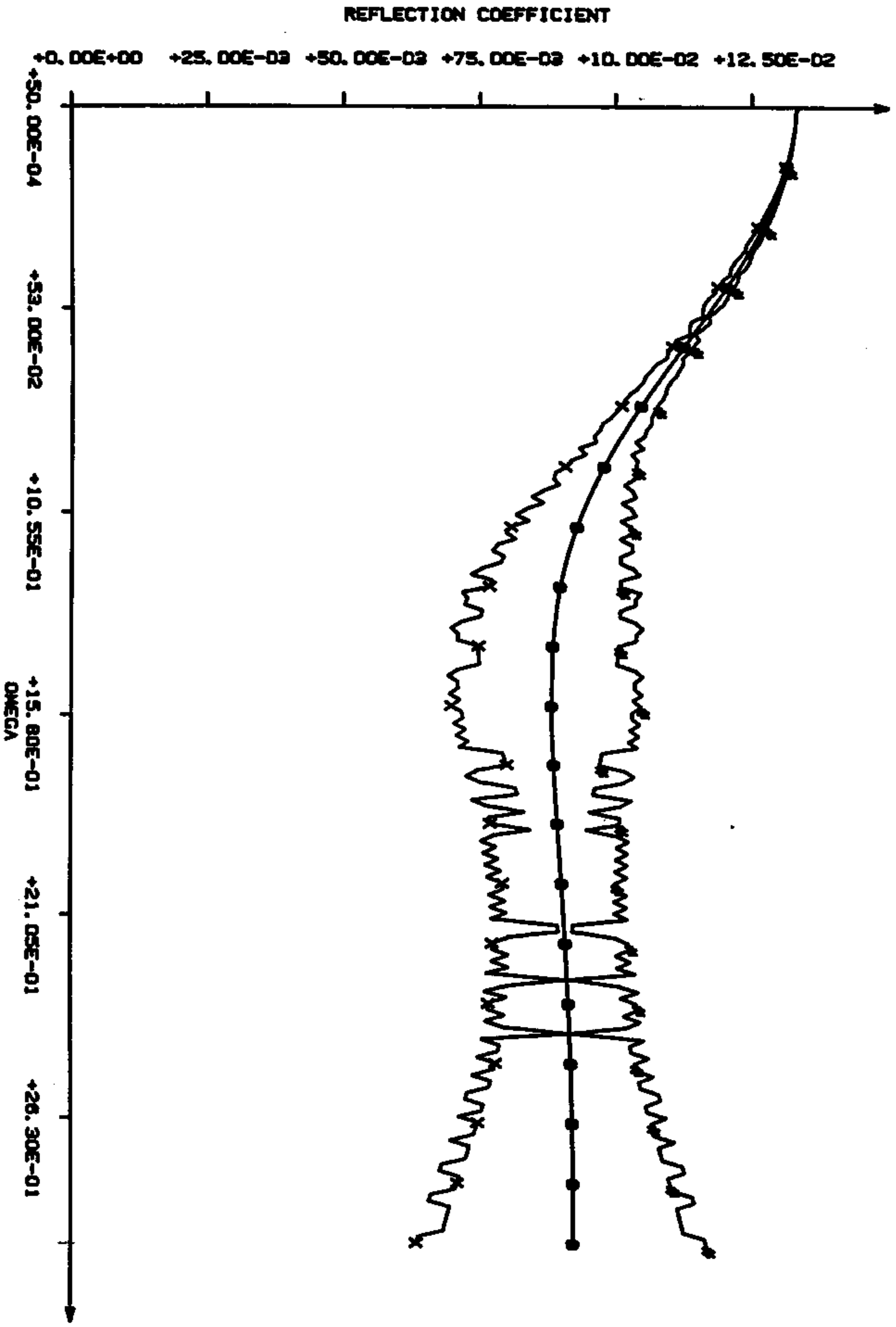
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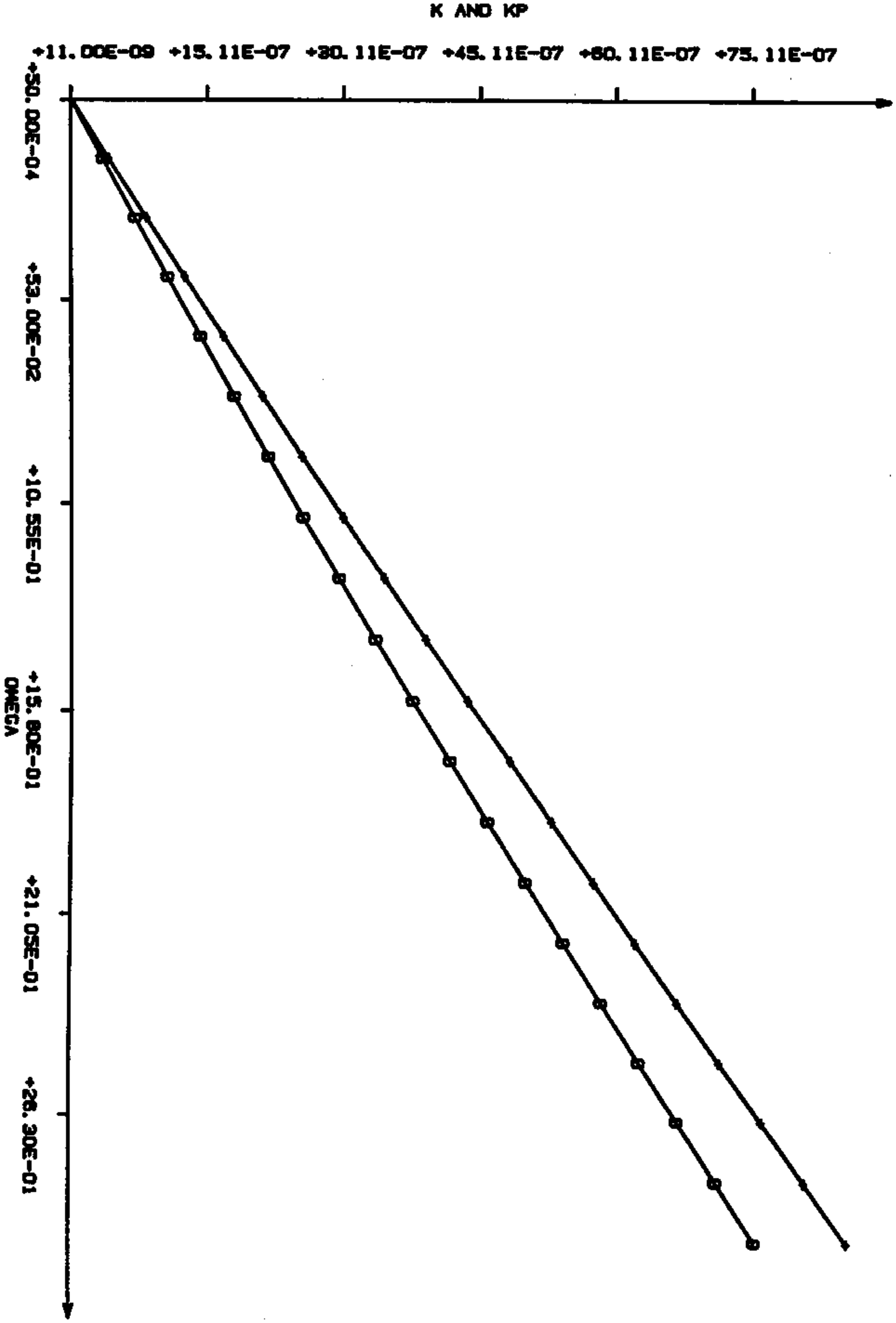
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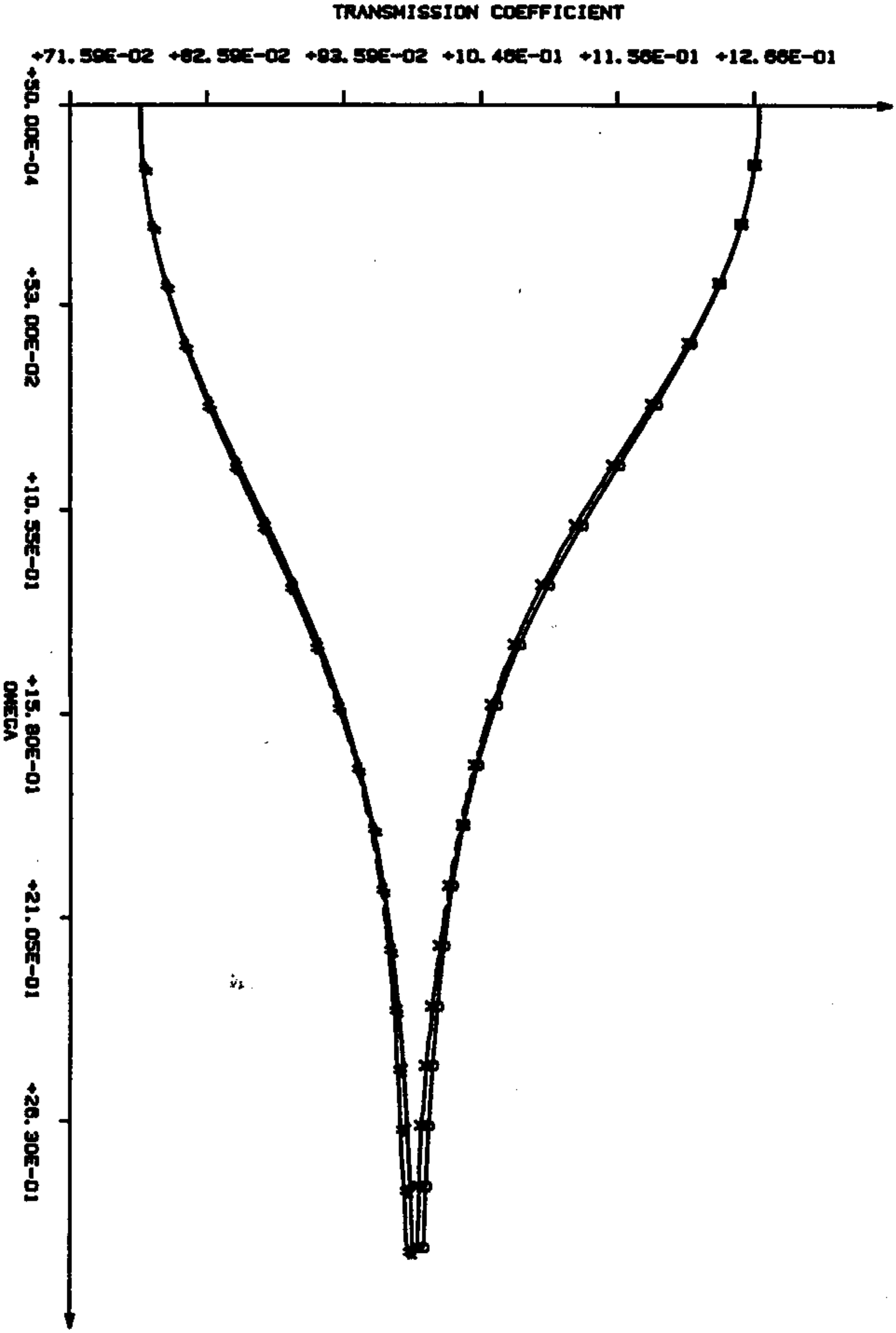
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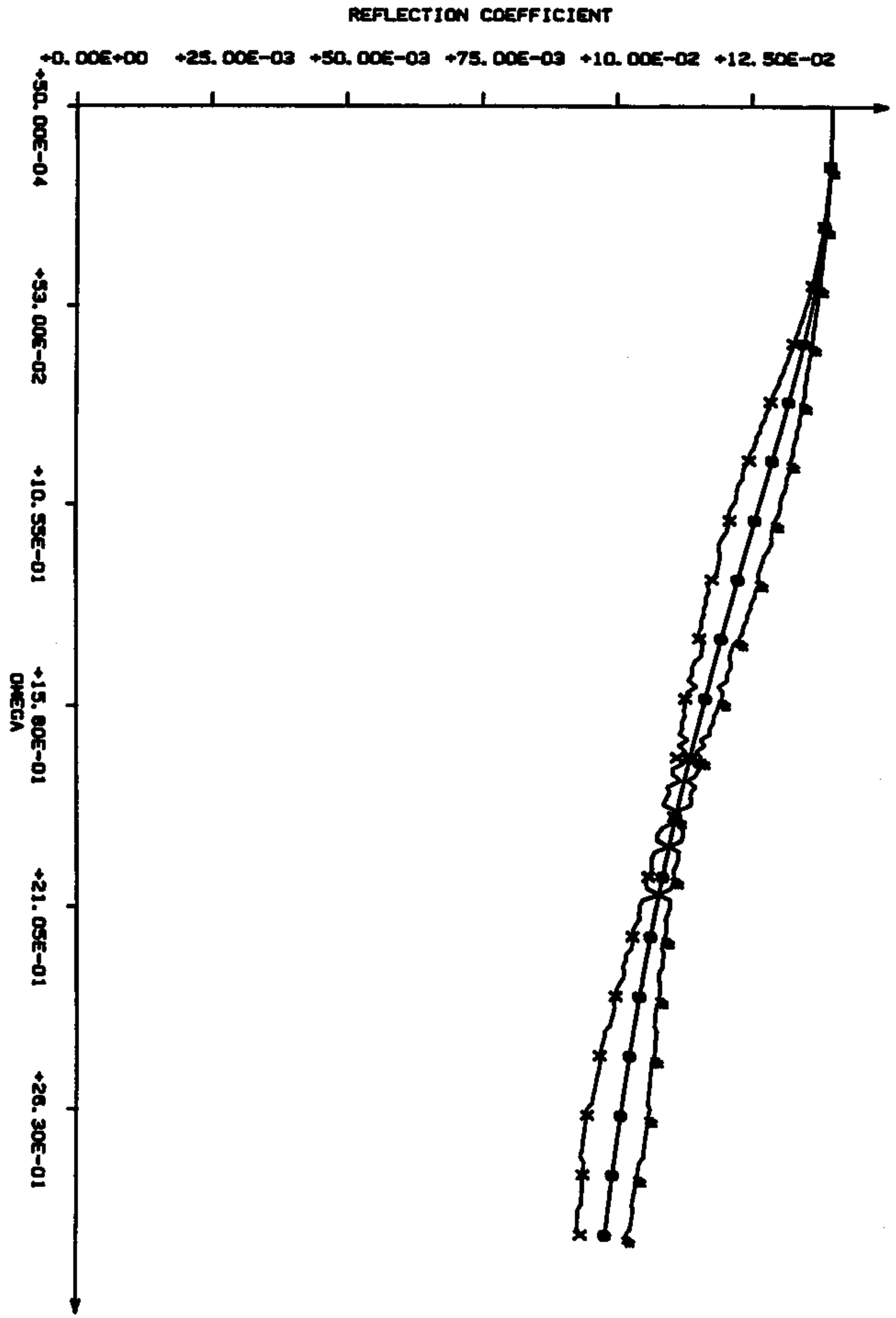
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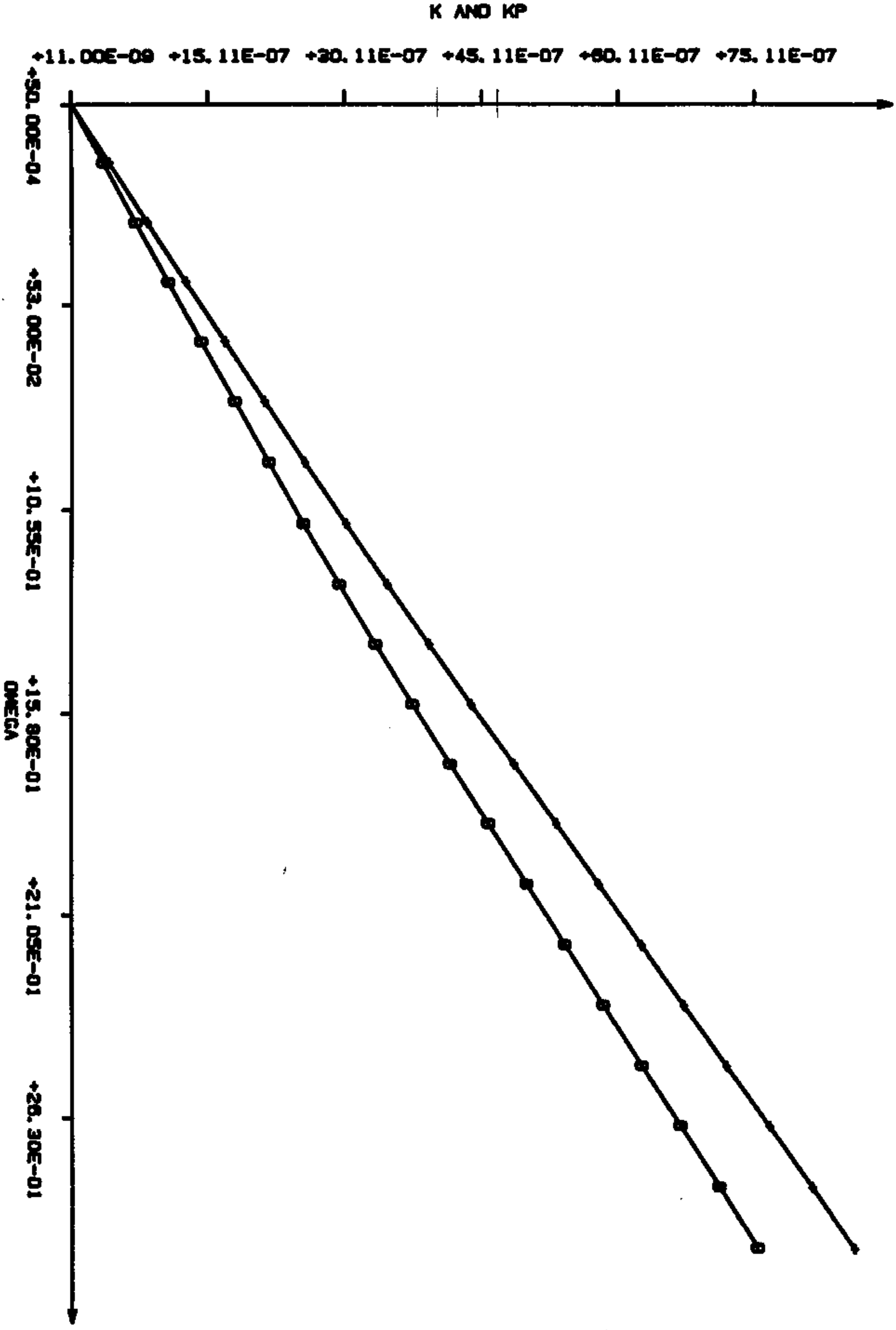
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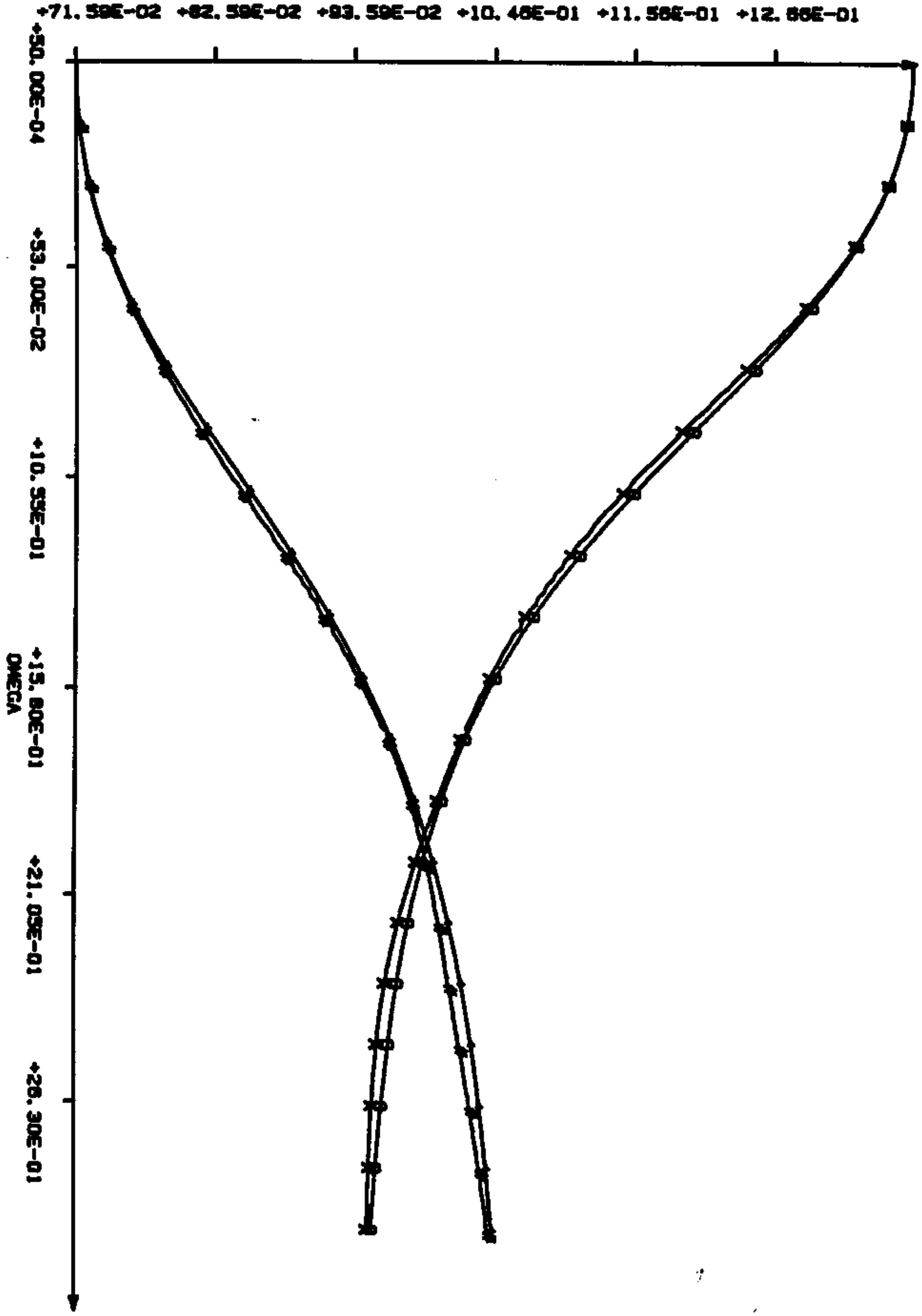

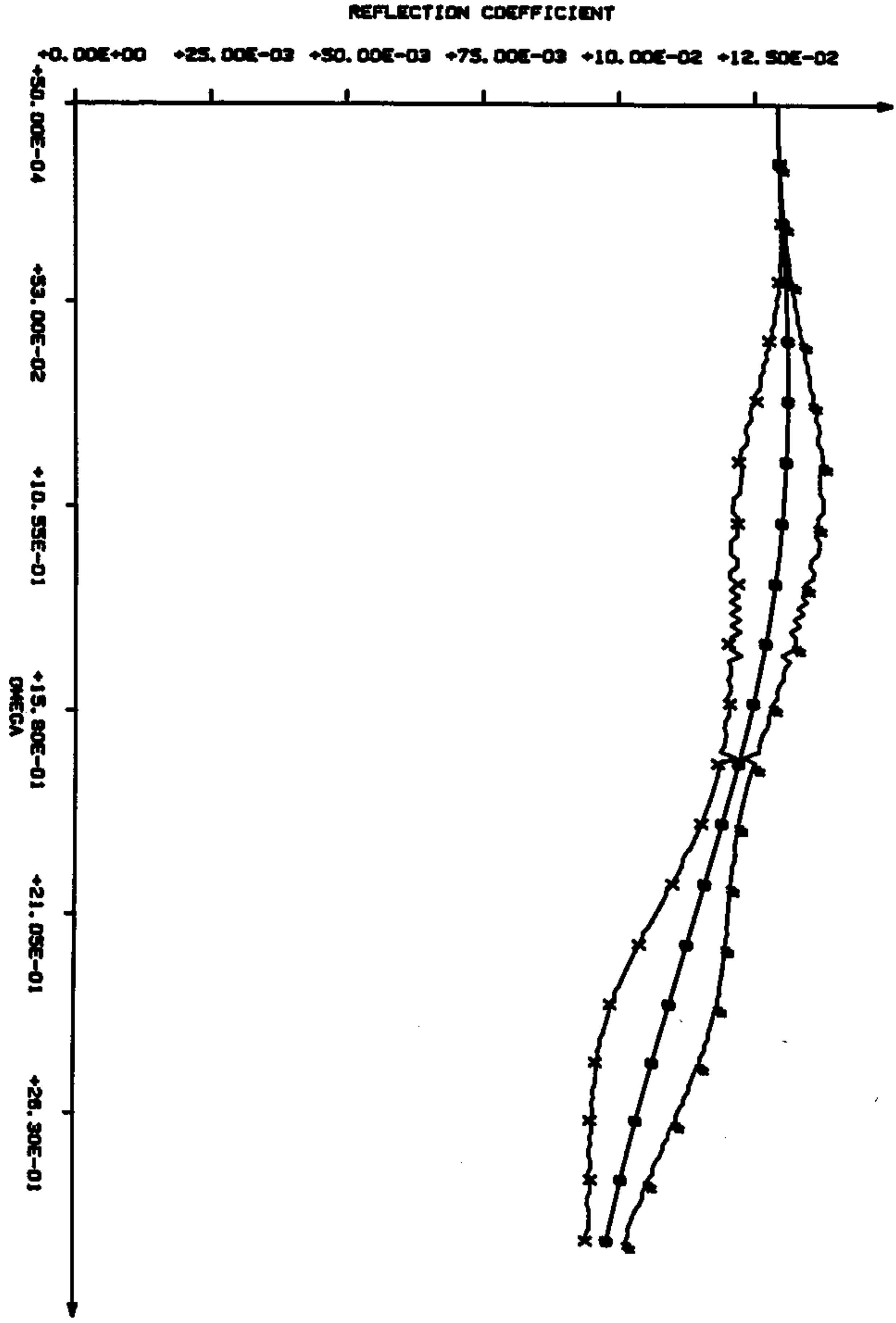
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