1. Use the $\epsilon - \delta$ definition to prove that $\lim_{x \to 2} \frac{1}{x} = \frac{1}{2}$.

2. Find the limit of each of the following:

   (a) $\lim_{x \to 5} \frac{x^2 - 25}{|x - 5|}$
   
   (b) $\lim_{x \to 1} \frac{x^3 - 1}{x^2 - 1}$
   
   (c) $\lim_{x \to 2} \frac{x^2 - x + 6}{x - 2}$
   
   (d) $\lim_{x \to 0^-} \left[ \frac{1}{x} - \frac{1}{|x|} \right]$}
   
   (e) $\lim_{x \to -\infty} \frac{\sqrt{9x^6 - x}}{x^3 + 1}$
   
   (f) $\lim_{x \to 0} \frac{\sin^3 x}{x^3}$
   
   (g) $\lim_{x \to 0^+} (1 + x)^{1/x}$
   
   (h) $\lim_{x \to 1} \sin^{-1} \left( \frac{1 - \sqrt{x}}{1 - x} \right)$
   
   (i) $\lim_{x \to \infty} \frac{e^{3x}}{x^4}$
   
   (j) $\lim_{x \to 0} \frac{\tan x - x}{x^3}$
   
   (k) $\lim_{x \to 0^+} (\cos x)^{1/x^2}$

3. Find the derivative $y'$ for each of the following:

   (a) $y = \sqrt{e^{2x} - \csc^3 x}$
   
   (b) $y = \frac{\sec^2 5x^2 + 1}{1 + \cos^{-1} x}$
   
   (c) $xy = \cot(xy)$
   
   (d) $yx + 1 = 3 \tan^{-1} y$
   
   (e) $y = \sin(\tan \sqrt{\sin x})$
   
   (f) $y = 10^{\sin x^2}$
4. If \( x^4 + y^4 = 16 \), show that \( y'' = -48 \frac{x^2}{y^4} \).

5. Suppose \( f \) is a one-to-one differentiable function and its inverse \( f^{-1} \) also differentiable. Use implicit differentiation to show that
\[
\frac{d}{dx} [f^{-1}(x)] = \frac{1}{f'([f^{-1}(x)])}, \quad \text{where} \quad f' \neq 0.
\]

6. Use the definition of derivative to show that \( \frac{d}{dx} [\log_b x] = \frac{1}{x \ln b} \), \( x > 0 \).

7. Show that the equation \( 4x^3 - 6x^2 + 3x - 2 = 0 \) has a real root between 1 and 2.

8. Show that the function \( f(x) = |x - 3| \) is continuous everywhere.

9. Given
\[
f(x) = \begin{cases} x^2 & \text{if } x \geq 0 \\ e^x & \text{if } x < 0. \end{cases}
\]
Discuss the continuity of \( f \) at \( x = 0 \).

10. Find the horizontal and vertical asymptotes of the graph of \( f(x) = \frac{\sqrt{2x^2 + 1}}{3x - 5} \).

11. Find the critical points of \( f(x) = 4x^{3/5} - x^{8/5} \).

12. Find the absolute max and absolute min of \( f(x) = x^4 - 2x^2 + 3 \) on \([-2, 3]\).

13. Sketch the graph of \( \frac{2x - 5}{x + 3} \).

14. State Rolle’s theorem and verify that the function \( f(x) = \sin 2\pi x \) satisfies the hypotheses of Rolle’s theorem on the interval \([-1, 1]\). Then find a number \( c \) that satisfies its conclusion on this interval.

15. Is it true that the equation \( y = y''' + 5y' - 6 \) is satisfied by \( y = x \)?

16. Is it true that the inverse function of \( y = \sin x \) is \( y = \frac{1}{\sin x} \)?

17. Is it true that the function \( y = \ln x \) is differentiable everywhere?

18. Is it true that if \( k(x) = f(g(x)) \), then \( \frac{d^2k}{dx^2} = f'(g) \cdot g'' + f''(g) \cdot (g')^2 \)?

19. What is the error in the following steps:
\[
\lim_{x \to 0} \sin x \frac{x^2}{x^2} = \lim_{x \to 0} \cos x \frac{x^2}{2x} = \lim_{x \to 0} -\sin x \frac{x}{2} = 0
\]
and determine the correct value of this limit.

20. Use local linear approximation to approximate \( \sin 29^\circ \).