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Abstract

This paper makes an attempt to justify a multivariate t-model and reviews some of the most important results of this model developed in recent years.

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1 Introduction

The multivariate t-distribution is a natural generalization of the univariate Student t-distribution. Cornish (1954) derived it in connection with a set of normal sample deviates.

The classical theory of statistical analysis is primarily based on the assumption that errors are normally distributed. Recently many authors have investigated as to how inferences are affected if the population model departs from normality. Many economic and business data e.g. stock return data exhibit fat-tailed distributions. The suitability of independent t-distributions for stock return data was assessed by Blattberg and Gonedes (1974). Soon after that Zellner (1976) considered analyzing stock return data by a univariate regression model under the assumption that errors have a multivariate t-distribution. However, errors in this model are uncorrelated but not independent. After a thorough investigation, Kelejian and Prucha (1985) proved that uncorrelated t-distributions are better able to capture heavy-tailed behavior than independent t-distributions.

The multivariate t-distribution is a viable alternative to the usual multivariate normal distribution and on the other hand results obtained under normality can be checked for robustness. In this paper we justify an uncorrelated multivariate t-model as the model for sample and review the most important theories developed recently for statistical analysis with this model.

2 The Probability Model

The probability density function (p.d.f.) of a p-variate t-distribution is given by

$$f(\boldsymbol{x}) = \frac{|\boldsymbol{\Sigma}|^{-1/2}}{C(\nu,p)\pi^{p/2}}\left[1+\frac{1}{\nu}(\boldsymbol{x}-\boldsymbol{\mu})'\boldsymbol{\Sigma}^{-1}(\boldsymbol{x}-\boldsymbol{\mu})\right]^{-(\nu+p)/2} \tag{1.1}$$

where $\boldsymbol{x}$ is the realized value of a $p\times 1$ random vector $\boldsymbol{X}$, $\boldsymbol{\mu}$ is a $p\times 1$ unknown mean vector and $\boldsymbol{\Sigma}$ a $p\times p$ positive definite matrix of scale parameters while the normalizing constant $C(\nu,p)$ is given by

$$\Gamma((\nu+p)/2)C(\nu,p) = \nu^{p/2}\,\Gamma(\nu/2). \tag{1.2}$$

The p-variate random variable $\boldsymbol{X}$ has a mean vector $\boldsymbol{\mu}$ and a covariance matrix $\boldsymbol{\Sigma}^* = \nu^*\boldsymbol{\Sigma}$ where $\nu^* = \nu/(\nu-2)$ and can well be represented by $T_p(\boldsymbol{\mu},\ \nu^*\boldsymbol{\Sigma})$ where the shape parameter $\nu(>4)$ is assumed to be known. If $p=1,\ \boldsymbol{\mu}=0, \boldsymbol{\Sigma}=1$, then the p.d.f. in (1.1) defines the univariate t-distribution. It is well-known that the multivariate t-distribution can be written as

$$f(\boldsymbol{x}) = \int_0^\infty \frac{|\tau^2\boldsymbol{\Sigma}|^{-1/2}}{(2\pi)^{p/2}} \exp(-(\boldsymbol{x}-\boldsymbol{\mu})'(\tau^2\boldsymbol{\Sigma})^{-1}(\boldsymbol{x}-\boldsymbol{\mu})/2)\ h(\tau)d\tau \tag{1.3}$$

which is the mixture of the multivariate normal distribution $N_p(\boldsymbol{\mu}, \tau^2\boldsymbol{\Sigma})$ and the distribution of τ having p.d.f.

$$h(\tau) = \frac{2(\nu/2)^{\nu/2}}{\Gamma(\nu/2)}\tau^{-(\nu+1)} \exp\left(\frac{-\nu}{2\tau^2}\right). \tag{1.4}$$

It is easy to check that $\nu\tau^{-2}$ has a chi-square distribution with ν degrees of freedom. Thus given τ, the random vector $\boldsymbol{X}$ has a multivariate normal distribution i.e.

$$\boldsymbol{X}|\tau \sim N_p(\boldsymbol{\mu}, \tau^2\boldsymbol{\Sigma}). \tag{1.5}$$

As $\nu \to \infty$, the random variable τ becomes a degenerate random variable with all the non-zero mass at the point unity and, consequently, the multivariate t-distribution in (1.1) converges to the multivariate normal distribution $N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. This also follows from the fact that as $\nu \to \infty$, we have $C(\nu,p) \to 2^{p/2}$ and $(1+u/\nu)^{-\nu} \to e^{-u}$. It is also worth mentioning that the uncorrelatedness of the components $\boldsymbol{X}_1, \boldsymbol{X}_2, \ldots, \boldsymbol{X}_N$ does not imply their mutual independence unless $\nu \to \infty$.

The joint p.d.f. of N independent observations each having a p-variate t-distribution is given by

$$f_0(\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_N) = f(\boldsymbol{x}_1)f(\boldsymbol{x}_2)\ldots f(\boldsymbol{x}_N) \tag{1.6}$$

which may be referred to as the *independent t-model.* However, recent interest is noticed in uncorrelated t-distributions. The joint p.d.f. of N uncorrelated multivariate t-distributions is given by

$$f_0(\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_N) = \frac{\boldsymbol{\Sigma}^{-N/2}}{C(\nu, Np)\pi^{Np/2}} \left(1 + \frac{Q}{\nu}\right)^{-(\nu+Np)/2} \tag{1.7}$$

where $Q = \sum_{j=1}^{N} (\boldsymbol{x}_j - \boldsymbol{\mu})'\boldsymbol{\Sigma}^{-1}(\boldsymbol{x}_j - \boldsymbol{\mu})$ and $\boldsymbol{x}_j$ $(j = 1, 2, \ldots, N)$ is the realized value of a p-variate random vector $\boldsymbol{X}_j$ $(j = 1, 2, \ldots, N)$ having the t-distribution $T_p(\boldsymbol{\mu}, \nu^*\boldsymbol{\Sigma})$ where $\nu^* = \nu/(\nu - 2)$. The p.d.f. in (1.7) will hereinafter be called the *uncorrelated t-model.*

Kelegian and Prucha (1985) proved that the tails of the uncorrelated t-model is relatively thicker than those of the independent t-model given by (1.6). It may be remarked that observations in (1.7) are independent if and only $\nu \to \infty$, in which case the p.d.f. in (1.7) will be the product of N independent p-dimensional normal distributions N_p $(\boldsymbol{\mu}, \boldsymbol{\Sigma})$.

A more general case would be to consider k-samples each of which

$$(\boldsymbol{X}_{g_1}, \boldsymbol{X}_{g_2}, \ldots, \boldsymbol{X}_{gN_g}), \qquad g = 1, 2, \ldots, k$$

is a sample of size N_g from $T_p(\boldsymbol{\mu}_g, \nu^*\boldsymbol{\Sigma})$, $g = 1, 2, \ldots, k$. The joint p.d.f. of observations of k-samples would be

$$f_0(\boldsymbol{x}_{11}, \boldsymbol{x}_{12}, \ldots, \boldsymbol{x}_{kN_k}) = \frac{|\boldsymbol{\Sigma}|^{-N/2}}{C(\nu, Np)\pi^{Np/2}} \left(1 + \frac{1}{\nu}\, Q\right)^{-(\nu+Np)/2} \tag{1.8}$$

where $Q = Q_1 + Q_2 + \cdots + Q_k = \sum_{g=1}^{k} Q_g$, $Q_g = \sum_{j=1}^{N_g} (\boldsymbol{x}_{g_j} - \boldsymbol{\mu}_g)'\boldsymbol{\Sigma}^{-1}(\boldsymbol{x}_{g_j} - \boldsymbol{\mu}_g)$ and $N = N_1 + N_2 + \cdots + N_k = \sum_{g=1}^{k} N_g$.

3 Moments and Characteristic Functions

By the use of the mixture representation in (1.5), it can be easily proved that $E(\boldsymbol{X}) = E(E(\boldsymbol{X}|\tau)) = \boldsymbol{\mu}$ and $\mathrm{Cov}(\boldsymbol{X}) = E[\mathrm{Cov}(\boldsymbol{X}|\tau)] + \mathrm{Cov}[E(\boldsymbol{X}|\tau)] = E(\tau^2\boldsymbol{\Sigma}) = \nu^*\boldsymbol{\Sigma}$ where $\nu^* = \nu/(\nu-2)$. The characteristic functions of the univariate and the multivariate t-distributions have been considered by many authors. See Joarder and Ali (1996) and the references therein. The characteristic function of multivariate t-distribution in (1.1) due to Joarder and Ali (1996) is presented below.

Let $\boldsymbol{X}$ have the p.d.f. given by (1.1). Then the characteristic function of $\boldsymbol{X}$ is given by

$$\phi_{\boldsymbol{X}}(\boldsymbol{t}) = E(e^{i\boldsymbol{t}'\boldsymbol{X}}) = e^{i\boldsymbol{t}'\boldsymbol{\mu}}\,\frac{\|(\nu\boldsymbol{\Sigma})^{1/2}\boldsymbol{t}\|^{\nu/2}}{2^{\nu/2-1}\,\Gamma(\nu/2)}\,K_{\nu/2}(\|(\nu\boldsymbol{\Sigma})^{1/2}\boldsymbol{t}\|) \tag{3.1}$$

where $K_{\nu/2}(\|(\nu\boldsymbol{\Sigma})^{1/2}\boldsymbol{t}\|)$ is the Macdonald function with order $\nu/2$ and argument $\|(\nu\boldsymbol{\Sigma})^{1/2}\boldsymbol{t}\|$.

The Macdonald function $K_\alpha(t)$ with order α and argument t may be defined by the following integral representation (see e.g. Watson, 1958, p. 172):

$$K_\alpha(t) = \left(\frac{2}{t}\right)^{\alpha}\frac{\Gamma(\alpha+1/2)}{\sqrt{\pi}}\int_0^\infty (1+u^2)^{-(\alpha+1)}\cos tu\,du, \qquad t>0,\quad \alpha>-1/2. \tag{3.2}$$

A series representation of the Macdonald function $K_\alpha(r)$ where $r>0$ and α a non-negative integer is well known (cf. Joarder and Ali, 1996). The quantity $\boldsymbol{Z} = \boldsymbol{\Sigma}^{-1/2}(\boldsymbol{X}-\boldsymbol{\mu})$ has a spherical t-distribution $T_p(\boldsymbol{0},\nu^*\boldsymbol{I})$ whose product moment is given by

$$E\left(\prod_{i=1}^{p} Z_i^{k_i}\right) = \begin{cases} 0 & \text{if at least one } k_i (i = 1, 2, \ldots, p) \text{ is odd} \\ \nu^{k/2} \dfrac{\Gamma((\nu-k)/2)}{2^k\,\Gamma(\nu/2)} \displaystyle\prod_{i=1}^{p} \dfrac{k_i!}{(k_i/2)!}, & \nu > k \\ \text{if all } k_i\text{'s } (i = 1, 2, \ldots, p) \text{ are even.} \end{cases} \tag{3.3}$$

where $k = \sum_{i=1}^{p} k_i$. The product moment can also be derived by using the stochastic representation $\boldsymbol{Z} = R\boldsymbol{U}$ where R^2/p has an $F(p, \nu)$ distribution, $R = (\boldsymbol{Z}'\boldsymbol{Z})^{1/2}$ and $\boldsymbol{U}$ has a uniform distribution on the surface of unit hypersphere in $\Re^p$ (See Theorem 2.8 of Fang, Kotz and Ng (1990) for details.)

It follows from (3.1) that the characteristic function of the univariate Student t-distribution with p.d.f.

$$f(x) = \frac{1}{C(\nu,1)\sqrt{\pi}}\left(1 + \frac{x^2}{\nu}\right)^{-(\nu+1)/2}, \quad \nu > 0$$

is given by

$$\phi_{\mathbf{X}}(t) = \frac{\nu^{\nu/4}|t|^{\nu/2}}{2^{\nu/2-1}\,\Gamma(\nu-2)} \quad K_{\nu/2}(\sqrt{\nu}|t|). \quad \nu > 2$$

It may be remarked that the characteristic function of $\boldsymbol{X}$ in (3.1) can also be written as

$$\phi_{\boldsymbol{X}}(\boldsymbol{t}) = E(e^{i\boldsymbol{t}'\boldsymbol{X}}) = e^{i\boldsymbol{t}'\boldsymbol{\mu}}\,\psi(\boldsymbol{t}'\boldsymbol{\Sigma}\boldsymbol{t}). \tag{3.4}$$

The covariance matrix and the kurtosis parameter can then be written as

$$Cov(\boldsymbol{X}) = -2\psi'(0)\boldsymbol{\Sigma} \text{ and } \kappa = \frac{\psi''(0)}{\{\psi'(0)\}^2} - 1 \tag{3.5}$$

respectively (Seo and Toyama, 1996).

4 Marginal and Conditional Distributions

It is well-known that linear combinations, marginal and conditional distributions of the components of $\boldsymbol{X}$ follow the multivariate t-distribution (see e.g. Sutradhar, 1984). Let $\boldsymbol{X}, \boldsymbol{\mu}, \boldsymbol{t}$ and $\boldsymbol{\Sigma}$ be partitioned as

$$\boldsymbol{X} = \begin{pmatrix} \boldsymbol{X}_1 \\ \boldsymbol{X}_2 \end{pmatrix}, \ \boldsymbol{\mu} = \begin{pmatrix} \boldsymbol{\mu}_1 \\ \boldsymbol{\mu}_2 \end{pmatrix}, \ \boldsymbol{t} = \begin{pmatrix} \boldsymbol{t}_1 \\ \boldsymbol{t}_2 \end{pmatrix} \quad \text{and} \quad \boldsymbol{\Sigma} = \begin{pmatrix} \boldsymbol{\Sigma}_{11} & \boldsymbol{\Sigma}_{12} \\ \boldsymbol{\Sigma}_{21} & \boldsymbol{\Sigma}_{22} \end{pmatrix}$$

where $\boldsymbol{X}_2, \boldsymbol{\mu}_2, \boldsymbol{t}_2 \in \Re^q \ \ (q < p)$ and $\boldsymbol{\Sigma}_2$ is a $q \times q$ positive definite matrix. By the use of the characteristic function of $\boldsymbol{X}$ given by (3.1), it may be easily checked that $\boldsymbol{X}_2 \sim T_q \ (\boldsymbol{\mu}_2, \ \nu^* \boldsymbol{\Sigma}_{22})$ where $\nu^* = \nu/(\nu \dot{-} 2)$. The c onditional distribution of $\boldsymbol{X}_1$ given $\boldsymbol{X}_2 = x_2$ is $T_{p-q} \, (\boldsymbol{\mu}_{1 \cdot 2}, \ \nu_{1 \cdot 2} \ \boldsymbol{\Sigma}^*_{11.2})$ where

$$\begin{aligned} \boldsymbol{\mu}_{1.2} &= \boldsymbol{\mu}_1 + \boldsymbol{\Sigma}_{12} \boldsymbol{\Sigma}_{22}^{-1} \, (\boldsymbol{x}_2 - \boldsymbol{\mu}_2) \\ \nu_{1.2} &= \nu/(\nu + q - 2), \quad \text{and} \\ \boldsymbol{\Sigma}^*_{11.2} &= \left(1 + (\boldsymbol{x}_2 - \boldsymbol{\mu}_2)' (\nu \boldsymbol{\Sigma}_{22})^{-1} \, (\boldsymbol{x}_2 - \boldsymbol{\mu}_2)\right) \boldsymbol{\Sigma}_{11.2} \end{aligned}$$

with $\boldsymbol{\Sigma}_{11.2} = \boldsymbol{\Sigma}_{11} - \boldsymbol{\Sigma}_{12} \boldsymbol{\Sigma}_{22}^{-1} \boldsymbol{\Sigma}_{21}$.

5 Distribution of the Sum of Products Matrix Based on the Uncorrelated T-model

The sum of product matrix based on the uncorrelated t-model is given by

$$\boldsymbol{A} = \sum_{j=1}^{N} \left(\boldsymbol{X}_j - \overline{\boldsymbol{X}}\right) \left(\boldsymbol{X}_j - \overline{\boldsymbol{X}}\right) = (a_{ik})$$

where $\overline{\boldsymbol{X}} = \sum_{j=1}^{N} \boldsymbol{X}_j / N$. It follows from (1.5) that for a given τ, the random matrix $\boldsymbol{A}$ has the usual Wishart distribution

$$A|\tau \sim W_p(m, \tau^2\boldsymbol{\Sigma}), \quad m = N-1 \tag{5.1}$$

i.e. the p.d.f. of $\boldsymbol{A}$ is given by

$$\int_0^\infty \frac{|\tau^2\boldsymbol{\Sigma}|^{-1/2}}{2^{mp/2}\Gamma_p(m/2)} |\boldsymbol{A}|^{(m-p-1)/2} \exp\left(-\frac{1}{2}\text{tr}(\tau^2\boldsymbol{\Sigma})^{-1}\boldsymbol{A}\right) \quad h(\tau)d\tau \tag{5.2}$$

where $\boldsymbol{A} > 0$, $m = N - 1 \geq p$ and the generalized gamma function $\Gamma_p(\alpha)$ is defined by

$$\Gamma_p(\alpha) = \pi^{p(p-1)/4} \prod_{i=1}^{p} \Gamma((2\alpha - i + 1)/2) \tag{5.3}$$

while $\nu\tau^{-2} \sim \chi^2_\nu$. The completion of integration in (5.2) results in the p.d.f. of $\boldsymbol{A}$ given by

$$\frac{|\boldsymbol{\Sigma}|^{-m/2}}{C(\nu, mp)2^{mp/2}\,\Gamma_p(m/2)} \; |\boldsymbol{A}|^{(m-p-1)/2} \left(1 + \frac{1}{\nu}\,\text{tr}\,\boldsymbol{\Sigma}^{-1}\boldsymbol{A}\right)^{-(\nu+mp)/2} \tag{5.4}$$

(cf. Sutradhar and Ali, 1989).

By the use of the mixture representation in (5.1), it is easy to derive the expected values of $|\boldsymbol{A}|^k, |\boldsymbol{A}|^k\boldsymbol{A},\ |\boldsymbol{A}|^k\boldsymbol{A}^{-1},\ (\text{tr}\ \boldsymbol{A})^2, \text{tr}(\boldsymbol{A}^2)$ etc. which are important in developing estimation strategies for functions based on the covariance matrix. See e.g. Joarder and Ali (1992) and Joarder (1995a).

6 Estimation of Parameters

The maximum likelihood estimators of the parameters $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ of the correlated t-model in (1.7) are given by $\hat{\boldsymbol{\mu}} = \overline{\boldsymbol{X}}$ and $\hat{\boldsymbol{\Sigma}} = \boldsymbol{A}/N$ respectively (see Fang and Anderson, 1990, pp. 201–216). But maximum likelihood estimators in this case are not appealing because most important properties of maximum likelihood estimators, follow from the independence of the observations which is not the case for the model in (1.7) for finite value of the shape parameter ν. The sample mean $\overline{\boldsymbol{X}}$ is obviously an unbiased and consistent estimator of $\boldsymbol{\mu}$. The unbiased estimator of $\boldsymbol{\Sigma}$ is given by $\hat{\boldsymbol{\Sigma}} = \boldsymbol{A}/(\nu^* m)$, where $\nu^* = \nu/(\nu - 2)$ and $m = N - 1$ (see Fang and Anderson, pp. 208).

Joarder (1995) considered the estimation of the scale matrix $\boldsymbol{\Sigma}$ of the uncorrelated t-model under a squared error loss function. It may be remarked that the scale matrix $\boldsymbol{\Sigma}$ determines the covariance matrix up to a known constant ν^*. Joarder and Ahmed (1996) developed estimation strategy for eigenvalues of $\boldsymbol{\Sigma}$ of the correlated t-model given in (1.7). The estimation of the trace of the scale matrix $\boldsymbol{\Sigma}$ under a squared error loss has been considered by Joarder and Beg (1998). The estimation of $\boldsymbol{\Sigma}$ under a entropy loss function was considered by Joarder and Ali (1997).

7 Robustness of Correlation

Fisher (1915) derived the exact sampling distribution of Pearsonian correlation coefficient R for a random sample drawn from a bivariate normal population $N_2(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. Since then many statisticians have tried to investigate the behavior of R for non-normal situations. Ali and Joarder (1991) proved that both null and non-null distribution of R remain robust in the entire class of elliptical distributions which accommodates the correlated t-model as a special cases. The result has been generalized by Joarder and Ali (1992a) for the correlation matrix $\boldsymbol{R}$.

8 Distribution of T^2-statistic

Consider a two-sample problem i.e. the case of $k = 2$ in the situation discussed in (1.8). The equality of mean vectors $\boldsymbol{\mu}_1$ and $\boldsymbol{\mu}_2$ can then be tested by $T^2 = \left(\overline{\boldsymbol{X}}_1 - \overline{\boldsymbol{X}}_2\right)\left(\frac{S_p}{N_1} + \frac{S_p}{N_2}\right)^{-1}\left(\overline{\boldsymbol{X}}_1 - \overline{\boldsymbol{X}}_2\right)$ where $(m_1 + m_2)S_p = m_1 S_1 + m_2 S_2$ with $m_1 = N_1 - 1$ and $m_2 = N_2 - 1$. The above result was derived by Sutradhar (1988) for a scaled correlated t-model obtained by reparametrizing $\nu^*\boldsymbol{\Sigma}$ by $\boldsymbol{\Sigma}$ in (1.7). The following dervation of T^2-statistic is based on the mixture representation of multivariate t-distribution (see e.g. Khan 1997).

By virtue of the mixture representation of (1.5), it follows that conditional on τ,

$$\frac{m}{p}\, T^2 \sim F_{p,m}(\delta_\tau) \tag{8.1}$$

where $F_{p,m}(\delta_\tau)$ denotes a noncentral F-distribution with parameters p, $m = m_1 + m_2 - p + 1$ and $\delta_\tau = (\boldsymbol{\mu}_1 - \boldsymbol{\mu}_2)'\,(\tau^2\boldsymbol{\Sigma})^{-1}(\boldsymbol{\mu}_1 - \boldsymbol{\mu}_2)$. The unconditional distribution of $\frac{m}{p}T^2$ can be obtained by completing the following integral

$$\int_0^\infty u_{p,m}(\delta_\tau)\; h(\tau)d\tau$$

where $u_{p,m}(\delta_\tau)$ is the p.d.f. of $F_{p,m}(\delta_\tau)$. It follows from (8.1) that under $H_0 : \boldsymbol{\mu}_1 = \boldsymbol{\mu}_2$

$$T^2 \sim \frac{p}{m}\, F_{p,m}.$$

The power function of the test $H_0 : \boldsymbol{\mu}_1 = \boldsymbol{\mu}_2$ against $H_1 : \boldsymbol{\mu}_1 \neq \boldsymbol{\mu}_2$ also follows from Sutradhar (1988).

9 Linear Regression Models

Zellner (1976) considered univariate linear regression model to analyze stock return data with errors having a univariate uncorrelated t-model. It is King (1980) who laid the rigorous mathematical foundation of linear regression anlysis under broader distributional assumptions of spherical symmetry which includes uncorrelated t-model as a special case. Prompted by the works of Zellner (1976) and King (1980), many authors used uncorrelated t-model for modeling real world data. Sutradhar and Ali (1986) generalized Zellner's model with errors having correlated t-model given by (1.7). Lange, Little and Taylor (1989) applied uncorrelated t-model to a variety of situations.

The null distribution of the usual F-statistic in a linear regression model under correlated t-model in (1.7) is robust but the power function depends on the form of (1.7); see e.g. Sutradhar(1988) and Sutradhar (1990) for a detailed proof. For the linear regression model with errors having an uncorrelated t-model, it is known (Singh, 1987) that the usual least square estimator of the vector of regression coefficients is not only the maximum likelihood estimator but also the unique minimum variance estimator. Singh (1988) also developed methods of estimation of error variance in linear regression models with errors having an uncorrelated t-model with unknown degrees of freedom.

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