King Fahd University Of Petroleum and Minerals

College of Sciences

Mathematics Department

Math 101

Major Exam I

Section 19

Name:.......................................... ID#:..............

NO CALCULATOR IS ALLOWED IN THE EXAM

SHOW ALL NECESSARY WORK

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Instructor: H. Al-Attas
1. (8 points) Consider the following graph. Find the following limits. (Discuss the limit from left and right when needed and explain your answer if the limit does not exist):

(a) \( \lim_{x \to 3} f(x) \)

(b) \( \lim_{x \to 0} f(x) \)

(c) \( \lim_{x \to -2} f(x) \)

(d) \( \lim_{x \to +\infty} f(x) \)

(e) \( \lim_{x \to -\infty} f(x) \)

2. Find the following limits, if they exist and explain your answer if the limit does not exist.

(a) (3 points) \( \lim_{x \to 2} \frac{2x}{x-2} \)
(b) (5 points) \( \lim_{x \to 2} \frac{x^2}{1 - \sqrt{x^2 - x - 1}} \)

(c) (5 points) \( \lim_{x \to -\infty} (\sqrt{x^2 + 2x + x}) \)

(d) (5 points) \( \lim_{x \to 5} \frac{25 - x^2}{\sqrt{(x-5)^2}} \)
(e) (5 points) \( \lim_{x \to \pi} \frac{\sin x}{x^2 - \pi x} \)

(f) (5 points) \( \lim_{x \to 0} \frac{4x + 3x - 4 \cos x}{5 \sin x} \)

(g) (4 points) \( \lim_{x \to +\infty} \frac{(2x+1)^4}{4(2x^3+2)(1+2x)} + \sqrt{\frac{5x^2-1}{1-2x^2}} \)
3. (6 points) State the intermediate value theorem and apply it to the function \( f(x) = x^4 + 4x^3 - 1 \) to show that \( f(x) \) has 2-real zeros on the interval \([-5, 2]\).

4. (5 points) Use \( \epsilon - \delta \) definition to show that \( \lim_{x \to 1} 3x - 5 = -2 \).
5. (7 points) If \( f(x) = \begin{cases} 
\frac{x^3 - 8}{x - 2} & \text{if } x > 2 \\
Ax - B + 1 & \text{if } 0 < x < 2 \\
\tan \frac{2x}{x} & \text{if } x < 0 
\end{cases} \)
is continuous everywhere, then find the values of \( A \) and \( B \).

6. (5 points) By using the squeezing theorem, show that \( \lim_{x \to 0} x^4 \sin \left( \frac{1}{x} \right) + \frac{1}{2} = 2 \).
7. (10 points) Consider \( f(x) = \frac{2x-8}{|x| - 4} \)

(a) Find the values of \( x \) (if any) for which \( f \) is not continuous.

(b) Determine whether each value is a removable discontinuity

(c) Can we redefine \( f \) so that it has no points of discontinuity? Explain your answer.
8. (6 points) Given $\epsilon = 0.005$ and $f(x) = \frac{3x-1}{x+4}$ and let $L = \lim_{x \to +\infty} f(x)$.
Find a positive number $N$ such that $|f(x) - L| < \epsilon$ if $x > N$.

9. (5 points) Let $f$ and $g$ be two continuous functions and assume that $f(x)$
passes through the point $(2, -1)$. If $\lim_{x \to 2} [2f^2(x) - 5g(x) + 1] = 3$,
then find $g(3)$ and justify your answer.
10. (6 points) Consider \( f(x) = \sqrt{5 - x} \) and find \( \lim_{x \to a} f(x) \), if it exists, where \( a \) is replaced by

(a) 0

(b) \( 5^+ \)

(c) 5

(d) \(-5\)

(e) \(-\infty\)

(f) \(\infty\)

11. (Bonus) (5 points) Find \( \lim_{x \to 0} \frac{\sin^2 3x}{2 - 8 \cos x + 4 \cos^2 x} \).