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Characterizations of $\alpha-$I$-$Open Sets and $\alpha-$I$-$Continuity

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Abstract

Given a space (X,τ,I) consisting of a nonempty set X with topology τ and an ideal I of subsets of X which has a heredity and finite additivity properties. The main objective of this paper is to introduce the notion of $\alpha-I-open$ set and discuss its numerous topological properties.

1. Introduction

In the beginning of ninetieth, Jankovic and Hamlett $[$*Jankonic et* al., 1992$]$ have defined the concept of $I-open$ set via the local function which was given by Vaidyanathaswamy $[$*Vaidyanathaswamy*, 1954$]$. The latter concept was also established utilizing the concept of an ideal whose topic in general topological spaces was treated in the classical text by $[$*Kuratowski*, 1933$]$ in 1933. In 1992, Abd El-Monsef et al. $[$*Abd El-Monsef et al.*, 1992$]$ studied a number of properties of $I-open$ sets as well as $I-closed$ sets and $I-continuous$ functions and investigated several of their properties. We devote this paper to generalize the $I-openness$ by presenting the class of $\alpha-I-open$ sets. The class of an $\alpha-I-continuous$ function have been also established and studied. The connections between these new concepts with the corresponding types are discussed. Some of their characterizations and other numerous properties are studied.

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2. Preliminaries

Throughout the present paper, (X,τ) and (Y,σ) are denoted by the topological spaces on which no separation axioms are assumed unless the needed ones will be explicitly stated. In (X,τ), the usual closure and the interior of any $W \subseteq X$ will be denoted by $Cl(W)$ and $Int(W)$, respectively. The notion of an ideal topological space (X,τ,I) means a topological space (X,τ) with an ideal I on X which is a nonempty subclass of a power set $P(X)$ having the heredity and finite additivity properties i.e. $I \subseteq P(X)$ is an ideal on X if $W \in I$ and $S \subseteq W$, then $S \in I$ and also if $W \in I$ and $S \in I$, then $W \cup S \in I$ $[$*Kuratowski*, 1933$]$. Sometimes an ideal is called a dual filter $[$*Jankonic et* al., 1990$]$. For any $W \subseteq X$ in (X,τ,I), $W^*(I,\tau)$ or simply $W^*(I)$ means the local function of W with respect to I and τ and is defined by $W^*(I,\tau) = \{x \in X : W \cap N \notin I, N \in \tau(x)\}$ $[$*Vaidyanathaswamy*, 1954$]$, where $\tau(x)$ denotes the open-neighbourhood of x. The simplest ideals are $\{\phi\}$ and $P(X)$ which satisfy $\{\phi\} \subseteq I \subseteq P(X)$, for any ideal I on X. Also, between the useful ideals is the collection of all nowhere dense subsets of (X,τ) which is an ideal and denoted by I_n. i.e., $I_n = \{W \subseteq X : Int[Cl(W)] = \phi\}$. For (X,τ,I), we say that the topology τ is compatible with an ideal I , denoted by $\tau \sim I$, if the following holds: for every $W \subseteq X$ and every $x \in W$ there exists $N \in \tau(x)$ such that $N \cap W \in I$, then $W \in I$ $[$*Njastad*, 1966$]$. $W \subseteq X$ is said to be $I-open$ $[$*Jankovic et* al., 1992$]$ (resp. semi-open $[$*Levine*, 1963$]$, $\alpha-open$ $[$*Njastad*, 1965$]$, $\beta-open$ $[$*Abd* El-Monsef et al., 1983$]$) set if $W \subseteq Int(W^*)$ (*resp.* $W \subseteq Cl[Int(W)]$, $W \subseteq Int[Cl(Int(W))]$, $W \subseteq Cl[Int(Cl(W))]$) while their complements are the corresponding types of closeness. The classes of all previous types of openness in (X,τ) are denoted by $IO(X,\tau)$, $SO(X,\tau)$, τ^{α} and $\beta O(X,\tau)$, respectively.

3. α–I–Open Sets

Definition 1. In an ideal topological space (X,τ,I), $W \subseteq X$ is said to be an $\alpha - I - open$ set if $W \subseteq Int\left[Cl\left(Int\left(W^{*}\right)\right)\right]$, The complement set $(X-W)$ is called $\alpha - I - closed$. The collection of all $\alpha - I - open$ $(resp.\ I - open)$ sets of (X,τ) will be denoted by $\alpha IO(X,\tau)$ $(resp.\ IO(X,\tau))$.

Remark 2. $IO(X,\tau) \subseteq \alpha IO(X,\tau)$.

Proposition 3. Arbitrary union of $\alpha - I - open$ sets is also $\alpha - I - open$.

Proof. Let (X,τ,I) be any ideal topological space and $W_i \in \alpha IO(X,\tau)$ for $i \in \nabla$, this means that for each $i \in \nabla$, $W_i \subseteq Int\left[Cl\left(Int\left(W^{*}\right)\right)\right]$ and so $\bigcup_{i\in\nabla} W_i \subseteq \bigcup_{i\in\nabla} Int\left[Cl\left(Int\left(W_i^{*}\right)\right)\right] \subseteq Int\left[\bigcup_{i\in\nabla} Cl\left[Int\left(W_i^{*}\right)\right]\right] \subseteq Int\left[Cl\left[\bigcup_{i\in\nabla} Int\left(W_i^{*}\right)\right]\right] \subseteq Int\left[Cl\left(Int\left(\bigcup_{i\in\nabla} W_i^{*}\right)\right)\right]$. Hence $\bigcup_{i\in\nabla} W_i \in \alpha IO(X,\tau)$.

Lemma 4. [*Jankonic et* al., 1990]. Let (X,τ) be a topological space with an ideal I_n on X. Then X coincides with its local function $(i.e.,\ X = X^{*})$.

Definition 5. Let X be a nonempty set and let $\tau \subseteq P(X)$. We say that τ is a supratopology on X if $\phi, X \in \tau$ and τ is closed under arbitrary union.

Theorem 6. For an ideal topological space (X,τ,I_n), the class $\alpha IO(X,\tau)$ forms a supratopology.

Proof. Follows by the fact $\phi^{*} = \{x \in X : \phi \cap N_x \notin I_n\} = \phi$ and both of Lemma 4 and Proposition 3.

The above Theorem leads us to present the following question: Does $\alpha IO(X,\tau)$ form a topology? The answer is no for the next remark.

Remark 7. A finite intersection of $\alpha - I - open$ sets need not in general $\alpha - I - open$, as Example 8 shows.

Example 8. Let $X = \{a,b,c,d\}$ and $\tau = \{X,\phi,\{a,b\},\{a,b,c\}\}$ with $I = \{\phi,\{c\},\{d\},\{c,d\}\}$. We deduce that the two sets $W_1 = \{a,b\}$ and $W_2 = \{b,c\}$ are $\alpha - I - open$ while their intersection is not.

The preceding remark turns our attention to establish the next result.

Proposition 9. In an ideal topological space (X,τ,I), if $U \in \tau$ and $W \in \alpha IO(X,\tau)$, then $U \cap W$ is $\alpha - I - open$.

Proof. By hypothesis and the fact that $U \cap Cl(W) \subseteq Cl(U \cap W)$, we have, $U \cap Int(W) \subseteq (U \cap W)$ and $(U \cap W^*) \subseteq (U \cap W)^*$, it follows that $U \cap W \subseteq U \cap Int\left[Cl\left(Int\left(W^*\right)\right)\right] \subseteq Int\left[U \cap Cl\left(Int\left(W^*\right)\right)\right] \subseteq Int\left[Cl\left(U \cap Int\left(W^*\right)\right)\right]$ $\subseteq Int\left[Cl\left(Int\left(U \cap W^*\right)\right)\right] \subseteq Int\left[Cl\left(Int\left(U \cap W\right)^*\right)\right]$. Hence the result.

Theorem 10. The intersection of an $\alpha - I - open$ set with an $\alpha - open$ set is a $\beta - open$ set.

Proof. Let (X,τ,I) be an ideal topological space, $W \in \alpha IO(X,\tau)$ and $G \in \tau^\alpha$. Then

$$W \cap G \subseteq Int\left[Cl\left(Int\left(W^*\right)\right)\right] \cap Int\left[Cl\left(Int(G)\right)\right] \subseteq Cl\left[Int\left(W^*\right) \cap Int\left[Cl\left(Int(G)\right)\right]\right]$$
$$= Cl\left[Int\left(Int\left(W^*\right)\right) \cap Cl\left(Int(G)\right)\right] \subseteq Cl\left[Int\left(Cl\left(W^* \cap Int(G)\right)\right)\right]$$

$([[Jankovic\ et\ al., 1990], Theorem(2.3)]) \subseteq Cl\left[Int\left(Cl\left(Cl(W) \cap Int(G)\right)\right)\right]$

$\subseteq Cl\left[Int\left(Cl\left(Cl(W \cap G)\right)\right)\right] = Cl\left[Int\left(Cl(W \cap G)\right)\right]$, which completes the proof.

Proposition 11. In an ideal topological space (X,τ,I), let $U \in \tau$ and $W \in \alpha IO(X,\tau)$. Then $U \cap W \in \alpha IO(U,\tau | U)$.

Proof. $U \cap W \subseteq U \cap Int\left[Cl\left(Int\left(W^*\right)\right)\right] \subseteq Int\left[Cl\left(U \cap Int\left(W^*\right)\right) \cap U\right]$

$= Int_U\left[Cl_U\left(U \cap Int\left(W^*\right)\right)\right]$. While $U \cap Int\left(W^*\right) \subseteq U$ which is open in U, i.e. $Int_U\left[U \cap Int\left(W^*\right)\right] = U \cap Int\left(W^*\right)$. Thus $U \cap W \subseteq Int_U\left[Cl_U\left(Int_U\left(U \cap Int\left(W^*\right)\right)\right)\right]$ $\subseteq Int_U\left[Cl_U\left(Int_U\left(U \cap W^*\right)\right)\right] \subseteq Int_U\left[Cl_U\left(Int_U\left(U \cap W^*\right)\right)\right]$ ([[*Jankovic et al.*, 1990], *Theorem*(2.3)]). Hence the result.

Proposition 12. Let (X,τ,I) be an ideal topological space and let $W \subseteq U \subseteq X$, $U \in \tau$ and $W \in \alpha IO(U,\tau|U)$. Then $W \in \alpha IO(X,\tau)$.

Proof. We notice that $W \subseteq Int_U\left[Cl_U\left(Int_U\left(W^*\right)\right)\right] \subseteq Int\left[Cl_U\left(Int_U\left(W^*\right)\right)\right]$. Since $Int_U\left(W^*\right)$ is an open set in U, there exists $V \in \tau$ such that $Int_U\left(W^*\right) = U \cap V \in \tau$. Hence $W \subseteq Int\left[Cl_U\left(U \cap V\right)\right] = Int\left[Cl\left(U \cap V\right) \cap U\right] \subseteq Int\left[Cl\left(U \cap V\right)\right]$ $= Int\left[Cl\left(Int_U\left(W^*\right)\right)\right] \subseteq Int\left[Cl\left(Int\left(W^*\right)\right)\right]$. Thus $W \in \alpha IO(X,\tau)$.

Jankovic and Hamlett [*Jankovic et al.*, 1992] have denoted by $I * J$ the extension of an ideal I via other one J which are given on the same nonempty set X with respect to (X,τ) and defined as: $I * J = \left\{A \subseteq X : A^*(I) \in J\right\}$. They showed also that $I * J$ is an ideal on X and $I \subseteq I * J$.

Lemma 13. [*Jankovic et al.*, 1992]. Let (X,τ,I) be an ideal topological space with $I_n \subseteq I$ and $\tau \sim I$. Then for any other ideal J on X, $\tau \sim (J * I)$.

For any (X,τ,I), the authors in [*Jankovic et al.*, 1992] denote $(I * I_n)$ by $\tilde{I}$ which is a compatible extension of I and it is also defined as $\tilde{I} = \left\{A \subseteq X : Int\left(A^*\right) = \phi\right\}$.

Theorem 14. (1) Given (X,τ,I), any $W \subseteq X$ is $\alpha - I - open$. Then $W \subseteq W^*(\tilde{I})$.

(2) Given (X,τ,I), $W \subseteq X$ such that $W \subseteq Int\left[W^*(\tilde{I})\right]$. Then W is $\alpha - I - open$.

Proof. (1) Let $x \notin W^*(\tilde{I})$. Then there exists an open neighbourhood N of x such that $Int(N \cap W)^*(I) = \phi$. Since $N \cap W^*(I) \subseteq (N \cap W)^*(I)$ $[Kuratowski, 1966]$, then $N \cap Int[W^*(I)] \subseteq Int(N \cap W)^*(I) = \phi$. This means that $x \notin Cl[Int(W^*)]$. Hence $x \notin Int[Cl(Int(W^*))]$. Since W is $\alpha - I - open$. So we get $x \notin W$.

(2) Since $I_n \subseteq \tilde{I}$, then for any $W \subseteq X$, $W^*(\tilde{I}) \subseteq W^*(I_n) = Cl[Int(Cl(W))]$ $[Vaidyanathaswamy, 1960]$. If we put W^* instead of W, the last inclusion gives $(W^*(\tilde{I}))^*(\tilde{I}) \subseteq Cl[Int(Cl(W^*(\tilde{I})))] = Cl[Int(W^*(\tilde{I}))]$, for $W^*(\tilde{I})$ is closed $[[Jankovic\ et\ al., 1990], Theorem(2.3)]$. By $\tilde{I} \sim \tau$, we get $W^*(\tilde{I}) = (W^*(\tilde{I}))^*(\tilde{I})$. Therefore, $W^*(\tilde{I}) \subseteq Cl[Int(W^*(\tilde{I}))] \subseteq W^*(\tilde{I})$. Hence $W \subseteq Int[W^*(\tilde{I})] \subseteq Int[Cl(Int(W^*(\tilde{I})))] \subseteq Int[Cl(Int(W^*(I)))]$, for $I \subseteq \tilde{I}$. Therefore, $W \in \alpha IO(X,\tau)$ and the proof is complete.

4. On $\alpha - I -$ Continuity

Definition 15. A function $f:(X,\tau,I) \to (Y,\sigma)$ is called $\alpha - I - continuous$ if for every $V \in \sigma$, $f^{-1}(V) \in \alpha IO(X,\tau)$.

The following Theorem gives some characterizations of $\alpha - I - continuity$.

Theorem 16. For a function $f:(X,\tau,I) \to (Y,\sigma)$, the following are equivalent:

(i) f is $\alpha - I - continuous$.

(ii) The inverse image of each closed set in (Y,σ) is $\alpha - I - closed$.

(iii) For each $x \in X$ and each $V \in \sigma$ containing $f(x)$, there exists $W \in \alpha IO(X,\tau)$ containing x such that $f(W) \subseteq V$.

Proof. Routine.

Definition 17. A function $f:(X,\tau)\to(Y,\sigma)$ is called $\beta-continuous$ $[Abd\ El\text{-}Monsef\ et\ al., 1983]$ (resp. $\beta-irresolute$ $[Mahmoud\ et\ al., 1989]$) if the inverse image of each open set $(resp.\ \beta-open\ set)$ in (Y,σ) is $\beta-open\ set$ $(resp.\ \beta-open\ set)$ in (X,σ).

Proposition 18. (1) Let $f:(X,\tau,I)\to(Y,\sigma,J)$ be $\alpha-I-continuous$ and $g:(Y,\sigma,J)\to(Z,\mu)$ be continuous. Then gof is $\alpha-I-continuous$.

(2) Let $f:(X,\tau,I)\to(Y,\sigma,J)$ be $\beta-irresolute$ and $g:(Y,\sigma,J)\to(Z,\mu)$ be $\alpha-I-continuous$. Then gof is $\beta-continuous.$

Proof. (1) Let $W\in\mu$. Then $(gof)^{-1}(W)=(f^{-1}og^{-1})(W)=f^{-1}(g^{-1}(W))$. Since $g^{-1}(W)$ is open as g is continuous. Now since f is $\alpha-I-continuous.$ So $f^{-1}(g^{-1}(W))$ is $\alpha-I-open$. Hence gof is $\alpha-I-continuous.$

(2) Let $W\in\mu$. Then $(gof)^{-1}(W)=(f^{-1}og^{-1})(W)=f^{-1}(g^{-1}(W))$. Since g is $\alpha-I-continuous$. So $g^{-1}(W)$ is $\alpha-I-open$ and hence $g^{-1}(W)$ is $\beta-open$. Now since f is $\beta-irresolute$. So $f^{-1}(g^{-1}(W))$ is $\beta-open$. Hence gof is $\beta-continuous.$

Theorem 19. The restriction of an $\alpha-I-continuous$ function to an open set is also $\alpha-I-continuous$.

Proof. Let $f:(X,\tau,I)\to(Y,\sigma)$ be an $\alpha-I-continuous$ function and $U\in\tau$. To show that $f|U$ is $\alpha-I-continuous$, let $V\in\sigma$, then $f^{-1}(V)\in\alpha IO(X,\tau)$. By Proposition 11, $(f|U)^{-1}(V)=U\cap f^{-1}(V)\subseteq U\cap Int\left[Cl\left(Int\left(f^{-1}(V)\right)^*\right)\right]$ $\subseteq$ $Int\left[Cl\left(U\cap Int\left(f^{-1}(V)\right)^*\right)\right]$ $\subseteq$ $Int\left[Cl\left(Int\left(U\cap\left(f^{-1}(V)\right)^*\right)\right)\right]$ $\subseteq$ $Int\left[Cl\left(Int\left(U\cap\left(f^{-1}(V)\right)\right)^*\right)\right]$ $[[Jankovic$ et al, 1990], $Theorem(2.3)]$. Hence $f|U$ is $\alpha-I-continuous$.

Definition 20. $\left[Abd\ El\text{-}Monsef\ \ et\ al., 1992\right]$ A function $f:(X,\tau,I)\to(Y,\sigma)$ is called $I-continuous$ if the inverse image of each open set in (Y,σ) is an $I-open$ set in (X,τ,I).

Theorem 21. A function $f:(X,\tau,I)\to(Y,\sigma)$ is $\alpha-I-continuous$ if its restrictions by an open cover $\{U_i : i\in\nabla\}$ of X are $I-continuous$.

Proof. Let $V\in\sigma$. Then $f^{-1}(V)=\bigcup_{i\in\nabla}\left[f^{-1}(V)\cap U_i\right]=\bigcup_{i\in\nabla}\left[\left(f|U_i\right)^{-1}(V)\right]$. By hypothesis, $\left(f|U_i\right)^{-1}(V)$ is $I-open$ and so is $\alpha-I-open$. Then by Proposition 3, $f^{-1}(V)$ is $\alpha-I-open.$ We conclude that f is $\alpha-I-continuous.$

Theorem 22. Prove that any function $f:(X,\tau,I)\to(Y,\sigma)$ is $\alpha-I-continuous$ if and only if its graph function is $\alpha-I-continuous.$

Proof. Let $f:(X,\tau,I)\to(Y,\sigma)$ be $\alpha-I-continuous,$ $x\in X$ and H an open set in $X\times Y$ containing $g(x)$, where $g:X\to X\times Y$ is defined by $g(x)=(x, f(x))$, for every $x\in X$. Then there exist $U\in\tau$ and $V\in\sigma$ such that $g(x)=(x, f(x))\in U\times V\subseteq H$. By hypothesis, there exists $W\in\alpha IO(X,x)$ such that $f(W)\subseteq V$. Proposition 9 shows that $x\in U\cap W\in\alpha IO(X,\tau)$ which is contained in U. So, $(U\cap W)\times V\subseteq U\times V\subseteq H$ and hence $g(U\cap W)\subseteq H$. This means that $g:(X,\tau,I)\to(X\times Y,\tau_{X\times Y})$ is $\alpha-I-continuous.$

Conversely, let $x\in X$ and $V\in\sigma$ containing $f(x)$, then $X\times V\in\tau_{X\times Y}$. $\alpha-I-continuity$ of g shows that there exists $W\in\alpha IO(X,x)$ such that $g(W)\subseteq X\times V$. This projection of both sides gives $f(W)\subseteq V$. Hence f is $\alpha-I-continuous$ and this completes the proof.

Definition 23. A space (X,τ) is $\alpha-I-compact$ if for every $\alpha-I-open$ cover $\{W_i : i\in\nabla\}$, there exists a finite subset ∇_0 of ∇ such that $X-\bigcup\{U_i : i\in\nabla_0\}\in I$.

Lemma 24. $[Newcomb,\ 1967]$. For any function $f:(X,\tau,I)\to(Y,\sigma)$, $f(I)$ is an ideal on Y.

Theorem 25. Let $f:(X,\tau,I)\to(Y,\sigma)$ be an $\alpha-I-continuous$ surjective function. Suppose that (X,τ) is $\alpha-I-compact$. Then (Y,σ) is $\alpha-f(I)-compact$.

Proof. Let $f:(X,\tau,I)\to(Y,\sigma)$ be an $\alpha-I-continuous$ surjection and $\{V_i : i\in\nabla\}$ be an open cover of Y. Then $\{f^{-1}(V_i): i\in\nabla\}$ is an $\alpha-I-open$ cover of X. From the assumption, there exists a finite subset ∇_0 of ∇ such that $X-\bigcup\{f^{-1}(V_i): i\in\nabla_0\}\in I$. Therefore, $Y-\bigcup\{V_i : i\in\nabla_0\}\in f(I)$ which shows that (Y,σ) is $\alpha-f(I)-compact$.

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