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APPLICATIONS OF FUZZY POINTS VIA FUZZY θ−OPEN SETS AND FUZZY θ−CLOSURE OPERATOR

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APPLICATIONS OF FUZZY POINTS VIA FUZZY $\theta-$OPEN SETS AND FUZZY $\theta-$CLOSURE OPERATOR

Saeid Jafari and Raja M. Latif

Abstract

In 1968, Veličko [22] introduced the notions of θ-open set and θ-closure operator. These notions have shown their fruitfulness and importance not only in General Topology but also in Fuzzy Topology. The objective of this paper is to offer some applications of fuzzy points via fuzzy θ-open sets and fuzzy θ-closure operator in fuzzy topological spaces. Moreover, we present and study some properties of θ-closed sets and some related separation axioms in fuzzy topological spaces.

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1 Introduction

The notions of θ-open subsets, θ-closed subsets and θ-closure were introduced by Veličko [22] for the purpose of studying the important class of H-closed spaces in terms of arbitrary filterbases. Dickman and Porter [5], [6], Joseph [9] continued the work of Veličko.
In 1965, Zadeh [23] generalized the usual notion of a set to what he called fuzzy set which proved to be of tremendous usage both in pure and applied mathematics. Mukherjee and Sinha [13] introduced the notion´s of fuzzy θ-open set and fuzzy θ-closure operator. These notions have been applied to define various new fuzzy topological notions and properties (see for example

[16], [14], [15], [7] and [3]). The purpose of this paper is to give some applications of fuzzy points via fuzzy θ-open sets and fuzzy θ-closure operator in fuzzy topological spaces. Moreover, we present and study some properties of θ-closed sets and some related separation axioms in fuzzy topological spaces.

Throughout this paper, the symbol I will denote the unit interval $[0, 1]$. A *fuzzy set* in X is a function with domain X and values in I, i.e. an element of I^X.

A member A of I^X is *contained* in a member B of I^X, denoted by $A \leq B$, if $A(x) \leq B(x)$ for every $x \in X$ (see [23]).

Let $A, B \in I^X$. We define the following fuzzy sets (see [23]):

(1) $A \wedge B \in I^X$ by $(A \wedge B)(x) = \min\{A(x), B(x)\}$ for every $x \in X$.

(2) $A \vee B \in I^X$ by $(A \vee B)(x) = \max\{A(x), B(x)\}$ for every $x \in X$.

(3) $A^c \in I^X$ by $A^c(x) = 1 - A(x)$ for every $x \in X$.

(4) Let $f : X \to Y$, $A \in I^X$ and $B \in I^Y$. Then $f(A)$ is a fuzzy set in Y such that $f(A)(y) = \sup\{A(x) : x \in f^{-1}(y)\}$, if $f^{-1}(y) \neq \emptyset$ and $f(A)(y) = 0$, if $f^{-1}(y) = \emptyset$. Also, $f^{-1}(B)$ is a fuzzy set in X, defined by $f^{-1}(B)(x) = B(f(x))$, $x \in X$.

The first definition of a fuzzy topological space is due to Chang (see [4]). According to Chang, a fuzzy topological space is a pair (X, τ), where X is a set and τ is a *fuzzy topology* on it, i.e. a family of fuzzy sets ($\tau \subset I^X$) satisfying the following three axioms:

(1) $\bar{0}, \bar{1} \in \tau$. By $\bar{0}$ and $\bar{1}$ we denote the characteristic functions $\mathcal{X}_\emptyset$ and $\mathcal{X}_X$, respectively.

(2) If $A, B \in \tau$, then $A \wedge B \in \tau$.

(3) If $\{A_j \mid j \in J\} \subset \tau$, then $\vee\{A_j \mid j \in J\} \in \tau$.

By using the notion of fuzzy set, Wong (see [19]) was able to introduce and investigate the notions of fuzzy points. In this paper we adopted Pu's definition of a fuzzy point. A fuzzy set in a set X is called a *fuzzy point* if it takes the value 0 for all $y \in X$ except one, say, $x \in X$. If its value at x is λ $(0 < \lambda \leq 1)$ we denote the fuzzy point by p_x^λ, where the point x is called its *support*, denoted by $supp(p_x^\lambda)$, that is $supp(p_x^\lambda) = x$. The class of all fuzzy points in X is denoted by $\mathcal{X}$.

The fuzzy point p_x^λ is said to be *contained* in a fuzzy set A or to belong to A, denoted by $p_x^\lambda \in A$, if $\lambda \leq A(x)$. Evidently, every fuzzy set A can be expressed as the union of all the fuzzy points which belongs to A (see [11]).

A fuzzy point p_x^λ is said to be *quasi-coincident* with A denoted by $p_x^\lambda q A$ if and only if $\lambda > A^c(x)$ or $\lambda + A(x) > 1$ (see [11]).

A fuzzy set A is said to be *quasi-coincident* with B, denoted AqB, if and only if there exists $x \in X$ such that $A(x) > B^c(x)$ or $A(x) + B(x) > 1$ (see [11]). If A does not quasi-coincident with B, then we write $A \not{q} B$.

Let f be a function from X to Y. Then (see for example [1], [2], [4], [10], [12], [14], [17], [20], and [21]):

(1) $f^{-1}(B^c) = (f^{-1}(B))^c$, for any fuzzy set B in Y.

(2) $f(f^{-1}(B)) \leq B$, for any fuzzy set B in Y.

(3) $A \leq f^{-1}(f(A))$, for any fuzzy set A in X.

(4) Let p be a fuzzy point of X, A be a fuzzy set in X and B be a fuzzy set in Y. Then, we have:

(i) If $f(p)\ q\ B$, then $p\ q\ f^{-1}(B)$.

(ii) If $p\ q\ A$, then $f(p)\ q\ f(A)$.

(5) Let A and B be fuzzy sets in X and Y, respectively and p be a fuzzy point in X. Then we have:

(i) $p \in f^{-1}(B)$ if $f(p) \in B$.

(ii) $f(p) \in f(A)$ if $p \in A$.

Let be a directed set and X be an ordinary set. The function $S : \ \to \mathcal{X}$ is called a *fuzzy net* in X. For every $\lambda \in$, $S(\lambda)$ is often denoted by s_λ and hence a net S is often denoted by $\{s_\lambda, \lambda \in \ \}$ (see [11]).

Definition 1 *[13]. A fuzzy point x_p in a fuzzy topological space X is said to be a fuzzy θ-cluster point of a fuzzy set λ if and only if for every fuzzy open q-neighborhood μ of x_p , $Cl(\mu)$ is q-coincident with λ. The set of all fuzzy θ-cluster points of λ is called the fuzzy θ-closure of λ and is denoted by $Cl_\theta(\lambda)$. A fuzzy set λ is fuzzy θ-closed if and only if $\lambda = Cl_\theta(\lambda)$. The complement of a fuzzy θ-closed set is called fuzzy θ-open. We denote the set of all fuzzy θ-open sets by $F\theta O(X)$. The θ-interior of λ denoted by $Int_\theta(\lambda)$ is defined as :*

$Int_\theta(\lambda) = \{x_p :$ *for some fuzzy open q-neighborhood* β *of* $x_p, Cl(\beta) \leq \lambda\}$.

Lemma 1 *Let λ be a fuzzy set in a fuzzy topological space X, then:*
1) λ is a fuzzy θ-open if and only if $\lambda = Int_\theta(\lambda)$.
2) $1 - Int_\theta(\lambda) = Cl_\theta(1 - \lambda)$ and $Int_\theta(1 - \lambda) = 1 - Cl_\theta(\lambda)$.
3) $Cl_\theta(\lambda)$ (resp. $Int_\theta(\lambda)$) is a fuzzy closed set (resp. fuzzy open set) but not necessarily a fuzzy θ-closed set (resp. fuzzy θ-open set).

Remark 1 . *(i) It should be noticed that* $Cl(\lambda) \leq Cl_\theta(\lambda)$ *and* $Int_\theta(\lambda) \leq Int(\lambda)$ *for any fuzzy set* λ *in a fuzzy topological space* X.
(ii) For a fuzzy closed (resp. fuzzy open) set λ *in a fuzzy topological space* X, $Cl(\lambda) = Cl_\theta(\lambda)$ *(resp.* $Int_\theta(\lambda) = Int(\lambda)$*).*

2 Fuzzy points, θ-closed sets and separation axioms

Definition 2 *A fuzzy set* A *in a fuzzy space* X *is called a fuzzy* θ*-neighborhood of a fuzzy point* p_x^λ *if there exists a* $V \in F\theta O(X)$ *such that* $p_x^\lambda \in V \leq A$. *A fuzzy* θ*-neighborhood* A *is said to be* θ*-open if* $A \in F\theta O(X)$.

Definition 3 *A fuzzy set* A *in a fuzzy space* X *is called a fuzzy* $Q{-}\theta$*-neighborhood of* p_x^λ *if there exists* $B \in F\theta O(X)$ *such that* $p_x^\lambda q B$ *and* $B \leq A$.

Remark 2 *A fuzzy* $Q{-}\theta$*-neighborhood of a fuzzy point generally does not contain the point itself. In what follows by* $\mathcal{N}_{Q-p-n}(p_x^\lambda)$ *we denote the family of all fuzzy* θ*-open* $Q{-}\theta$*-neighborhood s of the fuzzy point* p_x^λ *in* X. *The set* $\mathcal{N}_{Q-p-n}(p_x^\lambda)$ *with the relation* $\leq^*$ *(that is,* $U_1 \leq^* U_2$ *if and only if* $U_2 \leq U_1$*) form a directed set.*

Proposition 2 *Let* A *be a fuzzy set of a fuzzy space* X. *Then, a fuzzy point* $p_x^\lambda \in Cl_\theta(A)$ *if and only if for every* $U \in F\theta O(X)$ *for which* $p_x^\lambda q U$ *we have* UqA.

Proof. The fuzzy point $p_x^\lambda \in Cl_\theta(A)$ if and only if $p_x^\lambda \in F$, for every fuzzy θ-closed set F of X for which $A \leq F$. Equivalently $p_x^\lambda \in Cl_\theta(A)$ if and only if $\lambda \leq 1 - U(x)$, for every fuzzy θ-open set U for which $A \leq \bar{1} - U$. Thus $p_x^\lambda \in Cl_\theta(A)$ if and only if $U(x) \leq 1 - \lambda$, for every fuzzy θ-open set U for which $U \leq \bar{1} - A$. So, $p_x^\lambda \in Cl_\theta(A)$ if and only if for every fuzzy θ-open set U of X such that $U(x) > 1 - \lambda$ we have $U \not\leq \bar{1} - A$. Therefore by Proposition 2.1 of [11], $p_x^\lambda \in Cl_\theta(A)$ if and only if for every fuzzy θ-open set U of X such that $U(x) + \lambda > 1$ we have UqA. Thus, $p_x^\lambda \in Cl_\theta(A)$ if and only if for every fuzzy θ-open set U of X such that $p_x^\lambda q U$ we have UqA.

Definition 4 *Let* A *be a fuzzy set of a fuzzy space* X. *A fuzzy point* p_x^λ *is called a pre-boundary point of a fuzzy set* A *if and only if* $p_x^\lambda \in Cl_\theta(A) \wedge (\bar{1} - Cl_\theta(A))$. *By* $\theta Bd(A)$ *we denote the fuzzy set* $Cl_\theta(A) \wedge (\bar{1} - Cl_\theta(A))$.

Proposition 3 *Let A be a fuzzy set of a fuzzy space X. Then $A \vee \theta Bd(A) \leq Cl_\theta(A)$.*

Proof. Let $p_x^\lambda \in A \vee \theta Bd(A)$. Then $p_x^\lambda \in A$ or $p_x^\lambda \in \theta Bd(A)$. Clearly, if $p_x^\lambda \in \theta Bd(A)$, then $p_x^\lambda \in Cl_\theta(A)$. Let us suppose that $p_x^\lambda \in A$. We have

$$Cl_\theta(A) = \wedge\{F : F \in I^X,\ F\ \{\theta - \text{closed and}\}\ \text{A} \leq \text{F}\}.$$

So, if $p_x^\lambda \in A$, then $p_x^\lambda \in F$, for every fuzzy θ-closed F of X for which $A \leq F$ and therefore $p_x^\lambda \in Cl_\theta(A)$.

Definition 5 *A fuzzy space X is called θ-T_0 if for every two fuzzy points p_x^λ and p_y^μ such that $p_x^\lambda \neq p_y^\mu$, either $p_x^\lambda \notin Cl_\theta(p_y^\mu)$ or $p_y^\mu \notin Cl_\theta(p_x^\lambda)$.*

Definition 6 *A fuzzy space X is called θ-T_1 if every fuzzy point is fuzzy θ-closed.*

Remark 3 *Clearly, every θ-T_1 fuzzy space is θ-T_0 .*

Proposition 4 *A fuzzy space X is θ-T_1 if and only if for each $x \in X$ and each $\lambda \in [0,1)$ there exists a fuzzy θ-open set A such that $A(x) = 1 - \lambda$ and $A(y) = 1$ for $y \neq x$.*

Proof. $\Rightarrow$) Let $\lambda = 0$. We set $A = \bar{1}$. Then A is fuzzy θ-open set such that $A(x) = 1 - 0$ and $A(y) = 1$ for $y \neq x$. Now, let $\lambda \in (0,1]$ and $x \in X$. We set $A = (p_x^\lambda)^c$. The set A is fuzzy θ-open such that $A(x) = 1 - \lambda$ and $A(y) = 1$ for $y \neq x$.

$\Leftarrow$) Let p_x^λ be an arbitrary fuzzy point of X. We prove that the fuzzy point p_x^λ is fuzzy θ-closed. By assumption there exists a fuzzy θ-open set A such that $A(x) = 1 - \lambda$ and $A(y) = 1$ for $y \neq x$. Clearly, $A^c = p_x^\lambda$. Thus the fuzzy point p_x^λ is fuzzy θ-closed and therefore the fuzzy space X is θ-T_1 .

Definition 7 *A fuzzy space X is called a θ-Hausdorff space if for any fuzzy points p_x^λ and p_y^μ for which $supp(p_x^\lambda) = x \neq supp(p_y^\mu) = y$, there exist two fuzzy θ-open Q-θ-neighbourhoods U and V of p_x^λ and p_y^μ, respectively, such that $U \wedge V = \bar{0}$.*

Definition 8 *A fuzzy space X is called a θ-regular space if for any fuzzy point p_x^λ and a fuzzy θ-closed set F not containing p_x^λ, there exist $U, V \in F\theta O(X)$ such that $p_x^\lambda \in U$, $F \leq V$ and $U \wedge V = \bar{0}$.*

Definition 9 *A fuzzy space X is called a quasi θ-T_1 if for any fuzzy points p_x^λ and p_y^μ for which $supp(p_x^\lambda) = x \neq supp(p_y^\mu) = y$, there exists a fuzzy θ-open set U such that $p_x^\lambda \in U$ and $p_y^\mu \notin U$ and another V such that $p_x^\lambda \notin V$ and $p_y^\mu \in V$.*

Definition 10 *(see [8]) A fuzzy point p_x^λ is called weak (respectively, strong) if $\lambda \leq \frac{1}{2}$ (respectively, $\lambda > \frac{1}{2}$).*

Definition 11 *A fuzzy set A of a fuzzy space X is called θ-generalized closed (briefly $f\theta g$-closed) if $Cl_\theta(A) \leq U$ whenever $A \leq U$ and U fuzzy θ-open set of X.*

Proposition 5 *Let X be a fuzzy space X. Suppose that p_x^λ and p_y^μ be weak and strong fuzzy points, respectively. If p_x^λ is θ-generalized closed, then*

$$p_y^\mu \in Cl_\theta(p_x^\lambda) \Rightarrow p_x^\lambda \in Cl_\theta(p_y^\mu).$$

Proof. Suppose that $p_y^\mu \in Cl_\theta(p_x^\lambda)$ and $p_x^\lambda \notin Cl_\theta(p_y^\mu)$. Then $Cl_\theta(p_y^\mu)(x) < \lambda$. Also $\lambda \leq \frac{1}{2}$. Therefore $Cl_\theta(p_y^\mu)(x) \leq 1-\lambda$ and therefore $\lambda \leq 1-Cl_\theta(p_y^\mu)(x)$. So $p_x^\lambda \in (Cl_\theta(p_y^\mu))^c$.

But p_x^λ is pre-generalized closed and $(Cl_\theta(p_y^\mu))^c$ fuzzy θ-open. Thus

$$Cl_\theta(p_x^\lambda) \leq (Cl_\theta(p_y^\mu))^c.$$

By assumption we have $p_y^\mu \in Cl_\theta(p_x^\lambda)$. Thus

$$p_y^\mu \in (Cl_\theta(p_y^\mu))^c.$$

Now, we prove that this is a contradiction.

Indeed, we have

$$\mu \leq 1 - Cl_\theta(p_y^\mu)(y)$$

or

$$Cl_\theta(p_y^\mu)(y) \leq 1 - \mu.$$

Moreover, $p_y^\mu \in Cl_\theta(p_y^\mu)$. Thus

$$\mu \leq 1 - \mu.$$

But p_y^μ is a strong fuzzy point, that is $\mu > \frac{1}{2}$. This means that the above relation $\mu \leq 1 - \mu$ is a contradiction. Hence $p_x^\lambda \in Cl_\theta(p_y^\mu)$.

Proposition 6 *If X is a quasi θ-T_1 fuzzy space and p_x^λ a weak fuzzy point in X, then $(p_x^\lambda)^c$ is a fuzzy θ-neighborhood of each fuzzy point p_y^μ with $y \neq x$.*

Proof. Let $y \neq x$ and p_y^μ be a fuzzy point of X. Since the space X is a quasi θ-T_1 there exists a fuzzy θ-open set U of X such that $p_y^\mu \in U$ and $p_x^\lambda \notin U$. This implies that $\lambda > U(x)$. Moreover, $\lambda \leq \frac{1}{2}$. Thus $U(x) \leq 1 - \lambda$. Therefore $U(y) \leq 1 = (p_x^\lambda)^c(y)$, for every $y \in X \setminus \{x\}$. So $U \leq (p_x^\lambda)^c$. Hence the fuzzy point $\left(p_x^\lambda\right)^c$ is a θ-neighborhood of p_y^μ.

Proposition 7 *If X is a θ-regular fuzzy space, then for any strong fuzzy point p_x^λ and any fuzzy θ-open set U containing p_x^λ, there exists a fuzzy θ-open set W containing p_x^λ such that $Cl_\theta(W) \leq U$.*

Proof. Suppose that p_x^λ is any strong fuzzy point contained in $U \in F\theta O(X)$. Then $\frac{1}{2} < \lambda \leq U(x)$. Thus the complement of U, that is the fuzzy set U^c, is a fuzzy θ-closed set to which does not belong the fuzzy point p_x^λ. Thus, there exist $W, V \in F\theta O(X)$ such that $p_x^\lambda \in W$ and $U^c \leq V$ with $W \wedge V = \bar{0}$. Hence, we have $W \leq V^c$ and by Theorem 3.8 of [18] $Cl_\theta(W) \leq Cl_\theta(V^c) = V^c$. Now $U^c \leq V$ implies $V^c \leq U$. This means that $Cl_\theta(W) \leq U$ which completes the proof.

Proposition 8 *If X is a fuzzy θ-regular space, then the strong fuzzy points in X are $f\theta g$-closed.*

Proof. Let p_x^λ be any strong fuzzy point in X and U be a fuzzy open set such that $p_x^\lambda \in U$. By Proposition 2.23 there exists a $W \in F\theta O(X)$ such that $p_x^\lambda \in W$ and $Cl_\theta(W) \leq U$. Now, we have

$$Cl_\theta(p_x^\lambda) \leq Cl_\theta(W) \leq U.$$

Thus the fuzzy point p_x^λ is $f\theta g$-closed.

Definition 12 *A fuzzy space X is called a weakly θ-regular space if for any weak fuzzy point p_x^λ and a fuzzy θ-closed set F not containing p_x^λ, there exist $U, V \in F\theta O(X)$ such that $p_x^\lambda \in U$, $F \leq V$ and $U \wedge V = \bar{0}$.*

Observe that every θ-regular fuzzy space is weakly θ-regular.

Definition 13 *Let X be a fuzzy space. A fuzzy set U in X is said to be fuzzy θ-nearly crisp if $Cl_\theta(U) \wedge (Cl_\theta(U))^c = \bar{0}$.*

Proposition 9 *Let X be a fuzzy space. If for any weak fuzzy point p_x^λ and any $U \in F\theta O(X)$ containing p_x^λ, there exists a fuzzy θ-open and θ-nearly crisp fuzzy set W containing p_x^λ such that $Cl_\theta(W) \leq U$, then X is fuzzy weakly θ-regular.*

Proof. Assume that F is a fuzzy θ-closed set not containing the weak fuzzy point p_x^λ. Then F^c is a fuzzy θ-open set containing p_x^λ. By hypothesis, there exists a fuzzy θ-open and θ-nearly crisp fuzzy set W such that $p_x^\lambda \in W$ and $Cl_\theta(W) \leq F^c$. We set $N = Int_\theta(Cl_\theta(W))$ and $M = 1 - Cl_\theta(W)$. Thus N is fuzzy θ-open, $p_x^\lambda \in N$ and $F \leq M$. We are going to prove that $M \wedge N = \bar{0}$. Now assume that there exists $y \in X$ such that $(N \wedge M)(y) = \mu \neq \bar{0}$. Then $p_y^\mu \in N \wedge M$. Hence, $p_y^\mu \in Cl_\theta(W)$ and $p_y^\mu \in (Cl_\theta(W))^c$. This is a contradiction since W is θ-nearly crisp. Thus the fuzzy space X is weakly θ-regular.

Definition 14 *Let X be a fuzzy space. A fuzzy point p_x^λ in X is said to be well-θ-closed if there exists $p_y^\mu \in Cl_\theta(p_x^\lambda)$ such that $supp(p_x^\lambda) \neq supp(p_y^\mu)$.*

Proposition 10 *If X is a fuzzy space and p_x^λ is a $f\theta g$-closed but and well θ-closed fuzzy point, then X is not quasi θ-T_1 space.*

Proof. Let X be a fuzzy quasi θ-T_1 space. By the fact p_x^λ is well-θ-closed, there exists a fuzzy point p_y^μ with $supp(p_x^\lambda) \neq supp(p_y^\mu)$ such that $p_y^\mu \in Cl_\theta(p_x^\lambda)$. Then there exists an $U \in F\theta O(X)$ such that $p_x^\lambda \in U$ and $p_y^\mu \notin U$. Therefore $Cl_\theta(p_x^\lambda) \leq U$ and $p_y^\mu \in U$. But this is a contradiction and hence X can not be quasi θ-T_1 space.

Definition 15 *Let X be a fuzzy space. A fuzzy point p_x^λ is said to be just-θ-closed if the fuzzy set $Cl_\theta(p_x^\lambda)$ is again fuzzy point.*

Clearly, in a fuzzy θ-T_1 space every fuzzy point is just-θ-closed.

Proposition 11 *Let X be a fuzzy space. If p_x^λ and p_x^μ are two fuzzy points such that $\lambda < \mu$ and p_x^μ is fuzzy θ-open, then p_x^λ is just-θ-closed if it is $f\theta g$-closed.*

Proof. We prove that the fuzzy set $Cl_\theta(p_x^\lambda)$ is again a fuzzy point. We have $p_x^\lambda \in p_x^\mu$ and the fuzzy set p_x^μ is fuzzy θ-open. Since p_x^λ is $f\theta g$-closed we have $Cl_\theta(p_x^\lambda) \leq p_x^\mu$. Thus $Cl_\theta(p_x^\lambda)(x) \leq \mu$ and $Cl_\theta(p_x^\lambda)(z) \leq 0$, for every $z \in X \setminus \{x\}$. So the fuzzy set $Cl_\theta(p_x^\lambda)$ is a fuzzy point.

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