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**S. JAFARI, R. M. LATIF, S. P. MOSHOKOA AND N. RAJESH**

# ON TWO CLASSES OF IRRESOLUTE FUNCTIONS

S. JAFARI, R. M. LATIF, S. P. MOSHOKOA AND N. RAJESH

ABSTRACT. The purpose of this paper is to give two new types of functions called completely  $\delta$ - $\beta$ -irresolute functions and completely weakly  $\delta$ - $\beta$ -irresolute functions in topological spaces. We obtain their characterizations and basic properties.

## 1. INTRODUCTION

Functions and of course irresolute functions stand among the most important and most researched points in the whole of Mathematical science. In 1972, Crossley and Hildebrand [4] introduced the notion of irresoluteness. Many different forms of irresolute functions have been introduced over the years. Various interesting problems arise when one considers irresoluteness. Its importance is significant in various areas of Mathematics and its related sciences. Recently, Hatir and Noiri [12] have introduced a weak form of open sets called  $\delta$ - $\beta$ -open sets. The purpose of this paper is to give two new types of irresolute functions called completely  $\delta$ - $\beta$ -irresolute functions and completely weakly  $\delta$ - $\beta$ -irresolute functions in topological spaces. We obtain their characterizations and their basic properties.

## 2. PRELIMINARIES

Throughout this paper, spaces mean topological spaces on which no separation axioms are assumed unless otherwise mentioned and  $f : (X, \tau) \rightarrow (Y, \sigma)$  (or simply  $f : X \rightarrow Y$ ) denotes a function  $f$  of a space

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$(X, \tau)$  into a space  $(Y, \sigma)$ . Let  $A$  be a subset of a space  $X$ . The closure and the interior of  $A$  are denoted by  $\text{Cl}(A)$  and  $\text{Int}(A)$ , respectively. A subset  $A$  of a topological space  $(X, \tau)$  is said to be regular open [18] (resp. preopen [11]) if  $A = \text{Int}(\text{Cl}(A))$  (resp.  $A \subset \text{Int}(\text{Cl}(A))$ ). A set  $A \subset X$  is said to be  $\delta$ -open [20] if it is the union of regular open sets of  $X$ . The complement of a regular open (resp.  $\delta$ -open) set is called regular closed (resp.  $\delta$ -closed). The intersection of all  $\delta$ -closed sets of  $(X, \tau)$  containing  $A$  is called the  $\delta$ -closure [20] of  $A$  and is denoted by  $\text{Cl}_\delta(A)$ . A subset  $S$  of a topological space  $(X, \tau)$  is called  $\delta$ - $\beta$ -open [12] if  $S \subset \text{Cl}(\text{Int}(\text{Cl}_\delta(S)))$ . The complement of a  $\delta$ - $\beta$ -open set is called  $\delta$ - $\beta$ -closed [12]. The intersection of all  $\delta$ - $\beta$ -closed sets containing  $S$  is called the  $\delta$ - $\beta$ -closure of  $S$  and is denoted by  ${}_\beta \text{Cl}_\delta(S)$ . The  $\delta$ - $\beta$ -interior of  $S$  is defined by the union of all  $\delta$ - $\beta$ -open sets contained in  $S$  and is denoted by  ${}_\beta \text{Int}_\delta(S)$ . The set of all  $\delta$ - $\beta$ -open sets of  $(X, \tau)$  is denoted by  $\delta\beta O(X)$ . The set of all  $\delta$ - $\beta$ -open sets of  $(X, \tau)$  containing a point  $x \in X$  is denoted by  $\delta\beta O(X, x)$ .

### 3. COMPLETELY $\delta$ - $\beta$ -IRRESOLUTE FUNCTIONS

**Definition 3.1.** A function  $f : (X, \tau) \rightarrow (Y, \sigma)$  is said to be:

- (i) *strongly continuous* [8] if  $f^{-1}(V)$  is clopen in  $X$  for each subset  $V$  of  $Y$ ;
- (ii) *completely continuous* [3] if  $f^{-1}(V)$  is regular open in  $X$  for every open set  $V$  of  $Y$ ;
- (iii)  *$\delta$ - $\beta$ -irresolute* [7] (resp.  *$\delta$ - $\beta$ -continuous* [13]) if  $f^{-1}(V)$  is  $\delta$ - $\beta$ -open (resp.  $\delta$ - $\beta$ -open) in  $X$  for every  $\delta$ - $\beta$ -open (resp. open) subset  $V$  of  $Y$ .
- (iv) *completely  $\delta$ - $\beta$ -irresolute* if  $f^{-1}(V)$  is regular open in  $X$  for every  $\delta$ - $\beta$ -open subset  $V$  of  $Y$ .

**Remark 3.2.** Clearly, every strongly continuous function is completely  $\delta$ - $\beta$ -irresolute and every completely  $\delta$ - $\beta$ -irresolute function is  $\delta$ - $\beta$ -irresolute.

But the converse of the implications are not true in general as seen from the following examples.

**Example 3.3.** Let  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, X\}$ . Then the identity function  $f$  on  $X$  is completely  $\delta$ - $\beta$ -irresolute but not strongly continuous.

**Example 3.4.** Let  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, \{a\}, X\}$ . Then the identity function  $f$  on  $X$  is  $\delta$ - $\beta$ -irresolute but not completely  $\delta$ - $\beta$ -irresolute.

**Theorem 3.5.** The following statements are equivalent for a function  $f : X \rightarrow Y$ :

- (i)  $f$  is completely  $\delta$ - $\beta$ -irresolute;
- (ii)  $f^{-1}(F)$  is regular closed in  $X$  for every  $\delta$ - $\beta$ -closed set  $F$  of  $Y$ .

*Proof.* Clear. □

**Lemma 3.6.** [9] Let  $S$  be an open subset of a topological space  $(X, \tau)$ . Then the following hold:

- (i) If  $U$  is regular open in  $X$ , then so is  $U \cap S$  in the subspace  $(S, \tau_s)$ .
- (ii) If  $B \subset S$  is regular open in  $(S, \tau_s)$ , then there exists a regular open set  $U$  in  $(X, \tau)$  such that  $B = U \cap S$ .

**Theorem 3.7.** If  $f : (X, \tau) \rightarrow (Y, \sigma)$  is a completely  $\delta$ - $\beta$ -irresolute function and  $A$  is any open subset of  $X$ , then  $f|_A : A \rightarrow Y$  is completely  $\delta$ - $\beta$ -irresolute.

*Proof.* Let  $F$  be a  $\delta$ - $\beta$ -open subset of  $Y$ . By hypothesis  $f^{-1}(F)$  is regular open in  $X$ . Since  $A$  is open in  $X$ , it follows from the previous Lemma that  $(f|_A)^{-1}(F) = A \cap f^{-1}(F)$ , which is regular open in  $A$ . Therefore,  $f|_A$  is completely  $\delta$ - $\beta$ -irresolute. □

**Lemma 3.8.** [2] Let  $Y$  be a preopen subset of a topological space  $(X, \tau)$ . Then  $Y \cap U$  is regular open in  $Y$  for each regular open subset  $U$  of  $X$ .

**Theorem 3.9.** *If  $f : (X, \tau) \rightarrow (Y, \sigma)$  is completely  $\delta$ - $\beta$ -irresolute function and  $A$  is preopen subset of  $X$ , then  $f|_A : A \rightarrow Y$  is completely  $\delta$ - $\beta$ -irresolute.*

*Proof.* Similar to the Proof of Theorem 3.7 □

**Theorem 3.10.** *The following hold for functions  $f : (X, \tau) \rightarrow (Y, \sigma)$  and  $g : (Y, \sigma) \rightarrow (Z, \eta)$ :*

- (i) *If  $f$  is completely  $\delta$ - $\beta$ -irresolute and  $g$  is  $\delta$ - $\beta$ -irresolute, then  $g \circ f$  is completely  $\delta$ - $\beta$ -irresolute;*
- (ii) *If  $f$  is completely continuous and  $g$  is completely  $\delta$ - $\beta$ -irresolute, then  $g \circ f$  is completely  $\delta$ - $\beta$ -irresolute.*
- (iii) *If  $f$  is completely  $\delta$ - $\beta$ -irresolute and  $g$  is  $\delta$ - $\beta$ -continuous, then  $g \circ f$  is completely continuous function.*

*Proof.* The proof of the theorem is easy and hence omitted. □

**Definition 3.11.** *A space  $X$  is said to be almost connected [5] (resp.  $\delta$ - $\beta$ -connected) if there does not exist disjoint regular open (resp.  $\delta$ - $\beta$ -open) sets  $A$  and  $B$  such that  $A \cup B = X$ .*

**Theorem 3.12.** *If  $f : (X, \tau) \rightarrow (Y, \sigma)$  is completely  $\delta$ - $\beta$ -irresolute surjective function and  $X$  is almost connected, then  $Y$  is  $\delta$ - $\beta$ -connected.*

*Proof.* Suppose that  $Y$  is not  $\delta$ - $\beta$ -connected. Then there exist disjoint  $\delta$ - $\beta$ -open sets  $A$  and  $B$  of  $Y$  such that  $A \cup B = Y$ . Since  $f$  is completely  $\delta$ - $\beta$ -irresolute surjective,  $f^{-1}(A)$  and  $f^{-1}(B)$  are regular open sets in  $X$ . Moreover,  $f^{-1}(A) \cup f^{-1}(B) = X$ ,  $f^{-1}(A) \neq \emptyset$  and  $f^{-1}(B) \neq \emptyset$ . This shows that  $X$  is not almost connected, which is a contradiction to the assumption that  $X$  is almost connected. By contradiction,  $Y$  is  $\delta$ - $\beta$ -connected. □

**Definition 3.13.** *Let  $(X, \tau)$  be a topological space. A class  $\{G_i\}$  of  $\delta$ - $\beta$ -open subsets of  $X$  is said to be  $\delta$ - $\beta$ -cover of  $X$  if each point in  $X$  belongs to at least one  $G_i$  that is  $\bigcup_i G_i = X$ .*

**Definition 3.14.** A space  $X$  is said to be

- (i) *nearly compact* [14] if every regular open cover of  $X$  has a finite subcover;
- (ii) *nearly countably compact* [6] if every cover by regular open sets has a countable subcover;
- (iii) *nearly Lindelof* [5] if every cover of  $X$  by regular open sets has a countable subcover;
- (iv)  $\delta$ - $\beta$ -compact if every  $\delta$ - $\beta$ -open cover of  $X$  has a finite subcover;
- (v) *countably  $\delta$ - $\beta$ -compact* if every  $\delta$ - $\beta$ -open countable cover of  $X$  has a finite subcover;
- (vi)  $\delta$ - $\beta$ -Lindelof if every cover of  $X$  by  $\delta$ - $\beta$ -open sets has a countable subcover.

**Theorem 3.15.** Let  $f : (X, \tau) \rightarrow (Y, \sigma)$  be a completely  $\delta$ - $\beta$ -irresolute surjective function. Then the following statements hold:

- (i) If  $X$  is nearly compact, then  $Y$  is  $\delta$ - $\beta$ -compact.
- (ii) If  $X$  is nearly Lindelof, then  $Y$  is  $\delta$ - $\beta$ -Lindelof.
- (iii) If  $X$  is nearly countably compact, then  $Y$  is countably  $\delta$ - $\beta$ -compact.

*Proof.* (i) Let  $f : X \rightarrow Y$  be a completely  $\delta$ - $\beta$ -irresolute function of nearly compact space  $X$  onto a space  $Y$ . Let  $\{U_\alpha : \alpha \in \Delta\}$  be any  $\delta$ - $\beta$ -open cover of  $Y$ . Then,  $\{f^{-1}(U_\alpha) : \alpha \in \Delta\}$  is a regular open cover of  $X$ . Since  $X$  is nearly compact, there exists a finite subfamily,  $\{f^{-1}(U_{\alpha_i}) | i = 1, 2, \dots, n\}$  of  $\{f^{-1}(U_\alpha) : \alpha \in \Delta\}$  which covers  $X$ . It follows then that  $\{U_{\alpha_i} : i = 1, 2, \dots, n\}$  is a finite subfamily of  $\{U_\alpha : \alpha \in \Delta\}$  which covers  $Y$ . Hence, space  $Y$  is a  $\delta$ - $\beta$ -compact space.

The proof of other cases are similar. □

**Definition 3.16.** A space  $(X, \tau)$  is said to be:

- (i) *S-closed* [19] (resp.  $\delta$ - $\beta$ -closed compact) if every regular closed (resp.  $\delta$ - $\beta$ -closed) cover of  $X$  has a finite subcover;

- (ii) *countably  $S$ -closed-compact [1] (resp. countably  $\delta$ - $\beta$ -closed compact) if every countable cover of  $X$  by regular closed (resp.  $\delta$ - $\beta$ -closed) sets has a finite subcover;*
- (iii)  *$S$ -Lindelof [10] (resp.  $\delta$ - $\beta$ -Lindelof) if every cover of  $X$  by regular closed (resp.  $\delta$ - $\beta$ -closed) sets has a countable subcover.*

**Theorem 3.17.** *Let  $f : (X, \tau) \rightarrow (Y, \sigma)$  be a completely  $\delta$ - $\beta$ -irresolute surjective function. Then the following statements hold:*

- (i) *If  $X$  is  $S$ -closed, then  $Y$  is  $\delta$ - $\beta$ -closed compact.*
- (ii) *If  $X$  is  $S$ -Lindelof, then  $Y$  is  $\delta$ - $\beta$ -closed Lindelof.*
- (iii) *If  $X$  is countably  $S$ -closed, then  $Y$  is countably  $\delta$ - $\beta$ -closed compact.*

*Proof.* It can be obtained similarly as the previous Theorem. □

**Definition 3.18.** *A topological space  $X$  is said to be almost regular [15] (resp. strongly  $\delta$ - $\beta$ -regular) if for any regular closed (resp.  $\delta$ - $\beta$ -closed) set  $F \subset X$  and any point  $x \in X \setminus F$ , there exists disjoint open (resp.  $\delta$ - $\beta$ -open) sets  $U$  and  $V$  such that  $x \in U$  and  $F \subset V$ .*

**Definition 3.19.** *A function  $f : X \rightarrow Y$  is said to be  $\delta$ - $\beta$ -open if the image of each open set of  $X$  is  $\delta$ - $\beta$ -open in  $Y$ .*

**Theorem 3.20.** *If  $f$  is completely  $\delta$ - $\beta$ -irresolute  $\delta$ - $\beta$ -open from an almost regular space  $X$  onto a space  $Y$ , then  $Y$  is strongly  $\delta$ - $\beta$ -regular.*

*Proof.* Let  $F$  be a  $\delta$ - $\beta$ -closed set in  $Y$  with  $y \notin F$ . Take  $y = f(x)$ . Since  $f$  is completely  $\delta$ - $\beta$ -irresolute,  $f^{-1}(F)$  is regular closed and so closed set in  $X$  and  $x \notin f^{-1}(F)$ . By almost regularity of  $X$ , there exists disjoint open sets  $U$  and  $V$  such that  $x \in U$  and  $f^{-1}(F) \subset V$ . We obtain that  $y = f(x) \in f(U)$  and  $F \subset f(V)$  such that  $f(U)$  and  $f(V)$  are disjoint  $\delta$ - $\beta$ -open sets. Thus,  $Y$  is strongly  $\delta$ - $\beta$ -regular. □

**Definition 3.21.** *A topological space  $X$  is said to be:*

- (i) *almost normal* [16] if for each closed set  $A$  and each regular closed set  $B$  such that  $A \cap B = \emptyset$ , there exist disjoint open sets  $U$  and  $V$  such that  $A \subset U$  and  $B \subset V$ .
- (ii) *strongly  $\delta$ - $\beta$ -normal* if for every pair of disjoint  $\delta$ - $\beta$ -closed subsets  $A$  and  $B$  of  $X$ , there exist disjoint  $\delta$ - $\beta$ -open sets  $U$  and  $V$  such that  $A \subset U$  and  $B \subset V$ .

**Theorem 3.22.** *If  $f : X \rightarrow Y$  is completely  $\delta$ - $\beta$ -irresolute  $\delta$ - $\beta$ -open function from an almost normal space  $X$  onto a space  $Y$ , then  $Y$  is strongly  $\delta$ - $\beta$ -normal.*

*Proof.* Let  $A$  and  $B$  be two disjoint  $\delta$ - $\beta$ -closed subsets in  $Y$ . Since  $f$  is completely  $\delta$ - $\beta$ -irresolute,  $f^{-1}(A)$  and  $f^{-1}(B)$  are disjoint regular closed and so closed sets in  $X$ . By almost normality of  $X$ , there exist disjoint open sets  $U$  and  $V$  such that  $f^{-1}(A) \subset U$  and  $f^{-1}(B) \subset V$ . We obtain that  $A \subset f(U)$  and  $B \subset f(V)$  such that  $f(U)$  and  $f(V)$  are disjoint  $\delta$ - $\beta$ -open sets. Thus,  $Y$  is strongly  $\delta$ - $\beta$ -normal.  $\square$

**Definition 3.23.** *A topological space  $(X, \tau)$  is said to be:*

- (i)  $\delta$ - $\beta$ - $T_1$  [13] (resp.  $r$ - $T_1$  [5]) if for each pair of distinct points  $x$  and  $y$  of  $X$ , there exist  $\delta$ - $\beta$ -open (resp. regular open) sets  $U_1$  and  $U_2$  such that  $x \in U_1$  and  $y \in U_2$ ,  $x \notin U_2$  and  $y \notin U_1$ .
- (ii)  $\delta$ - $\beta$ - $T_2$  [13] (resp.  $r$ - $T_2$  [5]) for each pair of distinct points  $x$  and  $y$  in  $X$ , there exist disjoint  $\delta$ - $\beta$ -open (resp. regular open) sets  $A$  and  $B$  in  $X$  such that  $x \in A$  and  $y \in B$ .

**Theorem 3.24.** *If  $f : (X, \tau) \rightarrow (Y, \sigma)$  is completely  $\delta$ - $\beta$ -irresolute injective function and  $Y$  is  $\delta$ - $\beta$ - $T_1$ , then  $X$  is  $r$ - $T_1$ .*

*Proof.* Suppose that  $Y$  is  $\delta$ - $\beta$ - $T_1$ . For any two distinct points  $x$  and  $y$  of  $X$ , there exist  $\delta$ - $\beta$ -open sets  $F_1$  and  $F_2$  in  $Y$  such that  $f(x) \in F_1$ ,  $f(y) \in F_2$ ,  $f(x) \notin F_2$  and  $f(y) \notin F_1$ . Since  $f$  is injective completely  $\delta$ - $\beta$ -irresolute function, we have  $X$  is  $r$ - $T_1$ .  $\square$



**Theorem 3.25.** *If  $f : (X, \tau) \rightarrow (Y, \sigma)$  is completely  $\delta$ - $\beta$ -irresolute injective function and  $Y$  is  $\delta$ - $\beta$ - $T_2$ , then  $X$  is  $r$ - $T_2$ .*

*Proof.* Similar to the proof of Theorem 3.24. □

**Theorem 3.26.** *Let  $Y$  be a  $\delta$ - $\beta$ - $T_2$  space. Then we have the following*

- (i) *If  $f, g : X \rightarrow Y$  are completely  $\delta$ - $\beta$ -irresolute functions, then the set  $A = \{x \in X : f(x) = g(x)\}$  is  $\delta$ -closed in  $X$ .*
- (ii) *If  $f : X \rightarrow Y$  is a completely  $\delta$ - $\beta$ -irresolute function, then the set  $B = \{(x, y) \in X \times X : f(x) = f(y)\}$  is  $\delta$ -closed in  $X \times X$ .*

*Proof.* (i). Let  $x \notin A$ , then  $f(x) \neq g(x)$ . Since  $Y$  is  $\delta$ - $\beta$ - $T_2$ , there exist disjoint  $\delta$ - $\beta$ -open sets  $U_1$  and  $U_2$  in  $Y$  such that  $f(x) \in U_1$  and  $g(x) \in U_2$ . Since  $f$  and  $g$  are completely  $\delta$ - $\beta$ -irresolute,  $f^{-1}(U_1)$  and  $g^{-1}(U_2)$  are regular open sets. Put  $U = f^{-1}(U_1) \cap g^{-1}(U_2)$ . Then  $U$  is a regular open set containing  $x$  and  $U \cap A = \emptyset$ . Hence we have  $x \notin \text{Cl}_\delta(A)$ . This completes the proof.

(ii). Follows from (i). □

#### 4. COMPLETELY WEAKLY $\delta$ - $\beta$ -IRRESOLUTE FUNCTIONS

**Definition 4.1.** *A function  $f : (X, \tau) \rightarrow (Y, \sigma)$  is said to be completely weakly  $\delta$ - $\beta$ -irresolute if for each  $x \in X$  and for any  $\delta$ - $\beta$ -open set  $V$  containing  $f(x)$ , there exists an open set  $U$  containing  $x$  such that  $f(U) \subset V$ .*

**Remark 4.2.** *It is obvious that every completely  $\delta$ - $\beta$ -irresolute function is completely weakly  $\delta$ - $\beta$ -irresolute and every completely weakly  $\delta$ - $\beta$ -irresolute function is  $\delta$ - $\beta$ -irresolute. However, the converse may not be true in general as shown in the following example.*

**Example 4.3.** *Let  $X = \{a, b, c\}$ ,  $\tau = \{\emptyset, \{a\}, \{b, c\}, X\}$  and  $\sigma = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$ . Clearly the identity function  $f : (X, \tau) \rightarrow (X, \sigma)$  is completely weakly  $\delta$ - $\beta$ -irresolute but not completely  $\delta$ - $\beta$ -irresolute. Also the function  $f$  defined in Example 3.4 is  $\delta$ - $\beta$ -irresolute but not completely weakly  $\delta$ - $\beta$ -irresolute.*

**Remark 4.4.** *From the above arguments, we obtain the following implication diagram:*

$$\begin{array}{ccccc} \text{completely} & \rightarrow & \text{completely weakly} & \rightarrow & \delta - \beta - \text{irresolute} \\ \delta - \beta - \text{irresolute} & & \delta - \beta - \text{irresolute} & & \end{array}$$

**Theorem 4.5.** *For a function  $f : (X, \tau) \rightarrow (Y, \sigma)$ , the following statements are equivalent:*

- (i)  $f$  is completely weakly  $\delta$ - $\beta$ -irresolute;
- (ii) for each  $x \in X$  and each  $\delta$ - $\beta$ -open set  $V$  of  $Y$  containing  $f(x)$ , there exists an open set  $U$  of  $X$  containing  $x$  such that  $f(U) \subset V$ ;
- (iii)  $f(\text{Cl}(A)) \subseteq {}_{\beta}\text{Cl}_{\delta}(f(A))$  for every subset  $A$  of  $X$ ;
- (iv)  $\text{Cl}(f^{-1}(B)) \subseteq f^{-1}({}_{\beta}\text{Cl}_{\delta}(B))$  for every subset  $B$  of  $Y$ ;
- (v) for each  $\delta$ - $\beta$ -closed set  $V$  in  $Y$ ,  $f^{-1}(V)$  is closed in  $X$ ;
- (vi)  $f^{-1}({}_{\beta}\text{Int}_{\delta}(B)) \subset \text{Int}(f^{-1}(B))$  for every subset  $B$  of  $Y$ .

*Proof.* Clear. □

**Theorem 4.6.** *Let  $f : (X, \tau) \rightarrow (Y, \sigma)$  and  $g : (Y, \sigma) \rightarrow (Z, \eta)$  be any two functions. Then*

- (i) *If  $f$  is completely weakly  $\delta$ - $\beta$ -irresolute and  $g$  is  $\delta$ - $\beta$ -irresolute, then  $g \circ f : (X, \tau) \rightarrow (Z, \eta)$  is completely weakly  $\delta$ - $\beta$ -irresolute.*
- (ii) *If  $f$  is completely continuous and  $g$  is completely weakly  $\delta$ - $\beta$ -irresolute, then  $g \circ f$  is completely  $\delta$ - $\beta$ -irresolute.*
- (iii) *If  $f$  is strongly continuous and  $g$  is completely  $\delta$ - $\beta$ -irresolute, then  $g \circ f$  is completely  $\delta$ - $\beta$ -irresolute.*
- (iv) *If  $f$  and  $g$  are completely  $\delta$ - $\beta$ -irresolute, then  $g \circ f$  is completely  $\delta$ - $\beta$ -irresolute.*
- (v) *If  $f$  is completely  $\delta$ - $\beta$ -irresolute and  $g$  is completely weakly  $\delta$ - $\beta$ -irresolute, then  $g \circ f$  is completely  $\delta$ - $\beta$ -irresolute.*
- (vi) *If  $f$  is completely weakly  $\delta$ - $\beta$ -irresolute and  $g$  is  $\delta$ - $\beta$ -continuous, then  $g \circ f$  is continuous.*

(vii) *If  $f$  is  $\delta$ - $\beta$ -continuous and  $g$  is completely weakly  $\delta$ - $\beta$ -irresolute, then  $g \circ f$  is  $\delta$ - $\beta$ -irresolute.*

(viii) *If  $f$  is continuous and  $g$  is completely weakly  $\delta$ - $\beta$ -irresolute, then  $g \circ f$  is completely weakly  $\delta$ - $\beta$ -irresolute.*

*Proof.* Follows from their respective definitions.  $\square$

Recall that a function  $f : (X, \tau) \rightarrow (Y, \sigma)$  is said to be almost open if  $f^{-1}(V)$  is regular open in  $X$  for every open set  $V$  of  $Y$ .

**Theorem 4.7.** *If  $f : (X, \tau) \rightarrow (Y, \sigma)$  is almost open and  $g : (Y, \sigma) \rightarrow (Z, \gamma)$  is any function such that  $g \circ f : (X, \tau) \rightarrow (Z, \gamma)$  is completely  $\delta$ - $\beta$ -irresolute, then  $g$  is completely weakly  $\delta$ - $\beta$ -irresolute.*

*Proof.* Let  $V$  be a  $\delta$ - $\beta$ -open set in  $(Z, \gamma)$ . Since  $g \circ f$  is completely  $\delta$ - $\beta$ -irresolute,  $(g \circ f)^{-1} = f^{-1}(g^{-1}(V))$  is regular open in  $(X, \tau)$ . Since  $f$  is almost open surjection,  $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$  is open in  $Y$ . Therefore,  $g$  is completely weakly  $\delta$ - $\beta$ -irresolute.  $\square$

**Theorem 4.8.** *If  $f : (X, \tau) \rightarrow (Y, \sigma)$  is open surjection and  $g : (Y, \sigma) \rightarrow (Z, \gamma)$  is any function such that  $g \circ f : (X, \tau) \rightarrow (Z, \gamma)$  is completely weakly  $\delta$ - $\beta$ -irresolute, then  $g$  is completely weakly  $\delta$ - $\beta$ -irresolute.*

*Proof.* Similar to proof of Theorem 4.7.  $\square$

**Definition 4.9.** (i): *A filterbase  $\Omega$  is said to be  $\Omega$ -convergent to a point  $x$  in  $X$  if for any  $U \in \lambda O(X, x)$ , there exists  $B \in \Omega$  such that  $B \subset U$ .*

(ii): *A filterbase  $\Omega$  is said to be convergent to a point  $x$  in  $X$  if for any open set  $U$  of  $X$  containing  $x$ , there exists  $B \in \Omega$  such that  $B \subset U$ .*

**Theorem 4.10.** *If a function  $f : (X, \tau) \rightarrow (Y, \sigma)$  is completely weakly  $\delta$ - $\beta$ -irresolute, then for each point  $x \in X$  and each filterbase  $\Omega$  in  $X$  converging to  $x$ , the filterbase  $f(\Omega)$  is  $\delta$ - $\beta$ -convergent to  $f(x)$ .*

*Proof.* Let  $x \in X$  and  $\Omega$  be any filterbase in  $X$  converging to  $x$ . Since  $f$  is completely weakly  $\delta$ - $\beta$ -irresolute, then for any  $\delta$ - $\beta$ -open set  $V$  of  $(Y, \sigma)$  containing  $f(x)$ , there exists an open set  $U$  of  $X$  containing  $x$  such that  $f(U) \subset V$ . Since  $\Omega$  is converging to  $x$ , there exists  $B \in \Omega$  such that  $B \subset U$ . This means that  $f(B) \subset V$  and hence the filterbase  $f(\Omega)$  is  $\delta$ - $\beta$ -convergent to  $f(x)$ .  $\square$

Recall that for a function  $f : (X, \tau) \rightarrow (Y, \sigma)$ , the subset  $\{(x, f(x)) : x \in X\} \subset X \times Y$  is called the graph of  $f$  and is denoted by  $G(f)$ .

**Definition 4.11.** *A graph  $G(f)$  of a function  $f : (X, \tau) \rightarrow (Y, \sigma)$  is said to be contra  $\delta$ - $\beta$ -closed if for each  $(x, y) \in (X \times Y) \setminus G(f)$ , there exist open set  $U$  of  $X$  containing  $x$  and  $V \in \lambda C(Y, y)$  such that  $(U \times V) \cap G(f) = \emptyset$ .*

**Lemma 4.12.** *A graph  $G(f)$  of a function  $f : (X, \tau) \rightarrow (Y, \sigma)$  is contra  $\delta$ - $\beta$  closed in  $X \times Y$  if and only if for each  $(x, y) \in (X \times Y) \setminus G(f)$ , there exist open set  $U$  of  $X$  containing  $x$  and  $V \in \lambda C(Y, y)$  such that  $f(U) \cap V = \emptyset$ .*

*Proof.* It is an immediate consequence of Definition 4.11.  $\square$

**Theorem 4.13.** *If  $f : (X, \tau) \rightarrow (Y, \sigma)$  is a completely weakly  $\delta$ - $\beta$ -irresolute function and  $(Y, \sigma)$  is a  $\delta$ - $\beta$ - $T_1$ -space, then  $G(f)$  is contra  $\delta$ - $\beta$ -closed.*

*Proof.* Let  $(x, y) \in (X \times Y) \setminus G(f)$ . Then  $y \neq f(x)$ . Since  $Y$  is  $\delta$ - $\beta$ - $T_1$ , there exists a  $\delta$ - $\beta$ -open set  $V$  in  $Y$  such that  $f(x) \in V$  and  $y \notin V$ . Since  $f$  is completely weakly  $\delta$ - $\beta$ -irresolute, there exists open set  $U$  of  $X$  containing  $x$  such that  $f(U) \subset V$ . Therefore,  $f(U) \cap (Y \setminus V) = \emptyset$  and  $Y \setminus V$  is a  $\delta$ - $\beta$ -closed subset of  $Y$  containing  $y$ . This shows that  $G(f)$  is contra  $\delta$ - $\beta$ -closed.  $\square$

**Theorem 4.14.** *Let  $f : (X, \tau) \rightarrow (Y, \sigma)$  be a completely weakly  $\delta$ - $\beta$ -irresolute surjective function. Then the following statements hold:*

- (i) *If  $X$  is Lindelof, then  $Y$  is  $\delta$ - $\beta$ -Lindelof;*
- (ii) *If  $X$  is countably compact, then  $Y$  is countably  $\delta$ - $\beta$ -compact.*

*Proof.* (i): Let  $\{V_\alpha : \alpha \in I\}$  be a  $\delta$ - $\beta$ -open cover of  $Y$ . Since  $f$  is completely weakly  $\delta$ - $\beta$ -irresolute, then  $\{f^{-1}(V_\alpha) : \alpha \in I\}$  is an open cover of  $X$ . Since  $X$  is Lindelof, there exists a countable subset  $I_0$  of  $I$  such that  $X = \bigcup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$ . Thus,  $Y = \bigcup\{V_\alpha : \alpha \in I_0\}$  and hence  $Y$  is  $\delta$ - $\beta$ -Lindelof.

(ii): Similar to (i). □

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SAEID JAFARI, DEPARTMENT OF MATHEMATICS, COLLEGE OF VESTSJAELLAND SOUTH, HERRESTRÆDE 11, 4200 SLAGELSE, DENMARK  
*E-mail address:* [jafari@stofanet.dk](mailto:jafari@stofanet.dk)

RAJA M. LATIF, DEPARTMENT OF MATHEMATICS AND STATISTICS, KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS, DHAHRAN 31261, SAUDI ARABIA  
*E-mail address:* [raja@kfupm.edu.sa](mailto:raja@kfupm.edu.sa)

SEITHUTI P. MOSHOKOA, DEPARTMENT OF MATHEMATICAL SCIENCES, UNIVERSITY OF SOUTH AFRICA, P. O. BOX 392, PRETORIA 0003, SOUTH AFRICA  
*E-mail address:* [moshosp@unisa.ac.za](mailto:moshosp@unisa.ac.za)

N. RAJESH, DEPARTMENT OF MATHEMATICS, KONGU ENGINEERING COLLEGE, PERUNDURAI, ERODE-638 052, TAMILNADU, INDIA  
*E-mail address:* [nrajesh\\_topology@yahoo.co.in](mailto:nrajesh_topology@yahoo.co.in)