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**PROCEDURAL APPROACH TO SKETCHING ONE-
DIMENSIONAL GRAPHS USING MATHCAD**

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PROCEDURAL APPROACH TO SKETCHING ONE-DIMENSIONAL GRAPHS USING MATHCAD

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Abstract: Once their problems are correctly set up, students can use MathCAD as a tool for quickly calculating the correct answer or graphing one or more equations. This gives them rapid feedback on whether the algebraic calculations done by hand are correct. As a result of this rapid feed back, students may quickly reevaluate their written work. Repeating assigned questions becomes easier using MathCAD and repetition aids retention. By changing parameters values, students are able to instantly see changes in graphs [5:306]

1. Introduction:

MathCAD is a program that has been widely and intensively used in teaching various areas of Mathematics, Sciences and Engineering, especially at the university level. For instance, one can visit the site (http://www.zenex.fi/math/USA_Campus.htm) to see the intensity of this usage.

This wide acceptance of MathCAD perhaps, is as a result of its unique features of simplicity and user-friendliness in the classroom settings. MathCAD has been shown to be an effective way of promoting learning and creativity as it forces students to think carefully about how the problem should be set up and what goes into its solution [5]. Studies have shown that as the students get to see the different ways in which MathCAD is used to promote learning the desire to use it arises [2].

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Building mathematical model using MathCAD is not difficult. This is because the model is constructed using mathematical notations that are well known by all. This MathCAD feature of what-you-see-is-what-you-give (WYSIWYG) output instantly exposes the raw mathematics, and therefore, allows students to see if they have entered a problem correctly or not. This is shown to be helpful in making students appreciate and obey the *rigor* of mathematical statements [7]. And pedagogically speaking, it allows ‘even the weakest of students to focus on the real problem at hand’ [2:333].

After constructing a mathematical model using MathCAD, it is easy to play around with the model by simply changing parameters. This features is popularly known as “what – if”. Using this feature, students are able to instantly see changes in their models, easily solve large class of mathematical problems, obtain graphs, as well as verify whether or not their analytical methods done by hands are wrong or right. This indeed makes the student concentrate on the model rather than the language features, and it helps students explore the relationship between the model and the mathematics, which is very useful especially when models decide to misbehave.

These are some of the peculiar features of MathCAD. Although MathCAD may not have all the versatility of a high level programming language, it is still very flexible and user-friendly.

2. Sketching Graphs Using MathCAD

The use of computers and graphing calculators for teaching graphing techniques is getting more popular in many classrooms. These technologies have shown great promise to enhance teaching and learning through their visualization capabilities, and potential of analyzing the graphs for insight. Now, graphing of 2D and 3D is easily done using most

mathematics software and graphing utilities. Using MathCAD, graphing and other forms of mathematical computation are much easier since the mathematics is seen as it is done on the blackboard, or on paper.

However, what is not generally known is how to use these software to sketch one-dimensional graphs (that is graphs that lie on the Real line). This is in spite of the fact that we encounter many algebraic topics such as: linear inequality, absolute value inequality, rational inequality, distance etc, whose graphs are one-dimensional. And in most cases, students are required to display or represent these mathematical objects graphically. Although there are some attempts to address this problem (see [3]), this paper gives a closed-form solution to this problem. The approach is simple and can be used by the teacher in teaching almost all topics whose graphs and solutions lie in the Real line. The method will also help students to crosscheck their solutions quickly, and explore and obtain indebt knowledge of all classes of mathematical objects whose graphs lie in a Real Line.

3. Sketching One Dimensional Graphs Using MathCAD

MathCAD layout is 2-dimensional. So, to graph one dimensional object requires some little trick. However, we shall show this by simple illustration of how to sketch these graphs using Linear Inequality; which is a typical class of all classes of mathematical object whose graphs lie on the Real line, and then we shall give the generalizations to other objects as remarks. This paper assumed some little familiarity with MathCAD program.

The steps:

a) Define the general form of Linear Inequality together with all the variables involved as follows:

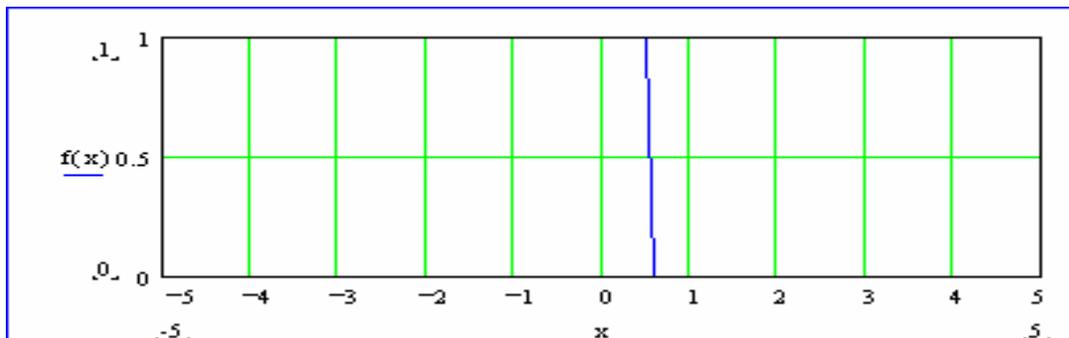
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$$x := -5, -4.9..5 \quad a := 8 \quad b := -2 \quad c := 2 \quad d := 1$$

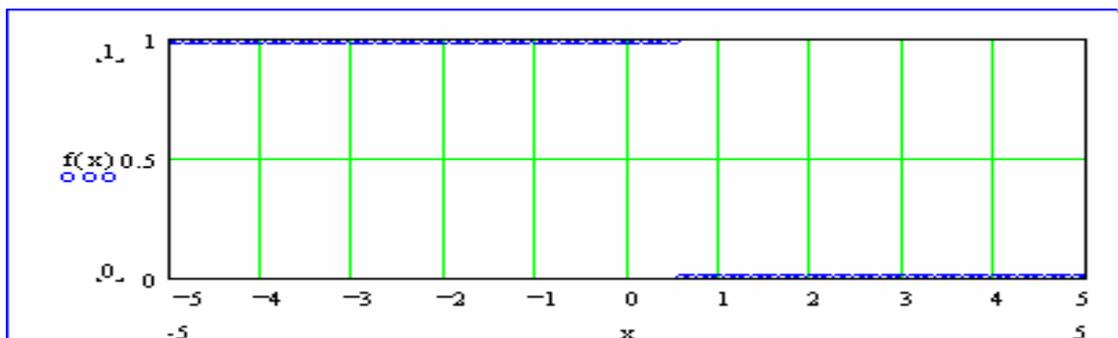
$$f(x) := a \cdot x + b < c \cdot x + d$$

b) To get the graph, Press @ and place $f(x)$ in the middle placeholder of y – axis and x in the middle placeholder of x – axis. Then press F9 or, Enter to see the plot.

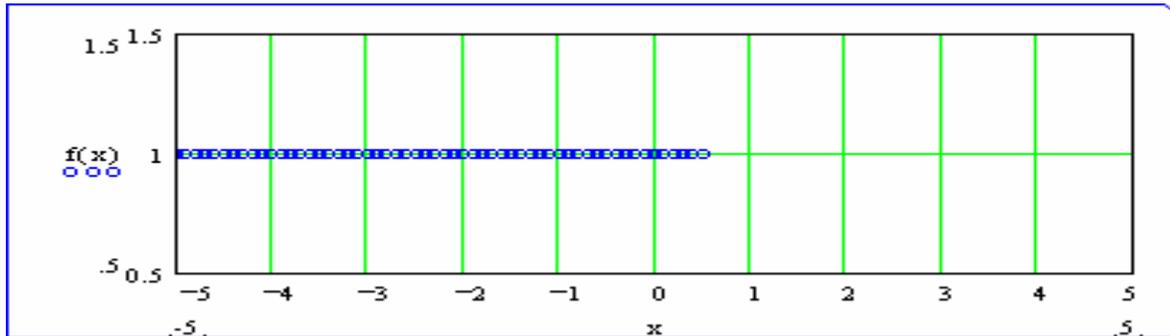
Therefore, the default graph will appear as:



c) To make the graph clearer, **double click** the graph, a dialog box will appear, click on **Traces** → **Type**, and then select **Points** in place of **lines**. Also, to make the line thicker, in the **Symbol**, select **O's** in place of **none**. Then Click **Apply** → **OK**. The new graph will appear as:



- d) Note that here MathCAD considers $f(x)$ as a logical statement that evaluates to 0 if **false** and 1 if **true**. Now change the range instead of 0 - 1 to -.5 - 1.5. This centered the number 1 (i.e., where the inequality is true). Our new graph then is:



Remark 3.1 Now with this setting, the “what if” features of MathCAD can be properly utilized for exploration. Therefore, one can graph and solve any linear inequality by just changing the variables a , b , c and d . By just changing parameters MathCAD automatically gives the new graph of the new equation. Hence, the new solution can be read directly from that.

Remark 3.2 It is note worthy that this graph setting is done once and for all for any class. Consequently, the teacher can teach all class of linear inequality with the above. Also, the student can use this to verify all his homework solutions or practice exercises. However, what will be changing is the function, domain of the graph, and the parameters in the function as we are going to see below.

Remark 3.3 Graph of Compound Inequalities

Some time the linear inequality is in compound form connected with OR and AND (union and intersections, respectively). The Union is indicated by plus in MathCAD,

while the Intersection is logically indicated by multiplication. With this construct for OR and AND, one can sketch all classes of compound in equality. For instance, to sketch the graph of

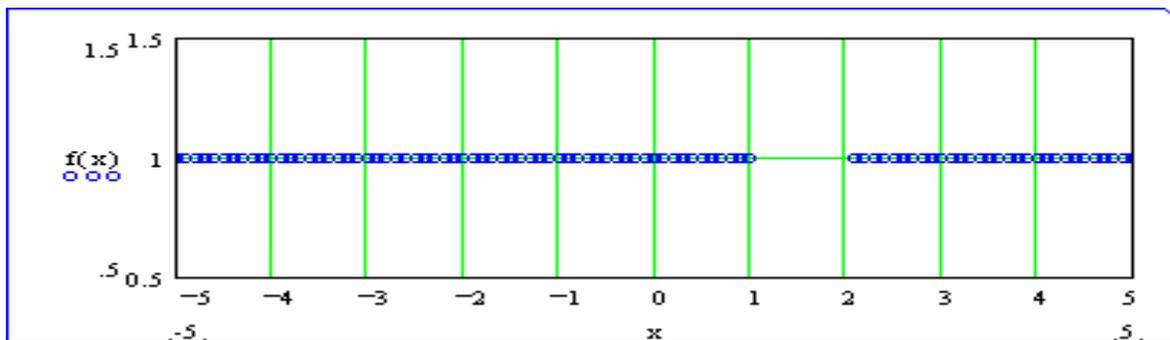
$$a.x+b < c \text{ OR } d.x+e > f, \text{ for any } a, b, c, d, e, \text{ and } f$$

we define these parameters first. For instance:

$$x := -5, -4.9..5 \quad a := 1 \quad b := 2 \quad c := 3 \quad d := 2 \quad e = -1 \quad f = 3$$

$$f(x) := (a.x+b < c) + (d.x+e > f)$$

this gives the graph:



Similarly, the graph of the class

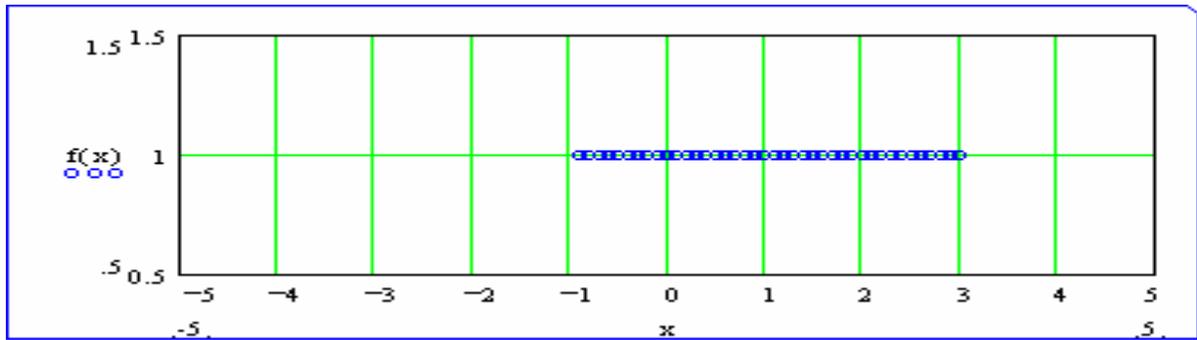
$$a.x+b < c \text{ AND } d.x+e > f, \text{ for any } a, b, c, d, e, \text{ and } f$$

can be obtained by defining the parameters in the same way.

$$x := -5, -4.9..5 \quad a := 1 \quad b := 0 \quad c := 3 \quad d := 1 \quad e = 0 \quad f = -1$$

$$f(x) := (a.x+b < c) . (d.x+e > f)$$

In this the graph is:



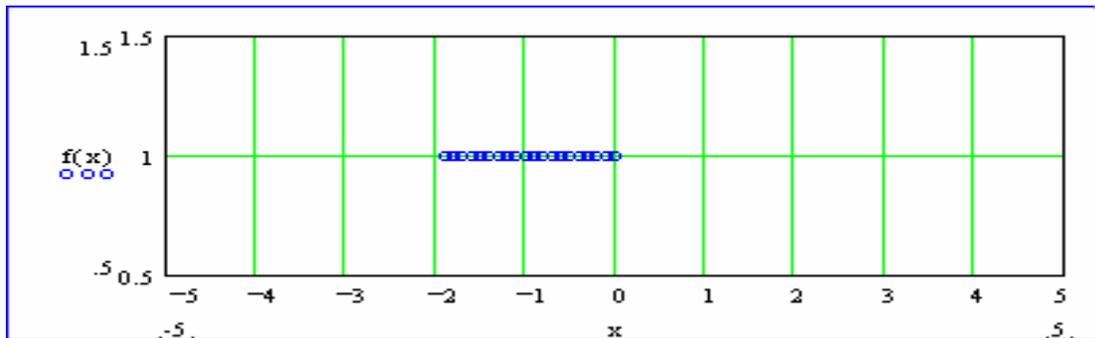
Remark 3.4 Graphs of quadratic inequalities

In a similar way, the graph of all class quadratic inequalities can also be drawn, and their solutions obtained. For this we define

$$x := -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5 \quad a := 1 \quad b := 2 \quad c := 0$$

$$f(x) := a.x^2 + b.x + c < 0$$

we obtain the graph



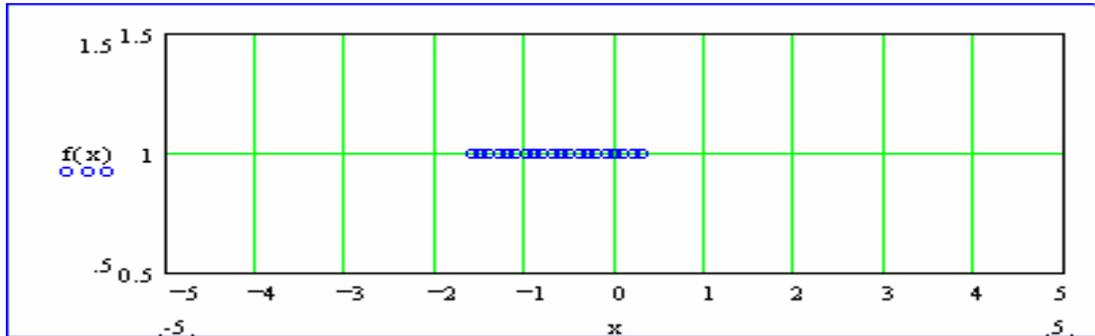
Remark 3.5 Absolute value Linear and Quadratic inequalities:

Absolute value inequalities are also another class of mathematical objects whose graphs are one-dimensional. Now, if we define:

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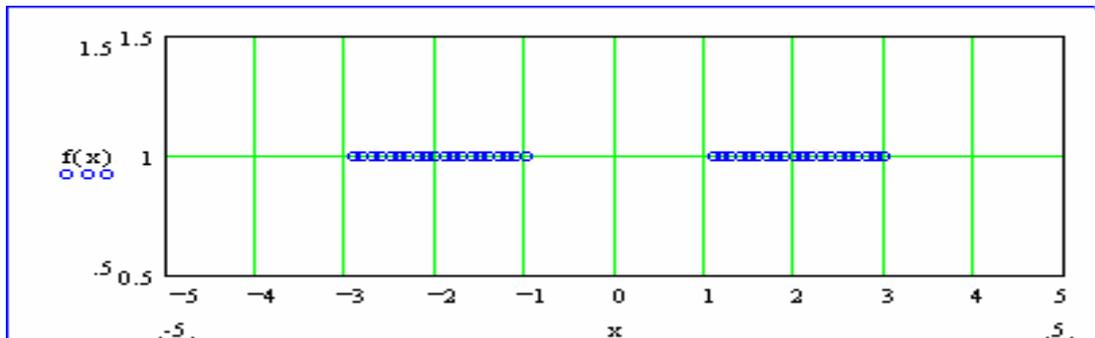
$$x := -5, -4.9..5 \quad a := 3 \quad b := 2 \quad c := 3$$

$$f(x) := |a.x + b| < c$$



The case is not different, if the absolute value is quadratic. For instance, if x is squared in the above example with $a := 1$ $b := -5$ $c := 4$

we obtain the graph:



Remark 3.6 Graphs of Rational inequalities (Linear and quadratic)

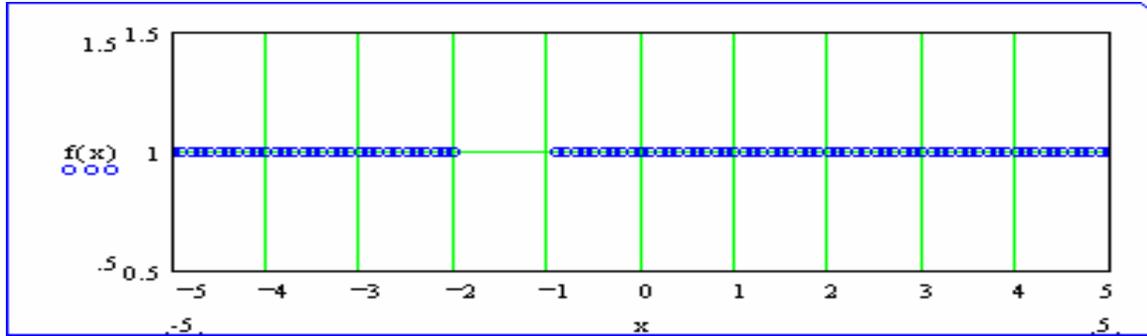
Also the class of rational (both linear and quadratic) inequities can be sketched and their solutions obtained in a similar fashion. For instance, define

$$x := -5, -4.9..5 \quad a := 1 \quad b := -1 \quad c := 1 \quad d := 1 \quad e := 3$$

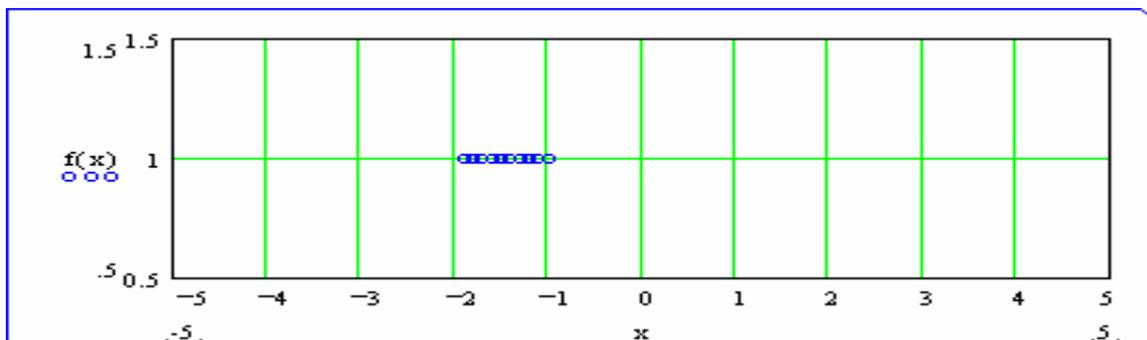
$$f(x) := \frac{a.x + b}{c.x + d} < e$$

MathCAD gives the following

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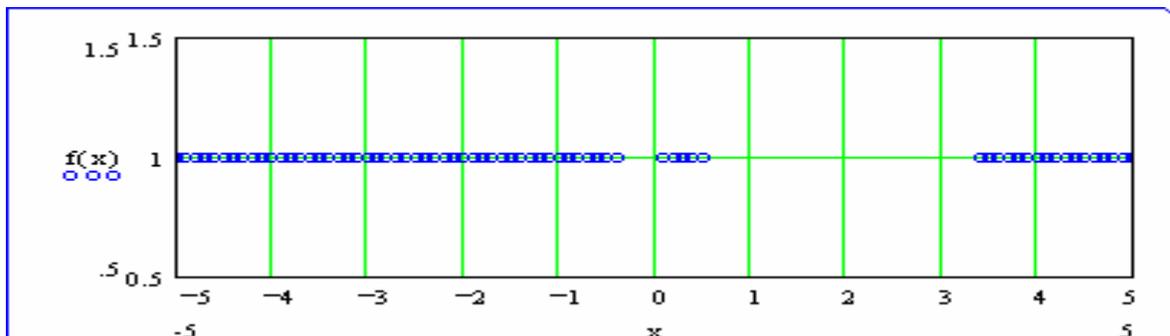


Similarly, if you invert the inequality, you get the following graph



It is interesting to note that in the same way, MathCAD can sketch the graph of quadratic rational inequalities: for instance the graph of $x := -5, -4.9..5$ $a := 1$ $b := 2$ $c := 1$ $d := 2$ $e := -1$ $f := 0$ $g := 1$

$$f(x) := \frac{a.x^2 + b.x + c}{d.x^2 + e.x + f} < g$$



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4. Pedagogical Notes

There are some few points that are worthy of taking note of while using this method.

1. In all our cases we used less than ($<$), but the situation works for all kind of inequalities. However, as we might have noticed MathCAD cannot distinguish between open interval and closed interval. In this case, the teacher in particular has two options to overcome this problem. One, he can make student take note of this from the question. The other option is for the teacher to write a small program that can show open and closed interval. Since MathCAD reads left to right and also allows pages of –screen to the right, the teachers can hide this program in a way that it will not interrupt the lesson or students' attention. The program below works fine for us for graphing different intervals $((-3, 5], [1, 2), [4, -5], (-1, 5), (-\infty, 6], (3, \infty))$. Therefore, can easily be modified for other purposes.

Is the interval open from the left := 1 (1 = Yes, 0 = No)

Is the interval open from the right := 0 (1 = Yes, 0 = No)

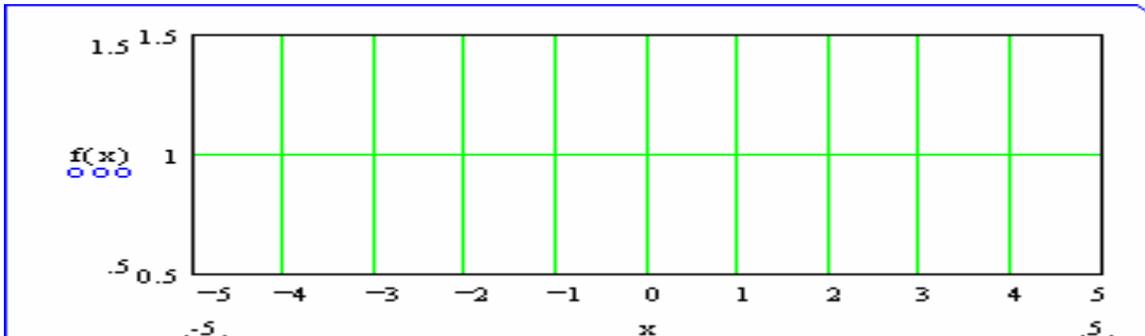
```
start := if(left=1, a + .2, a + .1)
step := start + .1
end := if(right=1, b - .2, b - .1)
x := start, step .. end
lc := if(left=1, a - .05, -20)
rc := if(right=1, b - .05, 20)
```

2. Using the graph one can find the solution set of all these inequalities as well as the solutions of the relevant equations.
 3. The domain of the graph is pre-defined with specifying the values of x . Although MathCAD will give you graph at the interval of your choice, some time the results
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might be misleading or confusing. This is especially if you want to find the solution set of the equation from the graph. For instance, if you want to sketch the graphs of

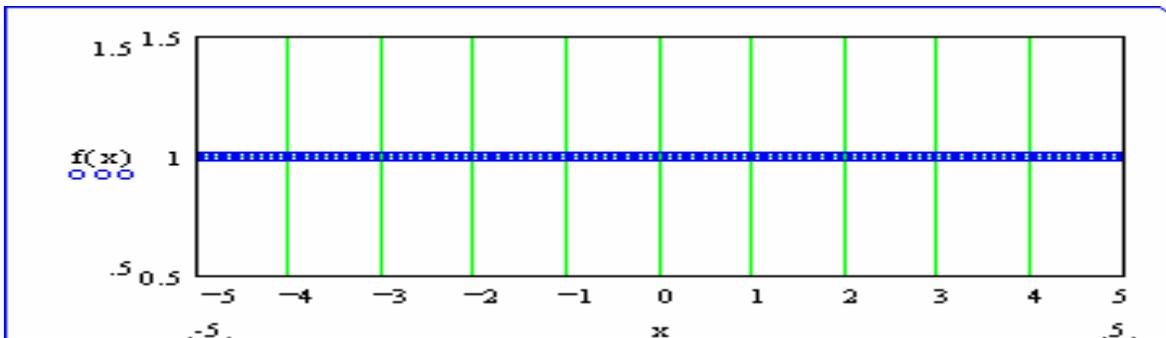
$$(a) 2x - 3 > 12 - x \quad \text{and} \quad (b) |x - 2| < -1$$

at the interval $[-5, 5]$, MathCAD gives the following graph for both (a) and (b)



While in the case of (a) the graph is indicating that the graph is far to the left of the specified domain (x -values). In the case of (b) the graph signifies “contradiction” meaning the graph doesn’t exist.

Similarly, while graphing (a) $2x + 5 > x - 12$ and (b) $|x - 2| > -1$, MathCAD gives the following graph for both expressions.



However, (a) can be interpreted to mean the graph covers the entire specified domain

$[-5, 5]$), while the interpretation of (b) is that the graph covers the set of all real numbers. In other words, the expression is true for any real number.

Therefore, this should be taken care of while using this approach in the classroom.

5. References:

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