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Abstract

The notion of semi-convergence of filters was introduced by Latif (1999) who investigated some characterizations related to semi-open continuous functions. In the spirit of Latif (1999), Min (2002) used the idea of semi-convergence of filters to introduce a new class of sets, called $\gamma - open$ sets, and the notions of $\gamma{-}closure$, $\gamma{-}interior$ and $\gamma{-}continuity$ and investigated some properties. In this paper, we apply the notion of $\gamma - open$ sets in topological spaces to present and study certain properties and characterizations of $contra - \gamma - continuity$ as a new generalization of $contra - continuity$ $[Dontchev,\ 1996]$.

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1. Introduction

The notion of $\gamma - open$ set (originally called $\gamma -$ $sets$) in topological spaces was introduced by Min $[Min, 2002]$. For these sets, in $[Latif, (TR\#331)\, 2005]$ we introduced the notions of $\gamma - derived$, $\gamma - border$, $\gamma -$ $frontier$, and $\gamma -$ $exterior$ of a set and showed that some of their properties are analogous to those for open sets. Also, we gave some additional properties of $\gamma - closure$ and $\gamma - interior$ of a set. We also continued to explore further properties and characterizations of $\gamma - continuous$, $\gamma - irresolute$ and $\gamma - open$ mappings. In $[Latif, (TR\#332)\, 2005]$ we also introduced and studied properties and characterizations of $\gamma - closed$, $pre - \gamma - open$ and $pre - \gamma - closed$ mappings.

In 1996, Dontchev [*Dontchev*, 1996] introduced a new class of functions called contra-continuous functions. Recently, Dontchev and Noiri [1999] introduced and studied, among others, a new weaker form of this class of functions called contra-semicontinuous functions. They also introduced the notion of RC-continuity [*Dontchev and Noiri*, 1999] which is weaker than contra-continuity and stronger than $\beta -$ *continuity* [*Tong*, 1998]. Jafari and Noiri [*Jafari and Noiri*, 1999] introduced and studied a new class of functions called contra-supper-continuous functions which lies between classes of RC-continuous functions and contra-continuous functions.

This paper is devoted to introduce and investigate a new class of functions called contra-γ-continuous functions which is weaker than contra-continuous functions.

2. Preliminaries

Throughout this paper, (X, τ) (simply X) always mean topological space on which no separation axioms are assumed unless explicitly stated. Let S be a subset of X. The closure (resp., *interior*) of S will be denoted by $Cl\,(S)$ (resp., $Int\,(S)$). A subset S of X is called a semi-open set $[Levine, 1963]$ (resp., $\alpha - open\ set$ [*Njastad*, 1965]) if $S \subseteq Cl\,[Int\,(S)]$ (*resp.*, $S \subseteq Int\,[Cl\,(Int\,(S))]$). The complement of a semi-open set (resp., $\alpha - open\ set$) is called semi-closed set (resp., $\alpha - closed\ set$).

The family of all semi-open sets (resp., $\alpha - open\ sets$) in a topological space (X, τ) will be denoted by $SO(X)$ (resp., τ^{α}). The complement of a semi-opem set S is called a semi-closed set. The intersection of all semi-closed sets containing A is called the semi-closure of A, denoted by $Cl_s(A)$. A subset $M(x)$ of a space X is called a semi-neighbourhood of a point $x \in X$ if there exists a semi-open set S such that $x \in S \subseteq M(x)$. In $[Latif,\ 1999]$ Latif introduced the notion of semi-convergence of filters and investigated some characterizations related to semi-open continuous function. Now, we recall the concept of semi-convergence of filters. Let $S(x) = \{A \in SO(X) : x \in A\}$ and let $S_x = \{A \subseteq X : there\ exists\ \mu \subseteq S(x)\ such\ that\ \mu\ is\ finite\ and \cap\mu \subseteq A\}$. Then, S_x is called the semi-neighbourhood filter at x. For any filter Γ on X, we say that Γ semi-converges to x if and only if Γ is finer than the semi-neighbourhood filter at x.

A subset U of X is called a $\gamma - open$ set if whenever a filter Γ semi-converges to x and $x \in U$, $U \in \Gamma$. The complement of a $\gamma-$ $open$ set is called a $\gamma-$ $closed$ set. The intersection of all $\gamma-$ $closed$ sets containing A is called the $\gamma-$ $closure$ of A, denoted by $Cl_{\gamma}(A)$. A subset A is also $\gamma-$ $closed$ if and only if $A = Cl_{\gamma}(A)$. We denote the family of all $\gamma-$ $open$ sets of (X, τ) by τ^{γ}. It is shown in $[Min, 2002]$ that τ^{γ} is a topology on X. In a topological space $(X,\ \tau)$, it is always true that $\tau \subseteq \tau^{\alpha} \subseteq SO(X) \subseteq \tau^{\gamma}$.

3. Contra -Gamma-Continuous Mappings

The purpose of this section is to explore properties and characterizations of contra$-\gamma-$ *continuous* mappings.

Definition 3.1. A function $f : (X, \tau) \longrightarrow (Y, \sigma)$ is called $contra-\ \gamma - continuous$ if $f^{-1}(V)$ is $\gamma - closed$ in X for each open set V of Y.

Definition 3.2. $[Dontchev,\ 1996]$. A function $f : (X, \tau) \longrightarrow (Y, \sigma)$ is called *contra*$-$ *continuous* if $f^{-1}(V)$ is closed in X for each open set V of Y.

Definition 3.3. $[Dontchev \,\&\, Noiri,\ 1999]$. A function $f : (X, \tau) \longrightarrow (Y, \sigma)$ is called $contra - semicontinuous$ if $f^{-1}(V)$ is semi-closed in X for each open set V of Y.

Remark 3.4. Every $contra - continuous$ function is $contra - \gamma - continuous$ but not conversely as the following example shows.

Example 3.5. Let $X = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}\}$ and $\sigma = \{X, \phi, \{b\}, \{c\}, \{b, c\}\}$. Then the identity function $f : (X, \tau) \longrightarrow (X, \sigma)$ is $contra-\ \gamma - continuous$ but not $contra - continuous$.

Definition 3.6. $[Mrsevic, 1986]$. Let A be a subset of a space (X, τ). The set $\cap \{U \in \tau : A \subseteq U\}$ is called the kernel of A and is denoted by $Ker(A)$.

Lemma 3.7. The following properties hold for subsets A, B of a space X :

(1) $x \in Ker(A)$ if and only if $A \cap F \neq \phi$ for any closed subset F of X such that $x \in F$.

(2) $A \subseteq Ker(A)$ and $A = Ker(A)$ if A is open in X.

(3) $A \subseteq B$, then $Ker(A) \subseteq Ker(B)$.

Theorem 3.8. The following are equivalent for a function $f : (X, \tau) \longrightarrow (Y, \sigma)$:

(1) f is $contra - \gamma - continuous$;

(2) for every closed subset F of Y, $f^{-1}(F) \in \tau^{\gamma}$;

(3) for each $x \in X$ and each closed subset F of Y containing $f(x)$, there exists $U \in \tau^{\gamma}$ such that $x \in U$ and $f(U) \subseteq F$;

(4) $f[Cl_{\gamma}(A)] \subseteq Ker[f(A)]$ for every subset A of X;

(5) $Cl_{\gamma}[f^{-1}(B)] \subseteq f^{-1}[Ker(B)]$ for every subset B of Y.

Proof The implications $(1) \Longleftrightarrow (2)$ and $(2) \Longrightarrow (3)$ are obvious.

$(3) \Rightarrow (2)$: Let F be any closed subset of Y and $x \in f^{-1}(F)$. Then $f(x) \in F$ and there exists $U_x \in \tau^\gamma$ such that $x \in U_x$ and $f(U_x) \subseteq F$. Therefore, we obtain $f^{-1}(F) = \cup \{U_x | x \in f^{-1}(F)\} \in \tau^\gamma$.

$(2) \Longrightarrow (4)$: Let A be any subset of X. Suppose that $y \notin Ker[f(A)]$. Then by Lemma 3.7 there exists F a closed subset of Y such that $y \in F$ and $f(A) \cap F = \phi$. Thus, we have $A \cap f^{-1}(F) = \phi$ and $Cl_\gamma(A) \cap f^{-1}(F) = \phi$. Therefore, we obtain $f[Cl_\gamma(A)] \cap F = \phi$ and $y \notin f[Cl_\gamma(f(A))]$. This implies that $f[Cl_\gamma(A)] \subseteq Ker[f(A)]$.

$(4) \Longrightarrow (5)$: Let B be any subset of Y. By (4) and Lemma 3.7 we have $f[Cl_\gamma(f^{-1}(B))] \subseteq Ker(B)$ and $Cl_\gamma[f^{-1}(B)] \subseteq f^{-1}[Ker(B)]$. $(5) \Longrightarrow (1)$: Let V be any open set of Y. Then, by *Lemma* 3.7 we have $Cl_\gamma[f^{-1}(V)] \subseteq f^{-1}[Ker(V)] = f^{-1}(V)$. This shows that $f^{-1}(V)$ is $\gamma - closed$ in X.

Theorem 3.9. A function $f : (X, \tau) \longrightarrow (Y, \sigma)$ is *contra* $-\ \gamma -$ *continuous* if and only if $f : (X, \tau^\gamma) \longrightarrow (Y, \sigma)$ is *contra* $-$ *continuous*.

Definition 3.10. A subset A of a topological space is called a $\Lambda - set$ if it is the intersection of open sets.

Theorem 3.11. A function $f : (X, \tau) \longrightarrow (Y, \sigma)$ is *contra* $-\ \gamma -$ *continuous* if and only if the inverse images of $\Lambda - sets$ are closed.

Definition 3.12 A function $f : (X, \tau) \longrightarrow (Y, \sigma)$ is said to be $(\gamma, s) - open$ if $f(U) \in SO(Y)$ for every $U \in \tau^\gamma$.

Theorem 3.13. Prove that a mapping $f : (X, \tau) \longrightarrow (Y, \sigma)$ is $(\gamma, s) - open$ if and only if for each $x \in X$, and $U \in \tau^\gamma$ such that $x \in U$, there exists a *semi* $-$ *open* set $V \subseteq Y$ containing $f(x)$ such that $V \subseteq f(U)$.

Proof. Follows directly from Definition 3.12.

Theorem 3.14. Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ be $(\gamma, s) - open$. If $W \subseteq Y$ and $F \subseteq X$ is a $\gamma - closed$ set containing $f^{-1}(W)$, there exists a $semi - closed$ set $H \subseteq Y$ containing W such that $f^{-1}(H) \subseteq F$.

Proof. Let $H = Y - f(X - F)$. Since $f^{-1}(W) \subseteq F$, we have $f(X - F) \subseteq (Y - W)$. Since f is $(\gamma, s) - open$, then H is $semi - closed$ and $f^{-1}(H) = X - f^{-1}[f(X - F)] \subseteq X - (X - F) = F$.

Corollary 3.15. Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ be $(\gamma, s) - open$ and let $B \subseteq Y$. Then $f^{-1}[Cl_s(Int_s(Cl_s(B)))] \subseteq Cl_\gamma[f^{-1}(B)]$.

Proof. $Cl_\gamma[f^{-1}(B)]$ is $\gamma - closed$ in X containing $f^{-1}(B)$. By Theorem 3.14, there exists $semi - closed$ $B \subseteq H \subseteq Y$ such that $f^{-1}(H) \subseteq Cl_\gamma[f^{-1}(B)]$. Thus, $f^{-1}[Cl_s(Int_s(Cl_s(B)))] \subseteq f^{-1}[Cl_s(Int_s(Cl_s(H)))] \subseteq f^{-1}(H) \subseteq Cl_\gamma[f^{-1}(B)]$.

Theorem 3.16. Prove that a function $f : (X, \tau) \longrightarrow (Y, \sigma)$ is $(\gamma, s) - open$ if and only if $f[Int_\gamma(A)] \subseteq Int_s[f(A)]$, for all $A \subseteq X$.

Proof. Necessity. Let $A \subseteq X$. Let $x \in Int_\gamma(A)$. Then there exists $U_x \in \tau^\gamma$ such that $x \in U_x \subseteq A$. So $f(x) \in f(U_x) \subseteq f(A)$ and by hypothesis, $f(U_x) \in SO(Y)$. Hence $f(x) \in Int_s[f(A)]$. Thus $f[Int_\gamma(A)] \subseteq Int_s[f(A)]$.

Sufficiency. Let $U \in \tau^\gamma$. Then by hypothesis, $f[Int_\gamma(U)] \subseteq Int_s[f(U)]$. Since $Int_\gamma(U) = U$ as U is $\gamma - open$. Also $Int_s[f(U)] \subseteq f(U)$. Hence $f(U) = Int_s[f(U)]$. Thus $f(U)$ is $semi - open$ in Y. So f is $(\gamma, s) - open$.

Theorem 3.17. Prove that a function $f : (X, \tau) \longrightarrow (Y, \sigma)$ is $(\gamma, s) - open$ if and only if $Int_\gamma[f^{-1}(B)] \subseteq f^{-1}[Int_s(B)]$, for all $B \subseteq Y$.

Proof. Necessity. Let $B \subseteq Y$. Since $Int_\gamma[f^{-1}(B)]$ is $\gamma - open$ in X and f is $(\gamma, s) - open$, $f[Int_\gamma(f^{-1}(B))]$ is $semi - open$ in Y. Also we have $f[Int_\gamma(f^{-1}(B))] \subseteq$

$f\left[f^{-1}(B)\right] \subseteq B$. Hence, $f\left[Int_{\gamma}\left(f^{-1}(B)\right)\right] \subseteq Int_{s}(B)$. Therefore $Int_{\gamma}\left[f^{-1}(B)\right] \subseteq f^{-1}\left[Int_{s}(B)\right]$.

Sufficiency.Let $A \subseteq X$. Then $f(A) \subseteq Y$. Hence by hypothesis, we obtain $Int_{\gamma}(A) \subseteq Int_{\gamma}\left[f^{-1}(f(A))\right] \subseteq f^{-1}\left[Int_{s}(f(A))\right]$. Thus $f\left[Int_{\gamma}(A)\right] \subseteq Int_{s}\left[f(A)\right]$, for all $A \subseteq X$. Hence, by Theorem 3.16, f is $(\gamma, s)-open$.

We remark that the equality does not hold in the preceding two theorems as the following example shows.

Example 3.18. Let $X = Y = \{1, 2\}$. Suppose τ be the antidiscrete topology on X and σ be the discrete topology on Y. Then $\tau^{\gamma} = \tau$ and $SO(Y) = \sigma$. Let $f = Id.$, $A = \{1\}$. Then $\phi = f\left[Int_{\gamma}(A)\right] \neq Int_{s}\left[f(A)\right] = \{1\}$. Let $B = \{1\} \subseteq Y$. Then $\phi = Int_{\gamma}\left[f^{-1}(B)\right] \neq f^{-1}\left[Int_{s}(B)\right] = \{1\}$.

Theorem 3.19. Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ be a mapping. Then a necessary and sufficient condition for f to be $(\gamma, s)-open$ is that $f^{-1}\left[Cl_{s}(B)\right] \subseteq Cl_{\gamma}\left[f^{-1}(B)\right]$ for every subset B of Y.

Proof. Necessity. Assume f is $(\gamma, s)-open$. Let $B \subseteq Y$. Let $x \in f^{-1}\left[Cl_{s}(B)\right]$. Then $f(x) \in Cl_{s}(B)$. Let $U \in \tau^{\gamma}$ such that $x \in U$. Since f is $(\gamma, s)-open$, then $f(U)$ is a $semi-open$ set in Y. Therefore, $B \cap f(U) \neq \phi$. Then $U \cap f^{-1}(B) \neq \phi$. Hence $x \in Cl_{\gamma}\left[f^{-1}(B)\right]$. We conclude that $f^{-1}\left[Cl_{s}(B)\right] \subseteq Cl_{\gamma}\left[f^{-1}(B)\right]$.

Sufficiency. Let $B \subseteq Y$. Then $(Y - B) \subseteq Y$. By hypothesis, $f^{-1}\left[Cl_{s}(Y - B)\right] \subseteq Cl_{\gamma}\left[f^{-1}(Y - B)\right]$. This implies $X - Cl_{\gamma}\left[f^{-1}(Y - B)\right] \subseteq X - f^{-1}\left[Cl_{s}(Y - B)\right]$. Hence $X - Cl_{\gamma}\left[X - f^{-1}(B)\right] \subseteq f^{-1}\left[Y - Cl_{s}(Y - B)\right]$. By applying Theorem 10 $[Latif, 1993]$, $Int_{\gamma}\left[f^{-1}(B)\right] \subseteq f^{-1}\left[Int_{s}(B)\right]$. Now from Theorem 3.17, it follows that f is $(\gamma, s)-open$.

Definition 3.20. A filter base Λ is said to be $\gamma-convergent$ $(resp.\ c-convergent)$

to a point x in X if for any $U \in \tau^{\gamma}$ such that $x \in U$ (*resp.* closed set $U \subseteq X$ *such* that $x \in U$), there exists $B \in \Lambda$ such that $B \subseteq U$.

Theorem 3.21. A function $f : (X, \tau) \longrightarrow (Y, \sigma)$ is $contra - \gamma - continuous$ if and only if for each point $x \in X$ and each filter base Λ in X $\gamma - converging$ to x, the filter base $f(\Lambda)$ is $c - convergent$ to $f(x)$.

Proof. Necessity. Let $x \in X$ and Λ be any filter base in X such that Λ $\gamma - converges$ to x. Since f is $contra - \gamma - continuous$, then for any closed subset V of Y such that $f(x) \in V$, there exists $U \in \tau^{\gamma}$ such that $x \in U$ and $f(U) \subseteq V$. Since Λ is $\gamma - converging$ to x, there exists a $B \in \Lambda$ such that $B \subseteq U$. This means that $f(B) \subseteq V$ and therefore the filter base $f(\Lambda)$ is $c - convergent$ to $f(x)$.

Sufficiency. Let $x \in X$ and V be a closed subset of Y such that $f(x) \in V$. Let $\Lambda = \{U \subseteq X : U \in \tau^{\gamma} \text{ and } x \in U\}$. Then Λ is a filter base which $\gamma - converges$ to x. Thus, there exists $U \in \Lambda$ such that $f(U) \subseteq V$.

4. Contra-Gamma-Closed Graph

In this section we define $contra - \gamma - closed$ graph and study some of its characteristics.

Definition 4.1. The graph $G(f) = \{(x, f(x)) \,|x \in X\}$ of a function $f : (X, \tau) \longrightarrow (Y, \sigma)$ is said to be $contra - \gamma - closed$ if for each $(x, y) \in (X \times Y) - G(f)$, there exists $U \in \tau^{\gamma}$ such that $x \in U$ and V a closed subset of Y such that $y \in V$ and we have $(U \times V) \cap G(f) = \phi$.

Lemma 4.2. The graph $G(f)$ of a function $f : (X, \tau) \longrightarrow (Y, \sigma)$ is $contra - \gamma - closed$ in $X \times Y$ if and only if for each $(x, y) \in (X \times Y) - G(f)$, there exists $U \in \tau^{\gamma}$ such that $x \in U$ and V a closed subset of Y such that $y \in V$ and we have $f(U) \cap V = \phi$.

Theorem 4.3. If $f:(X,\tau)\longrightarrow(Y,\sigma)$ is $contra-\gamma-continuous$ and Y is Urysohn, then $G(f)$ is $contra-\gamma-closed$ in $X\times Y$.

Proof. Let $(x,y)\in(X\times Y)-G(f)$. Then $y\neq f(x)$ and there exist open sets V , W such that $f(x)\in V$, $y\in W$ and $Cl(V)\cap Cl(W)=\phi$. Since f is $contra-\gamma-continuous$, there exists $U\in\tau^{\gamma}$ such that $x\in U$ and $f(U)$ $\subseteq Cl(V)$. Therefore, we obtain $f(U)\cap Cl(W)=\phi$. This shows that $G(f)$ is $contra-\gamma-closed$.

Definition 4.4. $[Min,\ 2002]$. A function $f:(X,\tau)\longrightarrow(Y,\sigma)$ is said to be $\gamma-continuous$ if $f^{-1}(V)\in\tau^{\gamma}$ for every $V\in\sigma$.

Theorem 4.5. If $f:(X,\tau)\longrightarrow(Y,\sigma)$ is $\gamma-$ $continuous$ and Y is T_1, then $G(f)$ is $contra-\gamma-closed$ in $X\times Y$.

Proof. Let $(x,y)\in(X\times Y)-G(f)$. Then $f(x)\neq y$ and there exists an open set V of Y such that $f(x)\in V$ and $y\notin V$. Since f is $\gamma-continuous$, there exists a $\gamma-open$ neighbourhood of x such that $f(U)\subseteq V$. Therefore, we obtain $f(U)\cap(Y-V)=\phi$ and $(Y-V)$ is a closed subset of Y such that $y\in(Y-V)$. This shows that $G(f)$ is $contra-\gamma-closed$ in $X\times Y$.

Definition 4.6. $[Dontchev,\ 1996]$. A space X is said to be strongly $S-closed$ if every closed cover of X has a finite subcover. A subset A of a space X is said to be strongly $S-closed$ if the subspace A is strongly $S-closed$.

Theorem 4.7. If $f:(X,\tau)\longrightarrow(Y,\sigma)$ has a $contra-\gamma-closed$ graph, then the inverse image of a strongly $S-closed$ set K of Y is $\gamma-closed$ in X.

Proof. Assume that K is a strongly $S-closed$ set of Y and $x\notin f^{-1}(K)$. For each $k\in K$, $(x,k)\notin G(f)$. By Lemma 4.2, there exist U_k a $\gamma-open$ neighbourhood of x and V_k a closed subset of Y with $k\in V_k$ such that $f(U_k)\cap V_k=\phi$. Since

$\{K \cap V_k | k \in K\}$ is a closed cover of the subspace K, there exists a finite subset $K_1 \subseteq K$ such that $K \subseteq \cup \{V_k | k \in K_1\}$. Then $U = \cap \{U_k | k \in K_1\}$ is a $\gamma - open$ neighbourhood of x and $f(U) \cap K = \phi$. Therefore $U \cap f^{-1}(K) = \phi$ and hence $x \notin Cl_\gamma [f^{-1}(K)]$. This shows that $f^{-1}(K)$ is $\gamma - closed$ in X.

Theorem 4.8. Let Y be a strongly $S - closed$ space. If a function $f : (X, \tau) \longrightarrow (Y, \sigma)$ has a $contra - \gamma - closed$ graph, then f is $contra - \gamma - continuous$.

Proof. Suppose that Y is strongly $S - closed$ and $G(f)$ is $contra - \gamma - closed$. First, we show that an open set of Y is strongly $S - closed$. Let V be an open set of Y and $\{H_\alpha : \alpha \in \nabla\}$ be a cover of V by closed sets H_α of V. For each $\alpha \in \nabla$, there exists a closed set K_α of X such that $H_\alpha = K_\alpha \cap V$. Then, the family $\{K_\alpha | \alpha \in \nabla\} \cup \{(Y - V)\}$ is a closed cover of Y. Since Y is strongly $S - closed$, there exists a finite subset $\nabla_0 \subseteq \nabla$ such that $Y = (\cup \{K_\alpha | \alpha \in \nabla_0\}) \cup (Y - V)$. Therefore we obtain $V = \cup \{K_\alpha | \alpha \in \nabla_0\}$. This shows that V is strongly $S - closed$. For any open set V, by Theorem 4.7 $f^{-1}(V)$ is $\gamma - closed$ in X and f is $contra - \gamma - continuous$.

5. Covering Properties

In this section we study the properties of compact and strongly $S - closed$ spaces under the $contra - \gamma - continuous$ functions.

Theorem 5.1. Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ be a $contra - \gamma - continuous$ function. Let K be a compact subset of (X, τ^γ). Then $f(K)$ is strongly $S - closed$ in Y.

Proof. Let $\{H_\alpha | \alpha \in \nabla\}$ be any cover of $f(K)$ by closed sets of the subspace $f(K)$. For each $\alpha \in \nabla$, there exists a closed set K_α of Y such that $H_\alpha = K_\alpha \cap f(K)$. For each $x \in K$, there exists $\alpha(x) \in \nabla$ such that $f(x) \in K_{\alpha(x)}$ and by Theorem 3.8, there exists U_x a $\gamma - open$ neighbourhood of x such that $f(U_x) \subseteq K_{\alpha(x)}$. Since the family $\{U_x | x \in K\}$ is a cover of K by $\gamma - open$ sets of X, there exists a finite subset K_0 of K such that $K \subseteq \cup \{U_x | x \in K_0\}$. Therefore we obtain $f(K) \subseteq \cup \{f(U_x) | x \in K_0\}$ which is a subset of $\cup \{K_{\alpha(x)} | x \in K_0\}$. Thus, $f(K) = \cup \{H_{\alpha(x)} | x \in K_0\}$ and hence $f(K)$ is strongly $S-closed$.

Corollary 5.2. If $f : (X, \tau) \longrightarrow (Y, \sigma)$ is a $contra - \gamma - continuous$ surjection and (X, τ^γ) is compact, then Y is strongly $S - closed$.

Theorem 5.3. If $f : (X, \tau) \longrightarrow (Y, \sigma)$ is $contra - \gamma - continuous$ surjection and (X, τ^γ) is a strongly $S - closed$ space, then (Y, σ) is compact.

Proof. Let $\{V_\alpha | \alpha \in \nabla\}$ be any open cover of Y. Then $\{f^{-1}(V_\alpha) | \alpha \in \nabla\}$ is a cover of X. Since f is $contra - \gamma - continuous$, $f^{-1}(V_\alpha)$ is $\gamma - closed$ in X for each $\alpha \in \nabla$. This implies that $\{f^{-1}(V_\alpha) | \alpha \in \nabla\}$ *is* a $\gamma - closed$ cover of the strongly $S - closed$ space X. We have $X = \cup \{f^{-1}(V_\alpha) | \alpha \in \nabla_0\}$ for some finite ∇_0 of ∇. Since f is surjective, $Y = \cup \{V_\alpha | \alpha \in \nabla_0\}$. This shows that (Y, σ) is compact.

6. Connected Spaces

In this section we study the properties of connected spaces under the $contra - \gamma - continuous$ functions.

Theorem 6.1. If $f : (X, \tau) \longrightarrow (Y, \sigma)$ is $contra - \gamma - continuous$ surjection and (X, τ^γ) is connected, then (Y, σ) is connected.

Proof. Suppose Y is not connected. There exist nonempty disjoint open sets V_1 and V_2 such that $Y = V_1 \cup V_2$. Therefore, V_1 and V_2 are clopen in Y. Since f is $contra - \gamma - continuous$, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are $\gamma - closed$ and $\gamma - open$ in X and hence $\gamma-$clopen in X. Moreover, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are nonempty disjoint and $X = f^{-1}(V_1) \cup f^{-1}(V_2)$. This shows that (X, τ^γ) is not connected.

Definition 6.2. [*Steen* and *Seebach*, 1970]. A topological space (X, τ) is said to be hyperconnected if the closure of every nonempty open set is the entire set X. It is well-known that every hyperconnected space is connected but not conversely.

Remark 6.3. In example 3.5, (X, τ) is hyperconnected and $f : (X, \tau) \longrightarrow (X, \sigma)$ is a $contra{-}\gamma{-}continuous$ surjection, but (X, σ) is not hyperconnected. This shows that $contra - \gamma - continuous$ surjections do not preserve hyperconnectedness.

Definition 6.4. $[Levine,\ 1961]$. A function $f : (X, \tau) \longrightarrow (Y, \sigma)$ is said to be weakly continuous if for each point $x \in X$ and each open set V of Y containing $f(x)$, there exists an open set U containing x such that $f(U) \subseteq Cl(V)$.

Theorem 6.5. $[Noiri, 1974]$. If $f : (X, \tau) \longrightarrow (Y, \sigma)$ is a weakly continuous surjection and X is connected, then Y is connected.

Remark 6.6. $Contra - \gamma - continuity$ and weak continuity are independent of each other. In Example 3.5, the function f is $contra{-}\gamma{-}continuous$ but not weakly continuous. The following example shows that not every weakly continuous function is $contra - \gamma - continuous$.

Example 6.7. Let $X = \{a, b, c, d\}$ and $\tau = \{\phi, X, \{b\}, \{c\}, \{b, c\}, \{a, b\}, \{a, b, c\}, \{b, c, d\}\}$. Define a function $f : (X, \tau) \longrightarrow (X, \tau)$ as follows: $f(a) = c$, $f(b) = d$, $f(c) = b$ and $f(d) = a$. Then f is weakly continuous $[Neubrunnova, 1980]$. However, f is not $contra - \gamma - continuous$ since $\{a\}$ is a closed set of (X, τ) and $f^{-1}(\{a\}) = \{d\}$ is not $\gamma - open$ in (X, τ).

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