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Mathematics: Conceptions, Learning and Teaching

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1. Introduction

Recently the concern of mathematics educators has been shifted to students learning rather than teachers teaching. As a result, students centered approach in teaching mathematics is getting more attention. The implication of this paradigm shift is not just in these changes in instructional practices, but also in the teachers' belief and conceptions of what mathematics is all about. Rene Thom (1972) noted that "All mathematical pedagogy, even if scarcely coherent, rest on a philosophy of mathematics" (p.204). Our aim in this report is to review literature on the nature of mathematics, its conceptions and their implications to instructional practices. The report is divided into five sections. After a brief introduction, the second section looks into the questions: What is mathematics and what is the nature of mathematics? In the third section various conceptions of mathematics are elaborated, and their implications to the teaching and learning of mathematics are outlined. This in turn helps us to form the basis of our discussion on the nature of successful mathematics teaching and learning in section four. The last but not the least section is the concluding remark.

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2.1 What is Mathematics?

There is no consensus on the question of what mathematics is all about. Even within the community of mathematicians, mathematics is defined differently, sometimes as if they are talking of entirely different objects. Kanser and Newman (1949) noted that:

> A large and varied body of thought which has grown up from the earliest times purports to answer this question. But upon examination, the opinions which range from those of Pythagoras to the theories of the most recent schools of mathematical philosophy reveal that it is easier to be clever than clear (Kanser & Newman, 1949 cited in Baron, 1972:21).

The lack of consensus in the definition of mathematics stem from the fact that people who attempt to define mathematics are, consciously or unconsciously, influenced by their background, experience, and area of specialization. For instance, some defined mathematics as a *method* used to discover certain ‘truths’, while others see it as the *truth* to be discovered (Baron, 1972).

> At one end of the spectrum, mathematical knowledge is seen as a body of facts and procedures dealing with quantities, magnitudes, and forms, and the relationships among them. At the other end, it is conceptualized as the “science of patterns,” an (almost) empirical discipline closely akin to science in its emphasis on pattern-seeking as the basis of empirical evidence (Schoenfeld, 1992:334-5).

Some other continuums from which mathematics has been looked upon are *utility* verses *cultural*, *application* verses *esoteric, invention* verses *discovery* etc. In fact, there are some that look at mathematics as a senseless game guided by some rules and regulations, which is being played by mathematicians. A statement attributed to Bertrand Russell (1917) attests to that. He was purported to defined mathematics as the “subject in which we never know what we are talking about, nor whether what we are saying is true” (quoted in the site www.mathsnet.net). In all of the above perspectives each one describes only some aspects of mathematics, and none of them gives full description of the subject. As a matter of fact, a complex field of endeavor like mathematics cannot be defined neatly in a few sentences or paragraphs. However, there is no doubt that the more a definition is inclusive of all perspectives the more it approximates the meaning of the subject (Davis & Hersh, 1980). As a result, one is tempted to look at mathematics as something that incorporates all the above perspectives with its nucleus rooted in what Wittmann (1995) called "specialized mathematics" (p.359). The main body includes, but is not restricted to, mathematics developed and used in sciences, engineering, economics, computer science, statistics, industry, commerce, craft, art, daily life and so forth (Wittmann, 1995). This unifying definition has taking note of the crucial importance of informal and social aspects of mathematical inquiry in the history and philosophy of mathematics (c.f. Ernest, 1991). It also allows the theoretical mathematicians to "do mathematics for mathematics sake", and the applied mathematicians to "use mathematics as a tool" to solve real problems. The definition is also considered as a good reference point for mathematics education (c.f. Wittmann, 1995).

2.2 What is the nature of Mathematics?

It is believed that in any attempt to discuss or define mathematics one has consciously or unconsciously, impose some assumptions on the nature of mathematics (Schwarzenberger, 1982). Similarly, it is unlikely that the controversy of what constitutes good mathematics teaching and learning can be resolved without first addressing the important issues about the nature of mathematics (Thompson, 1992). In view of this, Begle (1979) considers a clear understanding of the nature of mathematics as a prerequisite to any study on learning and teaching of mathematics. Unfortunately, like mathematics, the discussion on the nature of mathematics is challenging and controversial (Vergnaud, 1997). The controversy is perhaps due to the fact that the word mathematics can be used in many distinct and different senses. According to Dossey (1992), this controversy has been present since the time of Plato and Aristotle. Despite the controversy, there are many attempts by scholars to characterize people based on their conception of the nature of mathematics. Thompson (1992) in her analysis of the teacher's conceptions of mathematics reviewed five of these characterizations. The first characterization is by Lerman (1983) who broadly categorized people's conception of the nature of mathematics into two, which he called *absolutist* and *fallibilist.* The second characterization is by Ernest (1988) who distinguished three conceptions of mathematics: the *problem-solving* view, *platonist* view, and *instrumental* view. The third attempt is by Copes (1979) who looked at it from historical point of view. Copes proposed four types of conceptions: *absolutism, multiplism, relativism,* and *dynamism*. The fourth type is rather a scheme of intellectual and ethical development by Perry Jr. (1981) who identified “nine stages or ‘positions’ that describe the intellectual and ethical development of college students from a viewpoint of their conception of knowledge” (Thompson 1992:132). Perry’s scheme according to Thompson has been used by a number of researchers to analyze and characterize people’s conception of mathematics. The fifth categorization is due to Skemp’s (1978) work. Skemp proposed two conceptions of mathematics: *instrumental* and *relational*.

However, for the purpose of this report, we shall look at the nature of mathematics from Lerman (1983) perspectives (Absolutist and Fallibilist) with Ernest (1988; 1991 & 1996) articulations. For more detailed discussion on the other conceptions one can see Thompson (1992).

There are many perspectives in the philosophy of mathematics which can be termed *‘absolutist’* (see Ernest, 1996), but a common denominator among them is their view of mathematics as an “objective, absolute, certain and incorrigible body of knowledge, which rests on the firm foundations of deductive logic” (Ernest, 1996). This view of mathematics is based on the epistemology of logical positivism that are of the belief that the foundations of mathematical knowledge are not in any sense social in origin, but lie outside human action (Nickson, 1992). According to this school of thought, mathematics is ‘abstract’, consists of immutable truths and unquestionable certainty and hence removed from human activity and the context of everyday life (Nickson, 1992). Among the twentieth century perspectives in the philosophy of mathematics that fall into this category are: Logicism, Formalism, and to some extent Intuitionism and Platonism (Ernest, 1991).

However, this view of mathematical knowledge encountered problems at the beginning of the twentieth century when a number of paradoxes and contradictions were “invented” in mathematics, which show that something is wrong in the foundation of mathematics (Ernest, 1991). According to Ernest (1991), the emergence of *Logicism*, *Formalism*, and *Constructivism,* as a school of thought in the philosophy of mathematics is a result of attempts to remedy these problems and maintain the “certainty” of mathematical knowledge. However, despite all efforts, none of these schools of thought is without its ‘mathematical self-contradictions’ (Ernest, 1991). Consequently, the absolutist’s view of mathematics has been seriously criticized, and rethinking of what mathematics is all about was inspired by the seminal work of Lakatos (1976).

The thrust of Lakatos’ view is his opposition to the absolutist view of mathematics as the only fundamentally ‘true’ form of human knowledge. Indeed, he makes clear that both mathematical and logical viewpoints change historically and culturally, and the nature of ‘truth’ is influenced by both factors (Crowe & Zand, 2000).

This more or less recent position in the philosophy of mathematics is now popularly known as *fallibilism*. Fallibilists view mathematics as a human invention rather than a discovery, hence, fallible, corrigible, and eternally open to revision and corrections. In essence, the truth of mathematical knowledge can be challenged, discussed, explored and tested, and possibility of error and inconsistency in mathematics must always remain (Nickson, 1992; Ernest, 1991 & 1996). Wheeler (1970) puts this more bluntly:

> Mathematics is made by men and has all the fallibility and uncertainty that this implies. It does not exist outside the human mind, and it takes its qualities from the mind of men who created it. Because mathematics is made by men and exists only in their minds, it must be made or re-made in the mind of each person who learns it. In this sense mathematics can only be learnt by being created (Wheeler, 1970:2).

In view of this, the proponents of this school of thought consider the searching for a concrete foundation for mathematics in the absolutist approach as a misplaced priority. Rather, searching for the foundation of mathematics should be based on its contemporary practice, keeping in mind that the current practice is inherently fallible (Dossey, 1992). This view, according to Ernest (1996), “embraces as legitimate philosophical concerns of the practices of mathematicians, its history and applications, the place of mathematics in human culture, including issues of values and education”.

This view of mathematics is getting more popular among the community of mathematics educators and some mathematicians. It is considered as well grounded theoretically, as well as suited for school mathematics. As a result, all recent mathematics reforms movements have fallibilists conception of mathematics (see Cockcroft, 1982; NCTM Standard, 2000; Realistic Mathematics, etc).

Whatever might be one's conception of mathematics - Absolutism or Fallibilism, embedded in these conceptions are some ramifications to educational practices.

In the next two sections we intend to discuss the implication of these conceptions to the teaching and learning of mathematics.

3.1 The implication of mathematics conceptions to teaching

There are many models of teaching mathematics. Thompson (1992) reported four that are considered as "dominant and distinctive". Her review was based on the intensive work of Kuhs and Ball (1986). These models are:

1. *Learner-Focused*: mathematics teaching that focuses on the learner's personal construction of mathematical knowledge.
2. *Content-Focused with emphasis on conceptual understanding:* mathematics teaching that is driven by the content itself but emphasizes conceptual understanding.
3. *Content-Focused with emphasis on performance*: mathematics teaching that emphasizes student performance and mastery of mathematical rules and procedures.
4. *Classroom-Focused:* mathematics teaching based on knowledge of effective classroom and about an effective classroom.

All these models, as rightly noted by Thom (1973), are built based on some philosophy of mathematics. The statement also holds the other way round, ie., conceptions and beliefs of mathematics have practical implications for teaching practices (Hersh, 1979; Dossey, 1992; Ernest, 1996).

Study of the teacher's beliefs and conceptions of mathematics along with their implication to teaching practices is a relatively new area. However, a lot of work has been done, and much still remains to be done (Thompson, 1992; Ernest, 1996). For a synthesis of this research one can see Thompson (1992), Ernest (1991, 1994 & 1996), and Dossey (1992) (this review heavily depends on these works). All this research indicates the existence of a strong relationship between teachers' beliefs and conceptions of mathematics and their classroom practices. As might be expected, the relationship is complex and non-deterministic. As a result, it is difficult to connect neatly various conceptions and beliefs to their instructional practices (Ernest, 1991 & 1994). The difficulty arises as a result of reported cases of what Ernest (1994) called "mismatch", whereby the teacher's belief differs from his practice. According to Ernest, two main causes of mismatch are: "powerful influence of the social context" and "the teacher's level of consciousness of his or her own beliefs and practices".

The three key components of a teacher's belief are reported by Ernest (1994). They are: (a) the view or conception of the nature of mathematics, (b) the view of the nature of mathematics teaching, (c) the view of the process of learning mathematics. In the light of this discussion, we shall categorize teachers into two; Absolutists and Fallibilists and then see the implication of these conceptions to the teaching of mathematics.

3.1.1 Absolutist Perspective

Ernest (1994) placed mathematics teachers into three categories: *Instructors*, *Explainers* and *Facilitators*. The *Instructor* considers mathematics as "an accumulation of facts, rules and skills to be used in the pursuance of some external end". In addition, he is of the opinion that the "knowledge of mathematical facts, rules and methods as separate entities". The *Instructor* considers that his role as a teacher is to make students master skills with correct performance. On the other hand, the *Explainer* has a Platonist view of mathematics, whereby mathematics is considered as a "static but unified body of certain knowledge". The role of *Explainer* as a teacher therefore, is to make students conceptually understand and see mathematics as a body of unified "truth".

It is not difficult to see that *Instructor* and *Explainer* both fall in the category of absolutists in their conception of mathematics. These views of mathematics and approaches of teaching mathematics have dominated our classrooms till today in spite of the apparent failure of the approach to make mathematics student-friendly. The emphasis in this traditional approach is on procedures. Little attention is given to helping students develop their conceptual ideas, or even to connect the procedures they are learning with the concepts that show why the ideas work (Nickson, 1992; Hiebert, 1999). That is why “for far too long, far too many students have not connected the mathematics they study in school with the outside world. Their perception is that mathematics doesn’t make sense” (NCTM, making a living). Nunes & Bryant (1997) considered this as universal problem, and put it more concisely in the introduction to their book “Leaning and Teaching Mathematics: International perspective”:

> For us, in particular, learning mathematics was practicing a series of techniques to try to master them and using the same techniques over and over again in a series of problems. Although we grew up in different countries, our mathematics lessons were in a way very much the same. Mathematics was a collection of rules about how to set up numbers, how to expand or simplify equations, how to demonstrate theorems, and our task was to learn how to use these rules to solve the problems we were given. If this sounds boring to you, that is because it was boring (p.xiii).

Odili (1986) in his narration of history of mathematics in Nigeria observed that students found the so-called modern mathematics boring and lacking motivation with no good reason to study it. Students considered it more of memorization than understanding and therefore, made them developed a hatred of anything related to mathematics. This problem is global as rightly observed by Nunes & Bryant (1997). All these observations coupled with lack of success in achieving desirable results are believed to be a result of the type of mathematics taught in the schools and “the adoption of teaching methods reflecting this formalist perspective" (Nickson, 1992). It is not a coincidence that this conception of mathematics is what is being spread widely. Furthermore, it is what is generally accepted by the public, whereby mathematics is seen as “difficult, cold, abstract, theoretical, ultra-rational, but important and largely masculine,…..remote and inaccessible to all but a few super-intelligent beings with ‘mathematical minds’” (Ernest, 1996).

3.1.2 Fallibilist Perspective

The third teacher in Ernest's (1994) model is the *facilitator*. The conception of mathematics of a *facilitator* is that mathematics is "a dynamic, continually expanding field of human creation and invention, a cultural product…a process of enquiry and coming to know, not a finished product, for its results remains open to revision" (Ernest, 1994). Therefore, his role as a teacher is to make students confident problem posers and problem solvers. Clearly, the philosophical stance of *facilitator* is fallibilist. There is no doubt that if one conceives mathematics as dynamic and continually expanding field of human creation and invention, then to put this into practice, it is necessary for him to deviate from the traditional approaches of teaching. Furthermore, with this conception of mathematics the roles of the teacher, the learner and the learning environment must also be necessarily different in order to meet the new challenges. Under this construe, mathematics is now seen as dynamic, "...warm, human, personal, intuitive, active, collaborative, creative, investigational, cultural, historical, living, related to human situations, enjoyable, full of joy, wonder, and beauty" (Ernest, 1996). The teacher's role under fallibilists philosophy is not delivering a "finished product". Rather, the teacher is like a mentor with a carefully skilled plan that will facilitate a genuine mathematical learning process. The role of a student is no more listening but inventing and verifying mathematics at his or her own level.

Next section discusses the implication of mathematics conceptions to the learning of mathematics.

3.2 The implication of mathematics conceptions to learning

> It is difficult to conceive of teaching models without some underlying theory of how students learn mathematics, even if the theory is incomplete and implicit. There seems to be a logical, natural connection between the two (Thompson, 1992:135)

Whilst much progress has been made on the teacher's beliefs and conception of mathematics and their implication on the classroom practices, very few researchers have looked at the students' side (Shoenfeld, 1992). The influence of a mathematics teacher on the students' conceptions of mathematics is tremendous. The messages communicated to students about mathematics and its nature greatly influence students, and also affect the way they grow to view mathematics and its role in their world. Studies have shown that teachers' beliefs about mathematics teaching and learning are formed right during schooling years and are shaped by the teachers' experience as a student of mathematics (Thompson, 1992). This shows how much influential a teacher's view is to his students. Evidences of classroom practices have shown that, in general, teachers have an absolutist and instrumental view of mathematics (Dossey, 1992; Nickson, 1992; Thompson, 1992; Ponte, Matos, Guimaraes, Leal, & Canavaro, 1994). Considering mathematics as a linear subject mainly concerned with mechanistically teaching facts and skills. And the teaching concentrates in given students "mainly unrelated routine mathematical tasks which involve the application learnt procedures, and by stressing that every task has a unique, fixed and objectively right answer, coupled with disapproval and criticism of any failure to achieve this answer" (Ernest, 1996).

Studies have shown that this direct and deductive instructional approach of teaching mathematics causes fear and anxiety in students and consequently, they avoid mathematics (Tobias, 1981). Also, it makes many students develop some wrong beliefs about mathematics, and most of those beliefs have a very strong negative effect on the students' learning of mathematics (see Shoenfeld, 1987). In order to correct this bad image of mathematics and develop the mathematical confidence of as many students as possible, the last three decades have witnessed calls for reforms in the teaching and learning of mathematics. Mostly the reformers are calling for abandoning the traditional approach of teaching and learning mathematics. Alternatively, the reforms approach has shifted the role of a teacher to facilitator and that of a student to an "apprentice mathematician". Hence, students are fully involved and active participants in the invention and discovery of the mathematical object. This hopefully, may change the students' view of mathematics from a "divine" subject to something humanly invented by people like them, and can be reinvented by them.

Recent findings of certain longitudinal studies on the reform have shown some promises. For instance, Prof. Alan Schnoefeld (2002) gave an analysis of some data collected after a decade of implementation of the new approach to mathematics education in United States. Here, we summarize the outcomes in comparison to the traditional approach:

1. On tests of basic skills, no significant difference was found in students' performance between students who learn from traditional or reformed curricula.
2. On tests of conceptual understanding and problem solving, students who learn from reformed curricula consistently outperform by a wide margin students who learn from traditional curricula.
3. There are some encouraging evidences that reform curricula can narrow the performance gap between whites and underrepresented minorities.

Indeed, this is good news for the community of mathematics educators as well as the mathematicians.

In the next two sessions we shall look into the nature of successful mathematics teaching and learning.

4.1 The nature of successful mathematics learning

> "Tell me, and I will forget. Show me, and I may remember. Involve me, and I will understand." (old Chinese proverb)

Learning is a very complicated phenomenon that is largely taken for granted as a natural process. However, the existence of numerous definitions and theories of learning attests to the complexity of this process. Since the primary aim of all mathematics teachers is to make students learn mathematics, the necessity to have a good idea of how students learn mathematics and the nature of the learning process cannot be over emphasized. A brief look at psychology books reveals that "the complexity of understanding how humans learn is reflective of our complexity as biological, social and cognitive animals" (Forrester & Jantzie, 2002). Many theories exist, all focusing on different aspects of our make-up as humans. For instance;

> Sigmund Freud focused on our sub-conscious, Skinner on our observable behavior, cognitive psychologists on our mental processes, humanistic psychology on our social and interpersonal development. While others like Howard Gardner took a more holistic approach in describing our cognitive profiles (Forrester & Jantzie, 2002).

As a result, many learning theories have been developed. However, it has been claimed that the first major contribution to address learning problems scientifically is by researchers known as behavioral psychologists. Behaviorism originated from Pavlov's work, and based on his view about human learning. Latter, the area was developed by Watson, Hull and Thorndike. The behavioral school of thought reached its heyday in B.F. Skinner's work on operant psychology and reinforcement (Kelly, 1997; Ellingtone, 2002). The main thrust in this approach arises from the fact that since we cannot observe what is happening in the human brain, we should limit our measurements and theories to merely what is going in (the stimulus) and what is coming out (the response), hence, treating the human brain as a "black box" (Kelly, 1997). It is worth noting that behaviorists do not deny the existence of mental processes during learning. In fact, they acknowledge their existence as an unobservable indication of learning. However, what they deny is the ability to explain these complex processes. According to Kelly (1997), by mid-century, the stimulus/response S-R view was so powerful and had tremendously influenced educational thinkers, and also dominated many other fields of concern such as linguistics and sociology.

However, the discovery of Jean Piaget which shows that children go through stages of development that have no relation to external stimuli is a heavy blow to behaviorism. Indeed, it is this discovery that leads to the demise of the behavioral school of thought. Piaget's discovery proposed that the brain is not dormant; rather it is actively involved in the learning process. And now, with the advancement of research on how the brain processes information, it gives birth to a new area of study known as cognitive psychology. As a result of Piaget's discovery, we have recently witnessed what many considered as a distinct and major paradigm shift in educational thinking from behavioral to cognitive approach and latterly towards constructivism (Jonassen, Davidson, Collins, Campbell & Haag, 1995). Although constructivism is recognized as a unique learning theory in itself, it has something in common with cognitive psychology. That is, as a theory of learning, both focus on the learner's ability to mentally construct meaning of their own environment and to create their own learning (Kelly, 1997). As a result of this major discovery, it is claimed that Piaget was the first to take children's thinking seriously, which therefore, enabled him to lay a concrete foundation for genuine learning theories (Kelly, 1997).

The influence of Behaviorism learning theory in our educational system is vivid. As a matter of fact, the leaning theory of behaviorism is considered to be the guiding principle of absolutist instructional approaches (see Threlfall, 1996; Forrester & Jantzie, 2002), whereby students are regarded as passive recipients of knowledge (Bell, 1978). Here, the role of a student is to listen attentively and do their homework as ascribed by the teacher who is regarded as a 'knowledge giver'. It has been argued earlier that this approach of teaching mathematics is not very successful for the majority of students who are always left at the receiving end. They are bombarded with several rules and manipulative tricks, which most of the time do not make sense to them and neither do they know their sources.

Now with the advent of cognitive psychology, we have seen that constructivist learning theory has already been established. The findings on these theories are making the cognitive process involved in student learning of mathematics clearer in comparison to the behaviorist's 'stimuli-response' approaches. In addition, the possible cognitive-conflict that might hinder the process of learning mathematics is also becoming clearer (see for instance Tall, 1991). These findings are all pointing towards the fact that a genuine "learning proceeds takes place through *construction*, not *absorption*" (Romberg & Carpenter, 1986:868).

Although most of Piaget's experiments focused on the development of mathematical and logical concepts related to children, many of the theories by now have been extended to "Advanced Mathematical Thinking" (see Tall, 1991). For instance, the Piaget concept of "Reflective Abstraction" has been extended by Dubinsky (1991) to advanced mathematical thinking. In this work, Dubinsky has shown how reflective abstraction can be a powerful tool in the study of advanced mathematical thinking.

Moreover, how the concept can provide a theoretical basis that supports and contributes to our understanding of what this thinking is and how students can be helped to develop the ability of engaging in it (Dubinsky, 1991).

In essence, the constructivist view of mathematics learning gives us a way for analyzing the fact that the "extant mathematics instruction has not been sufficiently successful in promoting students' development of powerful mathematics ideas and useful conceptions of mathematics" (Simon, 1994:77). In fact, what the new approach requires is succinctly articulated by the United States' National Research Council (1989:84):

> Although the language of mathematics is based on rules that must be learned, it is important for motivation that students move beyond rules to be able to express things in the language of mathematics. This transformation suggests changes both in curricular and instructional style. It involves renewed effort to focus on
>
> - Seeking solutions, not just memorizing procedures;
> - Exploring patterns, not just memorizing formulas;
> - Formulating conjectures, not just doing exercises.
>
> As teaching begins to reflect these emphases, students will have opportunities to study mathematics as an exploratory, dynamic, evolving discipline rather than as a rigid, absolute, closed body of laws to be memorized. They will be encouraged to see mathematics as a science, not as a canon, and to recognize that mathematics is really about patterns and not merely about numbers.

4.2 The nature of successful mathematics teaching

> Although a social constructivist view of mathematics learning provides no model for instruction, it provides a foundation on which we can build such a model (Simon, 1994:77)

As noted earlier, behind every teaching model there is a learning theory, even if the theory is incomplete and implicit (Thompson, 1992). History has shown that the traditional pedagogical theory that dominated our classroom is from the artifact of behaviorism. This in a way considers knowledge as separate to the human mind, and so must be transferred to the learner through teacher. Teacher is seen as 'giver of knowledge'. Students are seen as empty vessels needing to be filled up with fact and skills. To learn is simply how much you are able to reproduce.

In this development, the concentration of mathematics educators then was on teacher preparation on how to pass-on this knowledge systematically. The teaching method revolves around knowledge of the subject with very little attention given to *how* students learn the subject.

However, as a result of Piaget discovery of human cognition, many teachers are now beginning to believe that students are not empty vessels to be filled with knowledge but rather active builders of their own knowledge. Therefore, teacher's role is to facilitate students in building their knowledge. In addition, the epistemological question, "What does it mean to know?" is receiving serious attention among mathematics educators. The traditional belief that to 'know' is the accumulation of bits of information (concepts and skills) arranged in some sequential order, is presently being challenged (Romberg, 1992:60).

The new trend views learning as mental process of the mind. The underlying philosophy here is constructivism, which asserts that students learn mathematics by active involvement with mathematical models that allow them to internally construct their own understandings and concepts.

It is worth noting that constructivism is a learning theory not an instructional approach. However, recently many mathematics instructional approaches came up in an attempt to fill-in this instructional gap. The most common name for them is Problem Solving (in every sense of the word). What is common among these new instructional approaches is that they are all student-centered, aimed at actively involving students at different levels in solving mathematics problems, and making mathematical sense out of that.

According to Koehler & Grouws (1992), the constructivist assumption about how students learn, automatically changes the assumption about what teacher actions or behaviors might be desirable. Therefore, "the goal is no longer one of developing pedagogical strategies to help students receive or acquire mathematical knowledge, but rather to structure, monitor, and adjust activities for the students to engage in" (p.119).

Contrary to the traditional approach, the proponents of the new approach conceive "mathematics learning as inherently social as well as a cognitive activity, and an essentially constructive activity instead of an absorptive one" (Schoenfeld, 1992:340). Therefore, a collaborative work is highly encouraged and recommended among students during their process of constructing mathematical meaning.

However, how much mathematics students learn, and how well they learn it, depends to a great extent on the dimensions of the quality or quantity of the mathematics instructional programs they encounter (NCTM 2000:21). Therefore, it is our belief that mathematics teaching can only be successful if it takes into cognizant of this recent development on how students learn mathematics. Furthermore, it must give students ample opportunities to learn mathematics, and make them have a good conception and appreciation of mathematics. The approach is a one that meaningfully "engages students in active exploration of mathematical situations….as mathematicians, creating mathematics, evaluating mathematics that has been created by members of mathematical community" (Simon, 1994:72). In this sense, learning is considered as a "peripheral participation" (Lave & Wenger, 1991) in mathematics activities.

Conclusion

In this review we have looked into the meaning and the nature of mathematics. The duo is difficult to define because mathematics as a subject is endowed with a "personality" that can be looked from different angles. Consequently, discussion on the issue remained inconclusive.

However, it has been argued that from whatever angle teachers' look at mathematics, their conceptions and belief of the subject determine in a way how they teach it.

Although studies have shown that students learn by construction rather than absorption, many students and teachers are yet to come to term with this reality. In one hand, many teachers are still seeing themselves, consciously or unconsciously, as the custodians of knowledge. On the other hand, many students are more comfortable in absorbing knowledge from the sources – teacher rather than "constructing it".

It is our belief that unless teachers and students are ready to face the challenges in the recent development in learning theories, in particular, students are psychologically ready to engage in mathematical activities and teacher willingly relinquish their authoritarian chair to a back sit, successful teaching and learning will remain a mirage.

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