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> One of the most fundamental aspects of all cultures is language, and it should be of serious concern that so many mathematics education researchers appear to have paid little more than lip service to the centrality of language factors in all aspects of mathematics teaching and learning (Ellerton & Clarkson, 1996:1017).

Despite the fact that mathematics has been portrayed mostly as a subject of abstract symbols and formalism, the centrality of language in the teaching and learning of mathematics cannot be over emphasized. Right from the outset, the development of mathematics is through language, and language is the means by which mathematics is communicated and learnt both in the formation of concepts and in the development of new forms of thought (Austin & Howson, 1979; Ernest, 1987; Durkin & Shire, 1991). However, it has been observed that the attention given to the crucial role of language in the teaching and learning of mathematics by mathematics educators and researchers in both research and practice domain is “little more than lip service” (Ellerton & Clarkson, 1996:1017). The reason for this, according to Ellerton & Clarkson (1996), is the “predetermined major themes chosen for mathematics education research, and the emphasis placed by major funding bodies on supporting theory-driven research in certain well-defined areas” (p.989). This is by no means saying that the importance of language as a means of developing and communicating mathematics has not been recognized. For instance, among the twelve components considered by the NCSM (National Council of Supervisors of Mathematics, USA) as “essential” is communicating mathematical ideas (Ellerton & Clarkson, 1996). And this position has been endorsed by the National Council of Teachers of Mathematics (NCTM-USA) in the *Principles and Standards for School Mathematics* (2000). There, communication has been emphasized as “an essential part of mathematics and mathematics education (p.60)." Also, this publication has elaborated that all students, and second-language learners in particular, need to have opportunities and be given encouragement and support for speaking, writing, reading and listening in mathematics classes. This will, in particular help second-language learners overcome barriers and will facilitate "communicating to learn mathematics and learning to communicate mathematically (NCTM, p.60)." Similar endorsement and recognition of the importance of language factors in mathematics teaching and learning are to be found in many other curriculum documents, especially in countries that are multicultural, such as USA, New Zealand, Australia and South Africa.

Here, we intend to look into some of the theoretical and empirical studies on the role of language in the teaching and leaning of mathematics. The work is divided into four sections: the first section looks into communication in the mathematics classroom; the second section dwells on the language and the learner. In the third section, the results of some empirical studies on bilingual students and their mathematics achievements will be displayed. In the fourth section, we shall give some instructional strategies and approaches that may assist teachers in alleviating the language constraints of mathematics students, especially bi- and multilingual students who are learning mathematics in their second or third language.

1. Communication in the Mathematics Classroom

> There are dual benefits when mathematics education is rich in communication: students communicate to learn mathematics and learn to communicate mathematically (NCTM, p.60).

The importance of communication and language in the mathematics classroom has long been recognized and emphasized in many curricula. However, the areas that need more effort include those of research and classroom practice. It is not difficult to recognize that most of the problems students face in mathematics are due to a lack of communication: between teachers and students, students and students, and students and recommended mathematics textbooks.

In the last few decades, the mathematics community has witnessed many movements that call for reforms in the mathematics classroom. In essence, the call is to change the way we “communicate” mathematics to our students. Prior to these developments, mathematics language was considered to be a specialist language that could only be understood by a ‘gifted’ few. The bulk of other folks are made to believe that mathematics is a ‘no go’ area for them. The current trend is to change the hitherto dominant belief and make mathematics a subject for all. According to Ellerton & Clarkson (1996), the publication of ‘mathematics and the Public’ which appeared in the first Year book published by NCTM, and ‘Mathematics in Modern life’ (the six Year book), are all as a result of the efforts and awareness to communicate the importance of mathematics in the school curriculum to wider audience, as well as to communicate with the stakeholders related with the teaching and learning of mathematics.

Although communication in the mathematics classroom refers to many different aspects of language, four important aspects can be deciphered from Ellerton & Clarkson (1996): Classroom Discourse, Language and Assessment, Semantics structure, and writing. We shall elaborate below on these topics. However, for more detailed discussion one can refer to book itself.

Classroom Discourse: This can superficially be viewed according to Ellerton & Clarkson (1996) as “merely the language exchanges which take place in the classroom” (p.994). These exchanges can be between teacher and student or students and students; nevertheless language is the means in which it is carried out. Researchers view classroom communication from different perspectives, such as from cognitive, social, and linguistics perspectives.

Language and Assessment: Assessment is the method teachers employ to evaluate how far their students are able to grasp what they were taught. Teachers use different kinds of examinations to evaluate their students’ comprehension. The most commonly used method of assessment throughout the world is the pencil-and-paper short-answer and multiple-choice tests. The dominance of these kinds of assessment and their acceptance in academic circles highlight the need to look at the language structure used in setting these examinations (Ellerton & Clarkson, 1996). As a matter of fact, some members of the mathematics education community have extended this need to a revisitation of the whole process teachers use in assessing their students, with a view to developing more reliable and authentic methods.

Semantic Structure

> No one would call a system a language, if it had no semantic component (Burton, 1988:4).

It has been predicted that semantics as a science of meaning in language, or the study of the relationships between signs and symbols and what they represent is likely to become a major focus in investigations into the language and mathematics interface (Ellerton & Clarkson, 1996). The data in the review done by Ellerton & Clarkson (1996) shows that semantic structure was the main factor contributing to the difficulty of many word problems, and that semantic structure has a much more important influence on learning and quality of participation in classroom discourse than other more obvious language variables such as vocabulary (Ellerton & Clarkson, 1996). For instance, the sentence written as "$I = prt$" is to be understood, within English as "interest is principal times interest-rate times time." Yet the student whose algebraic language is divorced from his or her natural language may well apprehend only the string of five symbols "$I = prt$" (Burton 1988:4). Therefore, with meaning missing in the string, there are many students who could not make sense out of it.

Writing in Mathematics

> The ability to think clearly and the ability to write clearly are inextricably linked (Krantz, 1996:4).

The nature of mathematical textbooks varies greatly from other types of writing in the richness of the content. In a mathematics text, the content is mostly stated in the most precise and economic way possible, with little attempt to convey the flavor of the ideas. This approach has recently been challenged (see Krantz, 1996). In addition, mathematics teachers and educators have being trying to develop avenue for students to express their understanding of mathematics in their own language. In this way, writing as an important aspect of language is being used as a means of learning and teaching of mathematics. It appears that the use of this approach in teaching and learning of mathematics has increased over the last decade. For a synthesis of different approaches that people use in this regards, and forms of 'writing mathematics', one can refer to Ellerton & Clarkson (1996).

2. Language and the Learner

> For the learner, mathematical concepts only have meaning within the linguistic and social context from which they were derived (Ellerton & Clarkson, 1996:994).

A great deal of research has been done and is being done in recent years on the concept of understanding in general, and the understanding of mathematics in particular. Ernest (1987) traced the origin of these works to what he called the "monumental research work of Piaget and his co-workers" (p.10). According to Ernest (1987), Piaget's work is "concerned with the elaboration of a theory of the growth of understanding in children" (p.10). However, some other researchers such as Bruner, Hart, Mellin-Olsen, and Skemp looked at understanding from different points of view (see Ernest, 1987). In all these works, language has been found to be a necessary condition for understanding. For instance, Bruner's fifth characteristic of growth according to Bell (1978) is that:

> teaching and learning are vastly facilitated through the use of language. Not only is language used by teachers to communicate information to students, language is necessary for the complete formulations of most concepts and principles. In mathematics classrooms, one of the primary ways for students to demonstrate knowledge and understanding of mathematical ideas is through the use of language to express their conceptions of the ideas (p:140).

As a result of this inextricable role of language in teaching, learning and understanding, some researchers like Corson (1985) have narrowed the main objective of schooling to just encouraging the complete mastery of the language of the culture, without which, according to him, students are "denied power and influence over their own affairs and an opportunity for success in education" (Ellerton & Clarkson, 1996:991).

In the last two decades, many frameworks have been developed in an attempt to link the various elements of language and mathematics. Among these frameworks is the conceptual framework developed by Gawned (1990). According to Ellerton & Clarkson (1996), Gawned's framework is based on a sociolinguistic premise. The framework acknowledged that the language of the classroom has a particularly important "formative effect on the learners' understanding of mathematics" (p.990), and that for a learner, "mathematical concepts only have meaning within the linguistic and social context from which they were derived" (p.994). Earlier, Austin & Howson (1979) in their pioneering work on language and mathematics set out a framework for discussion. According to them, the questions that will serve as starting points are whether the teacher and the students have (1) the same language (2) same culture (3) the same logic and reasoning system, and (4) whether there is a 'match' between the last three points and, if they are not the same, then (5) what is the 'distance' of the learner's language from the language in which he or she is asked to work mathematically, and from the language that is currently known as the language of mathematics.

The framework has touched on all the variables involved: the teacher, student, mathematics and language. We shall now look at these issues more critically, and see their link to language, and how this link directly or indirectly affects students in learning mathematics.

Language and Students:

Language and thought are two very complex issues, as also is the link between them (Brodie, 1989:42). Researchers have been trying to fine the relationship between language and thought, and the most recent work in this area owes much to Piaget and Vygostky. In particular, Vygostky "saw language occupying and important an integral part of thought" (Austing & Howson, 1979:166).

The question that has preoccupied researchers' minds in this area is been whether thought precedes language, or vice versa. The former would imply that language is "merely a means of expressing our thoughts" while the later would imply that "language determines and is a prerequisite for thoughts" (Brodie, 1989:42).

Einstein, in his response to Hadamard's (1945) informal survey, was quoted as having said that "the word or the language, as they are written or spoken, do not seem to play any role in my mechanism of thought" (Davis & Hersh, 1980:308). On the other hand, the Sapir-Whorf hypothesis states that "the language habits of our community predispose certain choices of interpretation" (Durkin & Shire, 1991:12). A corollary to this hypothesis according to Durkin & Shire (1991), is that "people think and perceive in ways made possible by the vocabulary and phraseology of their language, and concepts not encoded in their language will not be accessible to them, or at least will prove very difficult" (p.12). Although the Sapir-Whorf hypothesis has not been generally accepted especially in mathematics education domain, there is evidence to show that the language we speak has an influence on our thought patterns (Brodie, 1989; Durkin & Shire, 1991). Similarly, though it has been shown that the cognitive style of most mathematicians coincides with Einstein's revelation (see Hadamard, 1945), it is necessary to make a distinction between the process of creating mathematics by professional mathematicians and that of communicating mathematics in the mathematics classroom. While the former is dominated by thought, the later is mediated by language.

It has been observed that, in many classrooms around the world, in both developed and developing countries, many students are now learning mathematics in their second or third language (Austing & Howson, 1979; Ellerton & Clarkson, 1996). Secada (1991) has argued that this phenomenon is becoming the norm rather than the exception in many urban classrooms. Studies on the consequence of this bilingualism and multilingualism on student mathematics learning are inconclusive (Morrison & McIntyre 1972; Austing & Howson, 1979; Davidenko, 2000). According to Cummins's 'threshold hypotheses', for a learner who speaks two languages, the interplay in the learning process between the language codes may either assist or detract from learning. On one hand, if a bilingual student has reached a "threshold" of competence in the two languages, then the learner may have a cognitive advantage. On the other hand, those bilingual students who are not really fluent in either of the two languages tend to experience difficulty in mathematics (Ellerton & Clarkson, 1996).

Cummins's hypothesis is based on the assumption that there is a distinction between what he called basic interpersonal communicative skills (BICS) and cognitive academic language proficiency (CALP). This distinction exerted a significant impact on many educational policies and practices, especially in North America and the United Kingdom (see Cline & Frederickson, 1996). Also, many of the current empirical studies on the implications of bilingualism revolve around this distinction, and all point to the fact that linguistic factors have a serious effect on students' learning of mathematics. However, it has been observed that most of these studies are mostly carried out in developed countries. Therefore, the need for urgent research investigations, particularly in developing countries, to determine the extent and nature of any deprivation of bilinguals has been emphasized (Austing & Howson, 1979; Ellerton & Clarkson, 1996).

Although Cummins distinction has helped in specifying ways in which educators' misunderstanding of the nature of language proficiency contributed to the creation of academic difficulties among bilingual students, "at a theoretical level, the distinction is likely to remain controversial, reflecting the fact that there is no cross-disciplinary consensus regarding the nature of language proficiency and its relationship to academic achievement" (Cummins, in press)

Language and the Teacher

One of the problems of comprehension that a foreign-language learner of mathematics encounters is the difficulty in understanding the conversational language of the teacher and the textbooks. On the other hand, "one of the most difficult things for a monolingual teacher to decide is if errors by a student who is acquiring English reflect a lack of mathematical understanding or some problems with English" (Secada & Cruz, 2000). Unless if the proximity of the 'distance' of the learner's language and that of the teacher is minimal, or the learner has reached a 'threshold' fluency in the language of instruction, there might be a communication gap, the consequence of which will be a lack of understanding.

Language and Mathematics

There is no doubt that one of the major creations of mathematics is symbolism. This symbolism assists in the creation and development of mathematics (Ernest, 1987). However, "symbolism can accordingly cause considerable difficulties to those whose mother language has different structures" (Austing & Howson, 1979:176). As noted earlier, unless a student is able to know what letters in an algebraic expression represent in natural language, there is a tendency for the student to misunderstand the essence of that aspect of mathematics in his real life. Brodie (1989) noted that:

> Effective use of mathematical symbolism requires that the reader, or student, understands the relevant mathematical concept, translates it into a suitable symbolic representation, can manipulate the symbols, and then translate them back into meaningful concepts (Brodie, 1989:49)

It is clear that most of the mathematical activities in our classrooms concentrates on the middle of Brodie's submission, i.e. the symbol manipulation level. Very little effort is made to show where the symbols come from and their interpretation. However, unless the students know where the symbols come from, and are able to make a reasonable interpretation of the manipulated symbols, mathematics will remain inaccessible to a large number of them.

Similarly, one of the principal functions of language is to transmit meaning, and one of the principal problems of language in mathematics is that the meaning to be conveyed is often complex, and the words that teachers and textbooks use to convey them are often endowed with other meanings and, in most cases, the other meaning is completely different from their 'everyday' usage (for instance, see Brodie, 1989; Raborn, 1995; Varughese & Glencross, 1996; Barton & Barton, 2003). For instance the words: root, similar, power, odd, irrational, or, etc are words that have different meaning when used in mathematics.

> It may be difficult, even for students who are not bilingual, to determine which meaning of 'odd' is intended in a problem (odd as in something peculiar or odd as in numbers that are not divisible by two) (Raborn, 1995)

The problem is more severe and critical for the bilingual students who will have to cope with the difficulty in learning to understand the special terminology and syntax of mathematics (see Brodie, 1989; Durkin & Shire, 1991). Consequently, language has a crucial role to play in the understanding of mathematical concepts. And if a student has not reached the 'threshold' of language fluency, he or she might have generated many misconceptions about mathematical concepts.

Language and Culture

The relationship between curriculum, knowledge, culture and language is succinctly captured by Curson (1985) in the following sentence:

> A school curriculum is a selection of knowledge from the culture: all those things in the culture considered worth passing on through schooling. Since all forms of knowledge are 'filtered' through language, the chief item of knowledge in any culture is its *language.* The chief object of the school is to encourage the complete mastery of the language of the culture, since without this mastery children are denied power and influence over their own affairs and an opportunity for success in education (Ellerton & Clarkson, 1996:991).

The role of culture in the teaching and learning of mathematics is currently being given special attention by researchers at both the theoretical and practice level. Also, the long held belief that mathematical language is invariant of people, place and culture is being questioned (Nickson, 1992). Studies have shown that every distinct language has an associated unique counting system. Even when two different counting systems appeared to have the same structure (for instance the same base) there are always subtle differences (Ellerton & Clarkson, 1996:1014). Since counting systems are an integral part of language, and language is inextricably bound to culture, then culture has a big role to play in the extension of this concept. For instance, mathematics terms such as 'zero', 'infinity', 'octagon', 'expansion', 'factorization' etc. are shown to be foreign to some cultures and therefore, are in conflict with their worldviews, and this creates difficulties for students within that culture in both understanding the meaning and visualizing the concept (Brodie, 1989; Ellerton & Clarkson, 1996).

3. Some Empirical Studies on Bilingualism and Mathematics

Many empirical studies have been done and are being done to ascertain the extent, and nature of deprivation of bilingual students in understanding mathematics, and their mathematical achievements. Here, we shall briefly review some of the studies carried out in the last two decades. These studies are selected from many different cultural and educational backgrounds, with a special interest on the ones that deal with student achievement.

Taole (1981) investigated the effect of studying a selected secondary school mathematics topic in the vernacular on student achievement. The study was carried out in Lesotho, which has changed the language of instruction from Sesotho, the vernacular, to English. The subjects of the study were in the fifth year of a seven-year elementary school program. Taole tested the following hypotheses: (1) Pupils studying a selected topic in first year secondary mathematics in Sesotho will perform better than those studying the same topic in English. (2) The level of English proficiency of pupils who perform at a passing level in a selected mathematics topic after receiving instruction in English is higher than that of pupils whose performance is below a passing level. Four hundred and forty-four pupils and ten teachers from six secondary schools participated in the study. Within each school the pupils were divided into three groups. One group was taught in English, using the regular textbook; the second group was taught in Sesotho, using translated materials. The third group, which was taught bilingually in English and Sesotho, had access to both versions of the materials. An achievement test was administered to all pupils. Also, the measures of pupils' English proficiency and their mathematical aptitude were also obtained. The results showed that pupils taught in Sesotho performed slightly better than those taught in English, and pupils taught bilingually performed slightly better than those taught in the vernacular. It was found that among pupils taught in English those who passed the achievement test had a higher level of English proficiency than those who failed. The difference in English proficiency was statistically significant. On the other hand, the difference was not significant among pupils who were taught in Sesotho.

In another study, Chan (1982) investigated the differences in discourse patterns between bilingual and monolingual Mexican-American students when tutoring mathematics to bilingual Mexican-American students and the effects of these differences on achievement. After the pre-test and posttest, different statistical method was used to analyze the data. This included a time series analysis with t-tests and tests of effect sizes. The t-tests revealed no significant differences between the first and second sessions. Tests of significant effect sizes revealed that bilingual tutors used more general explanations and explanations with examples and counter-examples than monolingual tutors. Bilingual tutors also used and received more accepting, agreeing, or acknowledging responses. Monolingual tutors used and received more negating or rejecting responses and responses with questions. In this study, no significant differences were found in mathematics achievement. However, the researcher concluded that the differences in discourse patterns support the conclusion that more communication occurs when a bilingual is taught by another bilingual rather than a monolingual. The conclusion of this researcher is supported by many other researchers such as Setati (2003). In particular, Setati reported two studies, one by Rakgokong (1994), where the researcher argued that using English only as a language of learning and teaching in multilingual primary mathematics classrooms in South Africa, where English is not the main language of the learners, has a negative effect on the learners' 'meaning making' and problem solving ability. In this study, the researcher also observed that in classrooms where English was the only language used for teaching and learning, students were unable to engage in either procedural or conceptual discourse (Setati 2003). Another study, by Varughese and Glencross (1996), found that students at university level had difficulty in understanding mathematical terms such as integer, perimeter, and multiple. The subjects of the study were first-year mathematics students in a South African university who were learning mathematics in English, which was not their main language.

A study conducted by Dawei (1983) investigated the effect of teaching mathematics in English to students that have English as their second language. The subjects of the study were bilingual Punjabi, Mirpuri, Italian, and Jamaican children aged 11-13 growing up in England. The result revealed that first-language competence was an important factor in children's ability to do mathematical reasoning in English as a second language.

Ferro (1983) investigated the influence of language on the mathematical achievement of Capeverdean students. Three basic patterns of instruction were considered: teaching entirely in English, teaching in some mixture of Capeverdean and English, and teaching in some mixture of Portuguese and English. The research questions were: (1) What are the comparative results of the three instructional treatments? (2) Is there a statistically significant difference between male and female achievement in basic mathematics, geometry, and algebra? (3) What relationships, if any, exist between the dependent variables (achievement in basic mathematics, geometry, and algebra) and certain independent variables taken collectively? The subjects of the study were 89 bilingual Capeverdean students enrolled in the first year of a two-year course in basic mathematics. This study tends to support the hypothesis that students who have Capeverdean as a native language and are taught mathematics with the Capeverdean/English treatment will increase their mean achievement scores relatively more than those taught with English/English treatment or Portuguese/English treatment.

In his investigation of the predictive validity of selection measures and related variables used by the University of Petroleum and Minerals (now KFUPM) in Saudi Arabia, Aldoghan (1985) used high school total scores (HSTS), admission test scores (UPMAT), admission English, physics, mathematics, and chemistry subtest scores, and student's age (first-stage variables); and preparatory GPA and preparatory English, mathematics, mechanical engineering, and systems engineering scores (second-stage variables) as independent variables (predictors). The dependent variables (criteria of success) were preparatory GPA, freshman GPA, final GPA, and attrition status. The main validation study included 1,261 student records selected from the files of applicants admitted in 1978/79. The cross-validation samples included 344 student records selected from the files of applicants admitted in 1981. English skill was found in this study as a good predictor only to a certain level, above which differences in English skill had no major influence on success.

Cuervo (1991) studied the effects of mathematics instruction in two languages (English/Spanish) on the performance of bilingual Hispanic college students on the mathematics tests of CLAST competencies and on a mathematics final examination similar to the CLAST mathematics subtest. The research question was: To what extent is the language of instruction related to bilingual Hispanic college students' course achievement in mathematics? The sample consisted of bilingual Hispanic students enrolled in five sections of MGF 1113, at the South Campus of Miami Dade Community College, during the winter term of the 1990-91 academic year. Two bilingual sections, with 32 students, all Hispanic, made up the experimental group. Three regular sections, with 118 students, of which 62 were Hispanic, made up the control group. The experimental group participated in bilingual instruction (English/Spanish) and the control group in traditional instruction (English only). Both groups received the same mathematical instruction, with the same book. The same concepts, skills and algorithms were taught to both groups in each class meeting. The same tests were administered to both groups on the same dates. The only difference was the language of instruction. Students in the bilingual group took the bilingual version of the four partial tests, which had the same questions but written in English and Spanish. The final examination was in English only for both groups. The study found that the bilingual Hispanic college students who participated in bilingual instruction achieved significantly higher scores in the mathematical areas of logic, probability/statistics and geometry, but not in algebra. Scores on a final examination similar to the CLAST mathematics subtest were significantly higher for Hispanic students in the bilingual experimental group. Therefore, the researcher concluded that bilingual instruction (English/Spanish) was more effective than traditional instruction (English only) in promoting overall higher academic achievement for Hispanic students on CLAST mathematics competencies examinations.

This result was supported by two studies conducted by Clarkson (1992) in Papua New Guinea. In the first study, Clarkson (1992) found that the influence of English, the language used in the schooling of 227 sixth graders from Papua New Guinea, as well as the influence of their native language, Pidgin, have a significant impact upon their mathematical performance. In the second study, he found that bilingual students competent in both languages scored significantly higher on two different types of mathematical tests than both low-competent bilingual students and monolingual students.

Bearde (1993) investigated the correlation of oral language proficiency and mathematics achievement for students included in “norming” the Woodcock-Johnson Psycho-educational Battery-Revised (WJ-R) test. Subjects included 1494 students in grades three, five, eight, and eleven. The oral language proficiency cluster, grade placement, and gender were used as predictor variables in multiple regression analyses of mathematics achievement as measured by the basic mathematics skills and the mathematics reasoning clusters. Oral language proficiency was a strong predictor of both mathematical reasoning and basic skills. Since each of the five oral language tests measures a different aspect of language ability, each was used as a predictor variable in multiple regression analyses of mathematics achievement. Tests measuring deeper levels of language (context-reduced, word-meaning and analysis) were stronger predictors than tests measuring verbal attention and memory. The results of this study suggest that a surface level understanding of language is insufficient for mathematics achievement; a deeper level of language, involving an understanding of relationships, is needed.

Maro (1994) investigated the ability of Tanzanian secondary school students to reason in English as a second language. The study also tried to isolate particular variables which best discriminate high from low achievers on the tasks of reasoning and problem solving in mathematics among Tanzanian secondary students, for example, the language spoken at home, encouragement and socio-economic status. English and Kiswahili versions of a mathematics reasoning test were developed and used to test students' reasoning ability in the two languages. A test of logical connectives comprised of mathematical statements was also used to determine the relationship between reasoning in English and familiarity with English logical connectives. The fourth instrument was an adapted non-verbal test of intelligence used to sort students into different mathematical development levels based on Piaget's cognitive development levels. Lastly, students' language background was obtained, for this study, through the use of a questionnaire. The results revealed that the performance of

Tanzanian students on the tests of mathematical reasoning ability varies depending on the language used in the test with performance being better on the Kiswahili version.

Han (1998) investigated the relationship between the clarity of specific mathematical terms and students' mathematics achievement. Student subjects were volunteers from an urban junior high school. The subjects formed three testing populations. One group was constituted of newly immigrated, monolingual Chinese-speaking, ethnic Chinese students. The second group was composed of American-born, monolingual English-speaking, ethnic Chinese students. The third comprised bilingual Chinese/English-speaking students. Statistical analysis showed that Chinese language ability was a strong predictor of students' experimental mathematics scores. The clarity of Chinese mathematical terms did positively relate to achievement in the experimental mathematics test for those students who could read and write Chinese. The overall conclusion that can be drawn from this study is that English and Chinese are inherently different in the ways they express mathematical ideas. A persuasive interpretation is that the relative clarity of mathematical terms in Chinese contributed to the performance of the Chinese-speaking students.

Lim (1998) studied the relationship between language and mathematics among Korean-American students. The research inv estigated the associations between various background factors (such as reading skills, self-reported English proficiency, parents' educational background, Korean language school attendance, gender, and length of residence in the United States) and the students' mathematics achievement. The associations were examined in relation to two separate mathematics sections; problem-solving (written English) and computation (written numbers and operational symbols). Seventy-one Korean American students in seven high schools completed a self-administered background information questionnaire including items on language preference and parents' place of birth. Among the findings of the study were: (1) Language is associated with mathematics achievement, especially in tasks that require substantial amounts of language processing, as in the problem-solving section; (2) Background factors that are directly or indirectly related to language proficiency are also associated with scores on the problem-solving section. Lim concluded that “these findings suggest that bilingual students' success in problem-solving is inextricably interwoven with their level of proficiency in English and factors that relate to English proficiency. Greater exposure to the language of the classroom and the language of mathematics was recommended for limited English proficient students”.

Dakroub (2002) investigated the role of Arabic literacy in the academic achievement of middle school students in English. Arab-American middle school students in a suburban middle school in southeast Michigan were tested to determine their level of literacy in Arabic. A total of 105 students met the requirements to be included in this research study. Raw scores from the Terra Nova standardized achievement test (CTB, McGraw-Hill, 1998) were compared with raw scores from an Arabic literacy test to determine if there was a significant relationship between levels of literacy in Arabic and academic achievement in English reading, language and mathematics. Results from all the analyses confirmed a significant positive relationship between the achievement of Arab-American middle school students in English reading, language and mathematics and their level of literacy in the Arabic language. On measures of academic achievement in English reading, language and mathematics, subjects who were classified as having high levels of literacy in Arabic outscored subjects with low levels of Arabic literacy

In a study, Barton & Neville-Barton (2003) investigated the dynamics of learning mathematics at the university level for students who do not have English as their first language, or rather students having English as an additional language (EAL) (Barton & Neville-Barton, 2003). The subjects of their study were eighty first year undergraduate mathematics students. The researchers reported among their results that in comparison with the students for whom English is their first language, "EAL students experience a 10% disadvantage in overall performance through a lack of understanding mathematical text". In addition to this, the authors also found that "technical mathematical discourse is a more important factor than general English and that EAL students unjustifiably rely on symbolic modes to make up for textual disadvantages". Although the researchers put a caveat on generalizing these results to other domains, and that the second and concluding part of their investigation is in progress, they conclude that "the study provides an initial insight into how language affects mathematical understanding". This study also adds weight to many other studies that indicated that mathematics is not what Reborn (1995) called "language-free". The particular vocabulary, syntax and discourse mathematics presents is a challenge to many students, especially EAP learners. Therefore, the researchers called for universities offering courses to EAL students to pay special attention to this development.

As rightly noted by Begle (1979), the factors that contribute to students' achievement are multidimensional. Hence, it is very difficult to determine accurately the extent to which bilingualism affects student performance. It has been argued that the poor performance of bilingual students cannot be completely attributed to the learners' language proficiencies in isolation from the wider social, cultural, and political factors that infuse schooling (see review in Setati, 2003), however, these results indicate, at minimum, that language factor is one of the many causes of students poor performance in mathematics.

4. Pedagogical Strategies and Approaches That Can Address Language Problems in the Mathematics Classroom

In the previous sections, we have reviewed some empirical studies on the link between language and mathematics achievement. We have seen also that for learners of mathematics, especially in a second language, there are some linguistics constraints that might hinder their mathematical learning and understanding. In this section, we are going to look at some strategies that may help in making mathematics meaningful, especially for bilingual students, in spite of the linguistic constraints.

Appraise Math Abilities (Reborn, 1996**)**

Individual needs and strengths vary between all students, including language-minority students. Bilingual students may have high ability in mathematics and yet not be able to communicate that ability, due to a lack of English proficiency or a lack of communication skills in either language. Therefore, it is critical that mathematical ability be appraised as early as possible, and on the basis of cognitive ability and not assumed on the basis of the student's proficiency in English.

Bilingual Instructional Approach

The level of bilingual language proficiency should be measured to ascertain the level of language proficiency in both languages. If the student is stronger in the native language than, for instance, in English, many researchers recommend that instruction should continue in the native language. For instance, Cuevas and Beech (1983) recommended the use of native language in instruction for teaching mathematical concepts before transition to English. This point has been reiterated by many other independent researchers (see review in Setati, 2003). Even taking that into consideration, Raborn (1995) advised that:

> If it is determined that the native language will be used for instruction, then a teacher who is fluent at the academic level of that language should provide instruction. Just as the language proficiency of bilingual students can range dramatically, so can the language proficiency of bilingual teachers. If the teacher is highly proficient in the academic language of math, then the student will be more likely to learn that higher level of their native language. A teacher who is only fluent in the social aspects of the native language may struggle to communicate precisely with mathematical terms and expressions in that language, and may not be able to take their (sic) native language students to a higher level of learning.

However, if the language of instruction is the student's second language, a translation of key words into the student's first language can be very helpful (see Yushau & Bokhari, in press; Secada & Cruz, 2000; Setati & Adler, 2001). Also, from time to time, the classroom teacher might ask a more strongly bilingual student to translate what has been said from (say) English to Arabic so that the rest of the class can have an opportunity to participate in the conversation (see Secada & Crux, 2000). Also, teachers might have some of their curricular materials translated into the students' native language (say Arabic), or the document be rewritten in simplified language so that they are more accessible to students (Secada & Crux, 2000).

Contextualization of Mathematics

It has been observed that the major problem that students generally face with mathematics has to do with formalism. As rightly noted by Nickson (1992), the approach makes mathematics "on the whole remained inaccessible to teachers and hence to pupils". Similar concerns and possible consequences have been observed by Burton (1988):

> If the young student receives only a quick and abstract initial encounter with variables followed by practice in the essentially syntactic skills of manipulating algebraic expressions, the semantics component of the new language may never be realized (p. 6).

In this case, the student will have a lot of difficulty making sense out of the problem. Burton (1988) noted that in algebra, "students can easily solve linear equation, quadratic equation, and even literal equation with some degree of simplicity. This is because there is algorithm for doing that. The moment an expression is devoid of this procedures, most students are in a fix" (p. 4).

For instance, in the problem:

$$\lim_{x \to 0} \frac{1}{\sqrt{x}}$$

As Burton rightly observed, without an algorithm or procedures that students can follow to arrive at the correct answer, they have to "imagine the expression $\frac{1}{\sqrt{x}}$ as a variable expression, think of x as a positive number close to 0, and picture the reciprocal of its square root" (p.4). So, here the student finds himself in a dual dilemma; he is unable to encode meaning from natural language, where interesting problems arise, into algebraic symbolic language, and seem neither able to recognize meaning in algebraic language simply to carry out formal manipulations on patterns of symbols (Burton, 1988:4). Therefore, teachers should try to use a context or theme with which their students are familiar to serve as an umbrella out of which they generate mathematical activities and problems. In essence, the point we are trying to make is that since "mathematical language, like language in general, develops in context to support communication, the way to increase students' use of precise language is to use contexts that require such precision" (Secada & Cruz, 2000).

Localizing Mathematics

Tikunoff (1985) has found that effective bilingual teachers use their children's home cultures to support classroom management and student learning. Secada & Crux (2000) recommended a similar approach in the mathematics classroom. According to Bishop (1988), all indigenous societies have their own forms of (small m) mathematics, and this mathematics in a sense is different from the internationalized (capital M) Mathematics (Ellerton & Clarkson, 1996). Ellerton & Clarkson (1996) reviewed many research works, the results of which makes one notice the strong connectivity between various aspects of "indigenous mathematics, language and culture" (p.1014). The studies also suggest the importance of localizing educational theories and practices. This will make it possible for the curriculum to be directed to whom is being developed. And such a curriculum can be build upon the natural cognitive modes of the children in question, which are determined by their language (Brodie, 1989:51). Bishop (1992) concludes that:

> The cultural perspective requires us to culturalise the curriculum at each of the levels, and demonstrates that no aspect of mathematics teaching can be culturally neutral (Ellerton & Clarkson, 1996:1017).

Linguistic Approach

> A major component of students' difficulty with algebra is the inability to make sense of the algebraic symbols system as a language, and accordingly that remedies should sought by considering algebra in a linguistic sense (Burton, 1988:2).

Two linguistic strategies were suggested by Brodie (1989): The first strategies should be aim at "improving the linguistics skills and fluency in the foreign language of instruction" (p.49), while at the same time developing the skills and showing support for the mother tongue. The second strategy is to "involve language teachers who would spend a substantial portion of their teaching time concentrating on linguistic skills that are relevant to mathematics" (p.49).

Removing Reading Difficulties

> Whether the text is written in student's first or second language, an important factor is readability (Brodie, 1989:50)

With recent developments trying to make mathematics accessible to a majority of students, the need for developing the reading skill of students, and making mathematics textbooks simple and direct to the point cannot be over emphasized. This calls for a change from the dense and "stylized mathematical writing which tends to concentrate the reader's attention on the correctness of what is written rather than on the richness of meaning" (Austing & Howson, 1979:174).

Language In the Classroom

The classroom conversational language between teachers and students and between students and students has a great impact on students' learning of mathematics. The language of the classroom should be simple and straightforward so that it will not create a communication gap in the class. It has been noted that many students acquiring English receive little encouragement to speak about their ideas, in part due to the belief that they will find it too difficult to express themselves (Secada & Crux, 2000). Therefore, teachers should initiate discussion in the class and should encourage weak language students to participate in classroom discussion. In so doing, "teachers of students acquiring English may find that their students are struggling with new vocabulary in mathematics or that they may speak very little English in general. The teacher should always keep in mind that the real issue is to ensure that students actually understand each other" (Secada & Crux, 2000), and are not left behind in classroom discourse.

> In addition to simplifying oral language, teachers should expand on student responses and build on those when posing their next question. Some students pass through what is known as a silent period when they are just beginning to learn a second language; during this time, they are listening and trying to make sense of the rules for conversation in the classroom as well as in the larger world. Teachers will need to give students time and look for nonverbal cues as to whether they understand the gist of the lesson" (Secada & Crux, 2000).

Analogy and Metaphor

> Other texts, especially literary ones, convey ideas through imagery and metaphor, and often restate them in different ways in order to present a more complete picture (Brodie, 1989:49)

The use of analogy and metaphor is also suggested and shown to be a useful instructional tool in mathematics (Newby & Stepich, 1987; Dickmeyer, 1989; Wessels, 1990). Analogy and Metaphor, if used properly, can address the language constraints of students, and contextualized mathematical concepts

5. Concluding Remark

> While educators may consider math to be universal, there are factors related to language, culture, and cognition that must be considered in math education. With careful assessment, planning, and implementation, students with diverse learning characteristics can be successful in math" (Riborn, 1995).

To achieve this, the following recommendations of the National Council of Teachers of Mathematics (NTCM), for ways to address cases of mathematics for second-language learners are imperative. They are culled from *Principles and Standards for School Mathematics* (2000).

- Schools should provide second-language learners with support in their dominant language and English language while learning mathematics.

- Teachers, counselors, and other professionals who have expertise should carefully assess the language and mathematics proficiencies of each student in order to make curricular decisions and recommendations.

- Mathematics teaching, curriculum, and assessment strategies should be based on best practices and build on the prior knowledge and experiences of students and on their cultural heritage.

- The importance of mathematics and the nature of the mathematics program should be communicated, with appropriate language support, to both students and parents.

- To verify that barriers have been removed, educators should monitor enrollment and achievement data to determine whether second-language learners have gained access to, and are succeeding in, mathematics courses. Reviews should be conducted at school, district, state or provincial, and national levels.

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