Name: ________________________________  Section #: __________

ID #: ________________________________

• Show complete work for full credit.

• Use of graphic calculators and mobile phones or any other equipment is not allowed in this exam.

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1. Find each of the following limits, if it exists. (5 points each)

(a) \( \lim_{x \to 3}(x + 2) \cdot \frac{3 - x}{|3 - x|} \).

(b) \( \lim_{x \to 2} \frac{x^4 - 16}{x^2 - x - 2} \).

(c) \( \lim_{h \to 0} \frac{\sqrt{2 + h} - \sqrt{2}}{h} \).

(d) \( \lim_{x \to -\infty} \frac{-3x + 1}{(2x + 1)^2(x + 1)} \).

(e) \( \lim_{x \to 0^+} \left( \sqrt{x} - \frac{1}{\sqrt{x}} \right) \).
(f) \[ \lim_{x \to 0} \frac{\sin x - 7x}{x \cos x} . \]

(g) \[ \lim_{x \to +\infty} \frac{2x + x \sin 3x}{5x^2 - 2x + 1} . \]

(h) \[ \lim_{x \to 3} \frac{\sin(\pi x)}{x - 3} . \] (Hint: let \( t = \pi x - 3\pi \))

(i) \[ \lim_{x \to +\infty} \frac{\sin x}{x} . \] (Hint: Use the Squeezing Theorem)

(j) \[ \lim_{x \to +\infty} \sqrt{\frac{8 + x^2}{x(x + 1)}} . \]
2. (a) Use the limit definition to prove that

\[ \lim_{x \to 15} \sqrt{x + 1} = 4. \]  

(5 points)

(b) Find all asymptotes for the graph of

\[ f(x) = \frac{4x - 3}{\sqrt{x^2 + 1}}. \]  

(12 points)

(c) Prove that the equation

\[ x^5 - 3x^4 - 2x^3 - x + 1 = 0 \]

has a solution between 0 and 1.  

(5 points)
3. (a) Find all numbers at which \( f \) is discontinuous, where

\[
    f(x) = \begin{cases} 
        -x^2 & \text{if } x < 1 \\
        2 & \text{if } x = 1 \\
        -\frac{1}{x} & \text{if } x > 1 
    \end{cases}
\]

(8 points)

(b) Show that \( f(x) = \sqrt{16 - x} \) is continuous on the interval \((-\infty, 16]\). (5 points)

(c) Suppose that \( f \) is a continuous function, \( \lim_{x \to 4} f(x) = 0 \) and \( \lim_{x \to 4} g(x) = -3 \). Find

\[
    \lim_{x \to 4} \frac{xg(x)}{\sqrt{f(x)} + 1}.
\]

(5 points)
Consider the function \( f(x) = x^2 + 1 \) and the point \( P(2, f(2)) \).

(i) Find the slope of the graph of \( y = f(x) \) at the point \( P \).

(ii) Find the instantaneous rate of change of \( y \) with respect to \( x \) at the general point \( x = x_0 \).

4. **Extra Credits** (10 points)

   (a) If \( \lim_{x \to a} [f(x) + g(x)] = 4 \) and \( \lim_{x \to a} [f(x) - g(x)] = 1 \), find \( \lim_{x \to a} f(x)g(x) \).

   (b) Evaluate \( \lim_{x \to 1} \frac{\sqrt{x} - 1}{\sqrt{x} + 1} \).